PAPR Reduction Algorithm for MIMO-OFDM Based on Conjugate Symmetry Character

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Abstract: MIMO-OFDM is a key technology of the fourth generation mobile communication system. However, too high PAPR affects the performance of the MIMO-OFDM system. So we analyzed and proved that the signal through orthogonal space-time coding has the same PAPR with its conjugate signal and the sending symbols using STBC coding structure have the conjugate symmetry relationships in different symbol periods. We take the six antenna Alamouti structure as an example to demonstrate the PAPR character and its conjugate symmetry by using matrix theory. Based on these results, a low-complexity PAPR reduction algorithm for STBC MIMO-OFDM was put forward. Because it only need to be operated in the first, third, fifth, seventh symbol periods and the vector of optimal random phase sequence is obtained by Minimum Maximum criterion, the algorithm not only reduce the computation complexity significantly but also improve the PAPR reduction effect slightly. The simulation results verified that the proposed algorithm is effective and reliable.

Keywords: MIMO-OFDM; STBC; PAPR; random phase sequence; conjugate symmetry; symbol periods.

1. Introduction

In recent years, the multiple-input multiple-output orthogonal frequency division multiplexing(MIMO-OFDM) system has become a key technology for the next generation of high speed broadband wireless communication system due to its enhancement of system capacity and efficiency and also resistance of multi-path fading. However, MIMO-OFDM has the shortcoming of too high peak to the average power ration (PAPR) because of modulating the sending symbol sequence through $IFFT$ transform. This disadvantage affects the performance of MIMO-OFDM seriously. The existing MIMO-OFDM PAPR algorithms mostly apply the PAPR algorithm of OFDM to each antenna directly without taking into account of itself characters of the MIMO-OFDM[1,2,3]. The algorithm applying selective mapping (SLM) algorithm directly to every antenna of MIMO-OFDM is called independent SLM algorithm (ISLM)[1], which not only need large quantity of calculation but also increase the amount of sideband information transmission while reducing transmission efficiency. In fact , in space-time block coding (STBC) MIMO-OFDM system, the signal coded with orthogonal space-time coding has the same PAPR with its conjugate signal, and the sending symbols have the conjugate symmetry character in different symbol periods because of the conjugate symmetry character of orthogonal space-time coding structure. In this paper, a PAPR reduction algorithm with low-complexity is proposed for MIMO-OFDM based on the proof and analysis of the characters mentioned above. In the proposed algorithm, the compute complexity is reduced considerably, and PAPR reduction effect is improved slightly at the same time. Thus the system efficiency is improved well, and the performance is enhanced significantly.

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2. System Model and Description

The structure of MIMO-OFDM is shown in Fig.1. We consider a system with M antenna and N sub-carriers, a block of N sub-carriers which are orthogonal is transmitted on different antennas separately[4,5]. The frequency of sub-carriers is $f_n = n\Delta f$ where $\Delta f = NT$ and T is the symbol period. The complex baseband signal of the m th antenna can be expressed as:

$$x_m(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{m,n} e^{j2pf_n t} \quad (0 \le t \le NT) \qquad (1)$$

where $X_{m,n}$ is the symbol transmitted with the n th sub-carrier on the m th antenna. The PAPR of the transmitted signal in (1) can be defined as:

$$PAPR = \frac{\max\limits_{0 \le k \le NT} |x_m(k)|^2}{E\left[x_m(k)^2\right]} \qquad (2)$$

Where $E[\bullet]$ denotes mathematical expectation.

