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The Fekete Szegö Functional and Second Hankel Determinant for a Certain Sublass of Analytic Functions

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Abstract

Let S denote the class of functions that are analytic, normalized and univalent in the open unit disk $E=\{z:|z|<1\}$. Subclasses of S are the class of starlike and convex functions denoted by S^* and C respectively. A new subclass of analytic functions that generalize some known subclasses of analytic functions was defined and investigated. We obtained coefficient bounds, upper estimates for the Fekete-Szegö functional and the Hankel determinant.

Keywords

Analytic Functions, Coefficient Bounds, Fekete-Szegö Functional, Salagean Differential Operator and Hankel Determinant

1. Introduction

Let A denote the class of functions

$$f(z)=z+a_2z^2+a_3z^3+a_4z^4+\cdots \tag{1.1}$$

which are analytic in the open unit disk $U=\{z:|z|<1\}$ and satisfy the condition $f(0)=0$ and $f'(0)=1$.

Let S denote the subclass of A consisting of univalent in U. A function $f(z)\in S$ is said to be starlike in the unit disk if and only if

$$Re\frac{zf'(z)}{f(z)}>0,\quad z\in U \tag{1.2}$$

Also, a function $f(z)\in S$ is said to be convex in the unit disk if and only if

$$Re\left(1+\frac{zf''(z)}{f'(z)}\right)>0,\quad z\in U \tag{1.3}$$

Let $D^n : A \to A$ be defined by

$$D^0 f(z) = f(z)$$

$$D^1 f(z) = zf'(z)$$

$$D^n f(z) = z\left[D^{n-1} f(z)\right]'$$

which is equivalent to

$$D^n f(z) = z + \sum_{k=2}^{\infty} k^n a_k z^k,\ \left(n = \{0,1,2,\cdots\}\right),\ z\in U$$

D^n is the Salagean differential operator [1].

Fekete and Szegö [2] studied the estimate of a functional $\left|a_3 - \sigma a_2^2\right|$ known as Fekete-Szegö functional, where σ is real. Also, Noonan and Thomas [3] defined the q^{th} Hankel determinant of $f(z)$ for $q \ge 1, n \ge 0$ by

$$H_q(n) = \begin{vmatrix} a_n & a_{n+1} & \cdots & a_{n+q-1} \\ a_{n+1} & a_{n+2} & \cdots & a_{n+q} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n+q-1} & a_{n+q} & \cdots & a_{n+2q-2} \end{vmatrix} \qquad \left(a_1 = 1 \text{ for } f(z)\in S\right)$$

This determinant has been considered for specific values q and n by many authors. It is well established that the Fekete-Szegö functional given by $\left|a_3 - a_2^2\right| = H_2(1)$. Pommerenke [4] investigated the Hankel determinant of areally mean p-valent functions, univalent functions as well as starlike functions. Noor [5] investigated the Hankel determinant problem for the class of functions with bounded boundary rotation. Janteng *et al.* [6] studied the sharp upper bound for second Hankel determinant $H_2(2) = \left|a_2 a_4 - a_3^2\right|$ for univalent functions whose derivative has positive real parts. Also, Lee *et al.* [7] obtained bounds on second Hankel determinants belonging to the subclasses of Ma-Minda starlike and convex functions. Bansal [8] has obtained bounds on $H_2(2)$ for a new class of analytic functions.

In this paper, we obtained the coefficient bound, Fekete-Szegö functional and second Hankel determinant for the functions belonging to the subclass $C_n(\beta,\gamma)$.

Definition 1.1. A function $f(z)$ of the form (1.1) analytic and univalent in U is said to be in the $C_n(\beta,\gamma), \beta\in[0,1], \gamma\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ and $n\in\mathbb{N}_0$ if it satisfies the inequality

$$Re\left\{e^{i\gamma}\left(1-e^{-2i\gamma}\beta^2 z^2\right)\frac{D^{n+1} f(z)}{z}\right\}>0,\ z\in U. \tag{1.4}$$

Remark 1

(1) For $n=0, \beta=0$ the class $C_0(0,\gamma)$ gives

$$Re\left\{e^{i\gamma} f'(z)\right\}>0,\quad z\in U \tag{1.5}$$

studied in [9].

(2) For $n=0, \gamma=0$ gives

$$Re\left\{\left(1-\beta^2 z^2\right) f'(z)\right\} > 0,\ z \in U. \tag{1.6}$$

investigated by [10].

For $n=0$, the class gives

$$Re\left\{e^{i\gamma}\left(1-e^{-2i\gamma}\beta^2 z^2\right) f'(z)\right\} > 0,\ z \in U. \tag{1.7}$$

studied in [11].

2. Preliminary Lemmas

We need the following lemmas to prove our results.

Let P denote the class of Caratheodory functions.

$$p(z) = 1 + c_1 z + c_2 z^2 + c_3 z^3 + \cdots \quad (z \in U)$$

which are analytic and satisfy $p(0)=1$ and $\Re p(z) > 0$

Lemma 2.1. *Let* $p \in P$. *Then*

$$|c_k| \le 2 \quad (k \in \mathbb{N}) \quad [12] \tag{2.1}$$

Lemma 2.2. *Let* $p \in P$, *then for any real* λ

$$\left|c_2 - \lambda \frac{c_1^2}{2}\right| \le \begin{cases} 2(1-\lambda) & \text{if } \lambda \le 0 \\ 2 & \text{if } 0 \le \lambda \le 2 \quad [13] \\ 2(\lambda - 1) & \text{if } \lambda \ge 2 \end{cases} \tag{2.2}$$

Lemma 2.3. *Let* $p \in P$ *then*

$$2c_2 = c_1^2 + x\left(4 - c_1^2\right) \tag{2.3}$$

$$4c_3 = c_1^3 + 2c_1\left(4-c_1^2\right)x - c_1\left(4-c_1^2\right)x^2 + 2\left(4-c_1^2\right)\left(1-|x|^2\right)z \tag{2.4}$$

for some value of x, z, such that $|x| \le 1$ and $|z| \le 1$ [14].

3. Main Results

Theorem 3.1. *Let* $f(z) \in C_n(\beta,\gamma), \beta \in [0,1], \gamma \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ *and* $n \in \mathbb{N}_0$.

Then

$$|a_2| \le \frac{\cos\gamma}{2^n}$$

$$|a_3| \le \frac{2\cos\gamma + \beta^2}{3^{n+1}}$$

Proof:

Let $f(z) \in C_n(\beta,\gamma)$, then by [1.4]

$$Re\, e^{i\gamma}\left[\left(1-e^{-2i\gamma}\beta^2 z^2\right)\frac{D^{n+1} f(z)}{z}\right] > 0,\ \gamma \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right), 0 \le \beta \le 1, n \in \mathbb{N}_0, z \in U$$

Now,

$$e^{i\gamma}\left[\left(1-e^{-2i\gamma}\beta^2 z^2\right)\frac{D^{n+1}f(z)}{z}\right]$$
$$= e^{i\gamma}+q_1 z+q_2 z^2+\cdots=(\cos\gamma+i\sin\gamma)+\sum_{n=1}^{\infty}q_n z^n \tag{3.1}$$

Then

$$\exists\, q(z)=\cos\gamma+i\sin\gamma+\sum_{n=1}^{\infty}q_n z^n,\quad z\in U, n\in\mathbb{N}$$

$$e^{i\gamma}\left[\left(1-e^{-2i\gamma}\beta^2 z^2\right)\frac{D^{n+1}f(z)}{z}\right]=p(z)\cos\gamma+i\sin\gamma \tag{3.2}$$

that is

$$\cos\gamma+c_1\cos\gamma z+c_2\cos\gamma z^2+c_3\cos\gamma z^3+\cdots=\cos\gamma+q_1 z+q_2 z^2+q_3 z^3+\cdots \tag{3.3}$$

Comparing coefficients of (3.1) and (3.3) gives

$$a_2=\frac{c_1 e^{-i\gamma}\cos\gamma}{2^{n+1}} \tag{3.4}$$

$$a_3=\frac{c_2 e^{-i\gamma}\cos\gamma+\beta^2 e^{-2i\gamma}}{3^{n+1}} \tag{3.5}$$

$$a_4=\frac{c_3 e^{-i\gamma}\cos\gamma+c_1\beta^2 e^{-3i\gamma}\cos\gamma}{4^{n+1}} \tag{3.6}$$

Solving for the bounds of (3.4), (3.5), (3.6) and using lemma 2.1 give

$$|a_2|\le\frac{\cos\gamma}{2^n} \tag{3.7}$$

$$|a_3|\le\frac{2\cos\gamma+\beta^2}{3^{n+1}} \tag{3.8}$$

$$|a_4|\le\frac{2\cos\gamma+2\beta^2\cos\gamma}{4^{n+1}} \tag{3.9}$$

Remark 2

For $n=0$

$$|a_2|\le\cos\gamma$$

$$|a_3|\le\frac{2\cos\gamma+\beta^2}{3}$$

Theorem 3.2. *Let* $f(z)\in C_n(\beta,\gamma)$, *then for any real number* μ

$$\left|a_3-\mu a_2^2\right|\le\begin{cases}\dfrac{\beta^2+2\cos\gamma}{3^{n+1}}-\dfrac{\mu e^{-i\gamma}\cos^2\gamma}{2^{2n}} & \text{if } \mu\le 0\\[2ex] \dfrac{\beta^2+2\cos\gamma}{3^{n+1}} & \text{if } 0\le\mu\le\dfrac{2^{2n+2}}{3^{n+1}e^{-i\gamma}\cos\gamma}\\[2ex] \dfrac{\beta^2-2\cos\gamma}{3^{n+1}}+\dfrac{\mu e^{-i\gamma}\cos^2\gamma}{2^{2n}} & \text{if } \mu\ge\dfrac{2^{2n+2}}{3^{n+1}e^{-i\gamma}\cos\gamma}\end{cases}$$