3. Low-Complexity Papr Reduction Algori- Thm for Mimo-Ofdm

A. Papr Character and Conjugate Symmetry of Stbc Mimo-Ofdm

In this paper, we take the six antenna Alamouti structure as an example to demonstrate and analyze the PAPR character and its conjugate symmetry between the sending symbols of STBC MIMO-OFDM[6,7]. The six antenna STBC scheme can be written as:

$$S=\begin{pmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \\ -s_2 & s_1 & s_4 & -s_3 & s_6 & -s_5 \\ -s_3 & -s_4 & s_1 & s_2 & s_7 & s_8 \\ -s_4 & s_3 & -s_2 & s_1 & s_8 & -s_7 \\ -s_5 & -s_6 & -s_7 & -s_8 & s_1 & s_2 \\ -s_6 & s_5 & -s_8 & s_7 & -s_2 & s_1 \\ -s_7 & s_8 & s_5 & -s_6 & -s_3 & s_4 \\ -s_8 & -s_7 & s_6 & s_5 & -s_4 & -s_3 \\ s_1^* & s_2^* & s_3^* & s_4^* & s_5^* & s_6^* \\ -s_2^* & s_1^* & s_4^* & -s_3^* & s_6^* & -s_5^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* & s_7^* & s_8^* \\ -s_4^* & s_3^* & -s_2^* & s_1^* & s_8^* & -s_7^* \\ -s_5^* & -s_6^* & -s_7^* & -s_8^* & s_1^* & s_2^* \\ -s_6^* & s_5^* & -s_8^* & s_7^* & -s_2^* & s_1^* \\ -s_7^* & s_8^* & s_5^* & -s_6^* & -s_3^* & s_4^* \\ -s_8^* & -s_7^* & s_6^* & s_5^* & -s_4^* & -s_3^* \end{pmatrix} \quad (3)$$

where S is a STBC matrix, and the column and row of matrix represent transmitting antenna m and sending slot k respectively. s_n is the sent symbol on n th antenna of the first slot and s_n^* is the conjugate of s_n. In the STBC MIMO-OFDM, $\pm s_i(n)$ and $\pm s_i^*(n)$ are the symbol sequences of sub-data flow. In fact, the symbol sequence $\pm s_i(n)$ coded with orthogonal space-time coding has the same PAPR with its conjugate symbol sequence $\pm s_i^*(n)$ after $IFFT$ transform. The proof is given thereinafter:

Let $s_i(n)$ and $S(k)$ denote the signal sequence and its DFT transform respectively. Then

$$S(k)=DFT\left[s_i(n)\right]=\sum_{n=0}^{L-1} s_i(n)e^{\frac{-j2\pi nk}{L}} \qquad (4)$$

So DFT transform of $s_i^*(n)$, the conjugate of $s_i(n)$, can be written as:

$$DFT\left[s_i^*(n)\right]=\sum_{n=0}^{L-1} s_i^*(n)e^{\frac{-j2\pi nk}{L}}R_L(k)=\left[\sum_{0}^{L-1} s_i(n)e^{\frac{j2\pi nk}{L}}\right]^* R_L(k)$$
$$=S^*\left(\left(-k\right)\right)_L R_L(k)$$
$$=\left[\sum_{n=0}^{L-1} s_i(n)e^{\frac{-j2\pi n(L-k)}{L}}\right]^* R_L(k)=S^*\left(\left(L-k\right)\right)_L R_L(k) \quad (5)$$

Where $S^*\left(\left(-k\right)\right)_L$ and $S^*\left(\left(L-k\right)\right)_L$ denote the period extended sequences with period L, which have no effects on the calculation of PAPR. From (2), we can know that in calculating PAPR the peak power and the average power are based on the modul-value square of the complex signal. So from (4) and (5) we can know $s_i(n)$ and $s_i^*(n)$ have the same peak power and average power after *DFT* transform. There are symmetries between *DFT* and *DFT*, and *DFT* is the fast algorithm of *IDFT*, so $s_i(n)$ also has the same peak power and average power with $s_i^*(n)$ after *IFFT* transform [8,9]. Based on the above analysis, we can know $\pm s_i(n)$ has the same PAPR character with $\pm s_i^*(n)$.

The STBC structure also has conjugate symmetry taking six antenna structure as example, that is

$$S^T S=2\left(\left|s_1\right|^2+\left|s_2\right|^2+\left|s_3\right|^2+\left|s_4\right|^2+\left|s_5\right|^2+\left|s_6\right|^2+\left|s_7\right|^2+\left|s_8\right|^2\right)I_6 \quad (6)$$

where I_6 is a 6×6 identity matrix. So the sending symbols in STBC MIMO-OFDM system also have the conjugate symmetry relationship. Namely, the sending symbols in other slots are the combination of the symbols sent in the first, third, fifth, seventh slot and their conjugate respectively. Thus, in STBC MIMO-OFDM system we only need apply PAPR algorithm in the first, third, fifth, seventh sending periods so as to reduce the algorithm complexity greatly.

B. Low-Complexity Papr Reduction Algorithm For Stbc Mimo-Ofdm System

Based on the above demonstration and analysis, a low-complexity PAPR reduction algorithm for STBC MIMO-OFDM is proposed in the following passage taking six antenna structure as instance. In the first slot, according to the Minimum Maximum criterion[10], the optimal character PAPR (denote $PAPR^1(s)$) of the first symbol period is obtained by iterative computing of the vector from M -dimension set of random phase vector sequences $P^{(\mu)}=\left(p_0^{(\mu)}, p_1^{(\mu)}, \ldots p_{N-1}^{(\mu)}\right)$ $(\mu=1,2,\ldots M)$ in the parallel SLM algorithm, the vector $p(n)$ of optimal random phase sequence is generated too at the same time. Furthermore, based on the above results that $\pm s_i(n)$ has the same PAPR with $\pm s_i^*(n)$ and the sending symbols have the conjugate symmetry relationships in STBC MIMO-OFDM system, the vector $p_k(n)$ $(k=2,9,10)$ of optimal random phase sequences can be obtained through phase transformation of $p(n)$ in the second, ninth, tenth symbol period. Similarly the vector of optimal random phase sequence in other symbol period can also be obtained. Thus the sending symbols have the best PAPR property eliminating the effect of too high PAPR. In this algorithm, in order to keep the conjugate symmetry relationships between $\pm s_i(n)$ and $\pm s_i^*(n)$, the weight coefficient which the sending symbol sequence multiplies the vector $P^{(n)}$ and $P_k^{(n)}$ of optimal random phase sequence must satisfy some corresponding correlation shown in Figure 2.

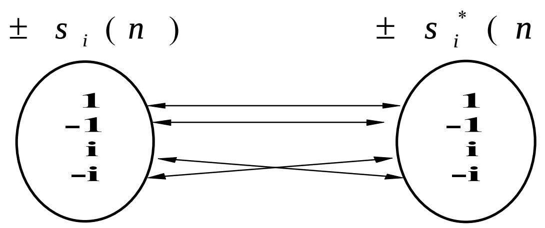

Figure2. The coefficient correlation of $P^{(n)}$ and $P_k^{(n)}$

The algorithm proposed above can be carried out as following steps.

Step1: In the first symbol period, the MIMO-OFDM sends the symbol sequence $s_i(n)$ through STBC coding $(i=1,2,3,4,5,6)$ $(n=0,\ldots,N-1)$, where i is the antenna ordinal and N is the length of symbol sequence. Let $P=(p_0,p_1,\ldots p_{N-1})$, we can obtain

$$s_{i,m}(n)=\{s_i\bullet P\}=\left\{\sum_{m,n=0}^{N-1}s_i(n)\,p_m\right\},\quad (i=1,2,3,4,5,6).$$

Where $p_m\in\{\pm j,\pm 1\}$. After operating N-point $IFFT$ transform on $s_{i,m}(n)$, we have

$$s_{i,m}(k)=IFFT\{s_{i,m}(n)\}=IFFT\left\{\left\{\sum_{m,n=0}^{N-1}s_i(n)\,p_m\right\}\right\}.$$

Then calculating $PAPR_i=\dfrac{\max\limits_{0\le k\le(N-1)T}|s_{i,m}(k)|^2}{E\left[s_{i,m}(k)^2\right]}$

$(i=1,2,3,4,5,6)$ by (2).

Step2: Let $PAPR^1(s)=\min\{PAPR_i\}$.