Proof:

Using (3.4) and (3.5) give

$$\begin{aligned}\left|a_3-\mu a_2^2\right| &= \left|\frac{c_2 e^{-i\gamma}\cos\gamma}{3^{n+1}}+\frac{\beta^2 e^{-2i\gamma}}{3^{n+1}}-\frac{\mu c_1^2 e^{-2i\gamma}\cos^2\gamma}{2^{2n+2}}\right| \\ &\le \frac{\beta^2}{3^{n+1}}+\frac{\cos\gamma}{3^{n+1}}\left|c_2-\frac{3^{n+1}\mu e^{-i\gamma}\cos\gamma}{2^{2n+1}}\frac{c_1^2}{2}\right|\end{aligned} \tag{3.10}$$

then using lemma (2.2) in (3.10) gives

$$\left|a_3-\mu a_2^2\right| \le \frac{\beta^2+2\cos\gamma}{3^{n+1}}-\frac{\mu e^{-i\gamma}\cos^2\gamma}{2^{2n}} \tag{3.11}$$

Let

$$0 \le \frac{3^{n+1}\mu e^{-i\gamma}\cos\gamma}{2^{2n+1}} \le 2$$

then by lemma 2.2 we obtain

$$\left|a_3-\mu a_2^2\right| \le \frac{\beta^2+2\cos\gamma}{3^{n+1}} \tag{3.12}$$

suppose

$$\frac{3^{n+1}\mu e^{-i\gamma}}{2^{2n+1}} \ge 2$$

then using lemma 2.2 gives

$$\left|a_3-\mu a_2^2\right| \le \frac{\beta^2-2\cos\gamma}{3^{n+1}}+\frac{\mu e^{-i\gamma}\cos^2\gamma}{2^{2n}} \tag{3.13}$$

Theorem 3.3 *Let* $f(z)\in C_n(\beta,\gamma), \beta\in[0,1], \gamma\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ *and* $n\in\mathbb{N}_0$

then

$$H_2(2)=\left|a_2a_4-a_3^2\right| \le \frac{\beta^4+4\beta^2\cos\gamma+4\cos^2\gamma}{3^{2n+2}}+\frac{\left(\beta^4+6\beta^2+9\right)\cos\gamma}{2^{3n+4}}$$

Proof:

Using (3.4), (3.5) and (3.6) give

$$\begin{aligned}&\left|a_2a_4-a_3^2\right| \\ &= \left|\frac{c_1 e^{-i\gamma}\cos\gamma}{2^{n+1}}\left(\frac{c_3 e^{-i\gamma}\cos\gamma+\beta^2 c_1 e^{-3i\gamma}\cos\gamma}{4^{n+1}}\right)-\left(\frac{c_2 e^{-i\gamma}\cos\gamma+\beta^2 e^{-2i\gamma}}{3^{n+1}}\right)^2\right|\end{aligned} \tag{3.14}$$

$$\begin{aligned}&\left|a_2a_4-a_3^2\right| \\ &= \left|\frac{c_1^4 e^{-2i\gamma}\cos^2\gamma}{2^{3n+5}}+\frac{c_1^2\left(4-c_1^2\right)e^{-2i\gamma}x\cos^2\gamma}{2^{3n+4}}-\frac{c_1^2\left(4-c_1^2\right)e^{-2i\gamma}x^2\cos^2\gamma}{2^{3n+5}}\right. \\ &\quad +\frac{c_1\left(4-c_1^2\right)\left(1-|x|^2\right)e^{-2i\gamma}\cos^2\gamma z}{2^{3n+4}}+\frac{c_1^2\beta^2 e^{-4i\gamma}\cos^2\gamma}{2^{3n+3}}-\frac{c_1^4 e^{-2i\gamma}\cos^2\gamma}{2^2\cdot 3^{2(n+1)}} \\ &\quad -\frac{x\left(4-c_1^2\right)c_1^2 e^{-2i\gamma}\cos^2\gamma}{2\cdot 3^{2n+2}}-\frac{c_1^2\alpha^2 e^{-3i\gamma}\cos\gamma}{3^{2n+2}}-\frac{x^2\left(4-c_1^2\right)^2 e^{-2i\gamma}\cos^2\gamma}{2^2\cdot 3^{2n+2}} \\ &\quad \left. -\frac{\beta^2 x\left(4-c_1^2\right)e^{-3i\gamma}\cos\gamma}{3^{2n+2}}-\frac{\beta^4 e^{-4i\gamma}}{3^{2n+2}}\right|\end{aligned} \tag{3.15}$$