Step3: Selecting different p_m^m $(m=0,\ldots,N-1;\mu=1,\ldots,M)$ from the M-dimension vector set of random phase sequences $P^{(\mu)}=\left(p_0^{(\mu)},p_1^{(\mu)},\ldots p_{N-1}^{(\mu)}\right)$ $(\mu=1,2,\ldots M)$, we can obtain the M-dimension set of $\{PAPR^i(s)\}$ $(i=1,2,\ldots M)$ by repeating Step1 and Step2.

Step4: Let

$$PAPR(s)=\min\{PAPR^1(s),PAPR^2(s),\ldots,PAPR^M(s)\},$$

then the vector of optimal random phase sequence in the first symbol period can be calculated as follows:

$$p(n)=\{p(0),p(1),\ldots p(N-1)\}$$
$$=\underset{\{p(0),p(1),\ldots p(N-1)\}}{\arg\min}\left\{\max\left[PAPR^1(s),PAPR^2(s),\ldots,PAPR^M(s)\right]\right\}$$

Step5: Converting the vector of optimal random phase sequence $p(n)\,(n=0,\ldots,N-1)$, which was obtained in step4 according to figure2, to the sequences

$$p_k(n)=\{p_k(0),p_k(1),\ldots p_k(N-1)\}\ (k=2,9,10).$$

Then the vectors of optimal random phase sequence in the second, ninth, tenth symbol periods can be obtained.

Step6: Repeating Step1 to Step4 in the third, fifth, seventh symbol periods. The vector of optimal random phase sequence in these symbol periods can be obtained.

Step7: Repeating step5, the optimal sequence of random phase vector in other symbol period can be obtained through symbol converting.

Step8: Repeating Step1 to Step7 until all the symbol sequences on each antenna are sent completely.

The above PAPR reduction algorithm only need to be operated in the first, third, fifth, seventh symbol periods, so its computation complexity are reduced significantly. Moreover, we will observe that the proposed algorithm can improve the effects of PAPR reduction slightly from the following simulation.

4. Simulation Results

In this section, we will evaluate the performance of the proposed algorithm by simulating on the six antenna STBC MIMO-OFDM system. We use the $N=128$ sub-carriers and sending signals constellated with QAM. The random vectors are chosen from the set of $\{\pm j,\pm 1\}$. Figure3 shows the comparison of the CCDF of PAPR between s_i and s_i^*. From this figure, we can see that there is the same PAPR property between symbols s_i and s_i^* in STBC MIMO-OFDM system. This character can lead to reduce the complexity of algorithm well.

Figure4 shows the comparison of the computational complexity between the proposed algorithm and the independent SLM algorithm. We can see that the proposed algorithm achieves significant computational complexity reduction compared with the independent SLM algorithm with the same number of sub-carriers.

Figue5 shows the comparison of the performances between the proposed algorithm and the original signal

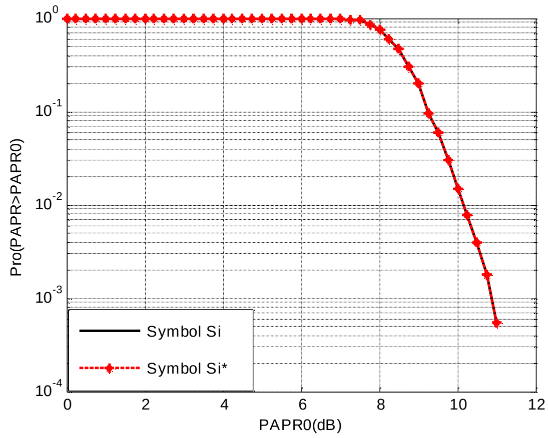

Figure3. PAPR performance comparison between S_i and S_i^*

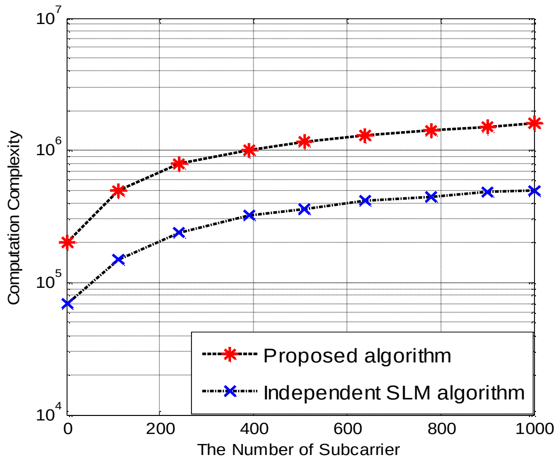

Figure 4. Comparison of computation complexity betweenproposed algorithm and independent SLM algorithm

as well as the independent SLM algorithm. We can see that the proposed algorithm improved the PAPR property significantly compared with the original signal and also have slight improvement in the effect of PAPR reduction compared with the independent SLM. In addition, the algorithm proposed in the paper reduced the transmission of side information to improve the efficiency of using bandwidth.

5. Conclusions

This paper analyzed and proved that the sending signals through STBC coding have the same PAPR with their conjugate signals and the sending symbols using STBC coding structure have the conjugate symmetry relation ships in different symbol periods. Based on these properties proof and analysis, a low-complexity algori -thm for STBC MIMO-OFDM system was put forward. The algorithm not only reduce the algorithm complexity

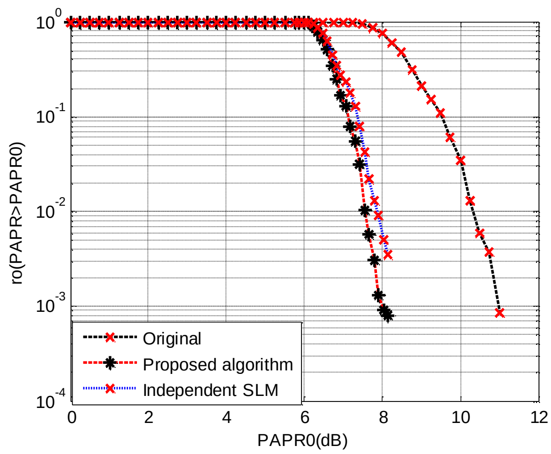

Figure 5. Comparison of PAPR performance in proposedalgorithm, original signal, and independent SLM algorithm

gnificantly but also improve the PAPR reduction effect slightly. Thus the MIMO-OFDM system efficiency was improved and its performance was enhanced. The simulation results verified that the proposed algorithm is effective and reliable.

References

[1] Y.L. Lee, Y.H. You, and W.G. Jeon. Peak-to-average power ratio in MIMO-OFDM systems using selective mapping[J]. *IEEE Communications Letters*, 2003, 7(12):575-577.

[2] D.W. Lim, S.J. Heo, and J.S. No. A new PTS OFDM scheme with low-complexity for PAPR reduction[J]. *IEEE Transactions on Broadcasting*, 2006, 52(1):77-82.

[3] S. H. Han, J. H. Lee. An overview of peak-to-average power ratio reduction techniques for multicarrier transmission[J]. *IEEE Wireless Communications*, 2005, 12(2): 56-65.

[4] Jankiraman M. Space-time codes and MIMO systems[M]. Newyork: Artech House, Aug, 2004.

[5] Ben Jmâa Ahmed Bassem. A PAPR reduction method for STBC MIMO-OFDM systems using SLM in combination with sub-band Permutation[C]. Guadeloupe: The Third International Conference on Wireless and Mobile Communications, 2007:84-88.

[6] Alamouti. S. M. A simple transmit diversity technique for wireless communications[J]. *IEEE Journal on Selected Areas Communic -ation*, 1998, 16(8): 1451–1458.

[7] V. Tarokh, H. Jafarkhani, and A. R. Calderbank. Space-time block codes from orthogonal designs[J]. *IEEE Transactions on Informa -tion Theory*, 1999, 45(5): 1456-1467.

[8] A. V. Oppenheim, R. W. Schafer, Discrete-time signal processing [M]. Prentice Hall: Englewood Cliffs, 1989.

[9] M.T. Zhou, Latinovi´c, and Y. Bar-Ness. STBC MIMO-OFDM Peak-to-Average power ratio reduction by cross-antenna rotation and inversion[J]. *IEEE Communication Letters*, 2005, 9(7): 592-594.

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