Suppose $c_1 = c$, and recall that $|c_1| \le 2$, and assuming without restriction that $c \in [0,2]$. Then, using triangle inequality (3.15) becomes

$$\begin{aligned}\left|a_2a_4-a_3^2\right| \le{}& \frac{c^4\cos^2\gamma}{2^{3n+5}}+\frac{c^2\left(4-c^2\right)|x|\cos^2\gamma}{2^{3n+4}}+\frac{c^2\left(4-c^2\right)|x|^2\cos^2\gamma}{2^{3n+5}}\\&+\frac{c\left(4-c^2\right)\left(1-|x|^2\right)\cos^2\gamma}{2^{3n+4}}+\frac{c^2\beta^2\cos^2\gamma}{2^{3n+3}}+\frac{c^4\cos^2\gamma}{2^2\cdot 3^{2n+2}}\\&+\frac{|x|\left(4-c^2\right)c^2\cos^2\gamma}{2\cdot 3^{2n+2}}+\frac{|x|^2\left(4-c^2\right)^2\cos^2\gamma}{2^2\cdot 3^{2n+2}}\\&+\frac{c^2\beta^2\cos\gamma}{3^{2n+2}}+\frac{\beta^2|x|\left(4-c^2\right)\cos\gamma}{3^{2n+2}}+\frac{\beta^4}{3^{2n+2}}\end{aligned} \tag{3.16}$$

Now, putting $\psi = |x| \le 1$ then

$$\begin{aligned}&\left|a_2a_4-a_3^2\right|\\&\le\left\{\frac{c^4\cos^2\gamma}{2^{3n+5}}+\frac{c\left(4-c^2\right)\cos^2\gamma}{2^{3n+4}}+\frac{c^2\beta^2\cos^2\gamma}{2^{3n+3}}+\frac{c^4\cos^2\gamma}{2^2\cdot 3^{2n+2}}+\frac{c^2\beta^2\cos\gamma}{3^{2n+2}}+\frac{\beta^4}{3^{2n+2}}\right\}\\&+\left\{\frac{c^2\left(4-c^2\right)\cos^2\gamma}{2^{3n+4}}+\frac{\left(4-c^2\right)c^2\cos^2\gamma}{2\cdot 3^{2n+2}}+\frac{\beta^2\left(4-c^2\right)\cos\gamma}{3^{2n+2}}\right\}\psi\\&+\left\{\frac{c^2\left(4-c^2\right)\cos^2\gamma}{2^{3n+5}}-\frac{c\left(4-c^2\right)\cos^2\gamma}{2^{3n+4}}+\frac{\left(4-c^2\right)^2\cos^2\gamma}{2^2\cdot 3^{2n+2}}\right\}\psi^2=F(c,\psi)\end{aligned} \tag{3.17}$$

Differentiating $F(c,\psi)$ partially with respect to ψ in the closed interval $0 \le \psi \le 1$

$$\begin{aligned}\frac{\partial F(c,\psi)}{\partial\psi}={}&\left\{\frac{c^2\left(4-c^2\right)\cos^2\gamma}{2^{3n+4}}+\frac{\left(4-c^2\right)c^2\cos^2\gamma}{2\cdot 3^{2n+2}}+\frac{\beta^2\left(4-c^2\right)\cos\gamma}{3^{2n+2}}\right\}\\&+\left\{\frac{c^2\left(4-c^2\right)\cos^2\gamma}{2^{3n+5}}-\frac{c\left(4-c^2\right)\cos^2\gamma}{2^{3n+4}}+\frac{\left(4-c^2\right)^2\cos^2\gamma}{2^2\cdot 3^{2n+2}}\right\}\psi\\&>0\end{aligned} \tag{3.18}$$

for $0 \le \psi \le 1$, therefore $F(c,\psi)$ is an increasing function. Hence, it attains maximum point at $\psi = 1$. Thus,

$$\begin{aligned}\max_{0\le\psi\le1}F(c,\psi)=F(c,1)\le{}&\frac{c^4\cos^2\gamma}{2^{3n+5}}+\frac{c\left(4-c^2\right)\cos^2\gamma}{2^{3n+4}}+\frac{c^2\beta^2\cos^2\gamma}{2^{3n+3}}\\&+\frac{c^4\cos^2\gamma}{2^2\cdot 3^{2n+2}}+\frac{c^2\beta^2\cos\gamma}{3^{2n+2}}+\frac{\beta^4}{3^{2n+2}}+\frac{c^2\left(4-c^2\right)\cos^2\gamma}{2^{3n+4}}\\&+\frac{\left(4-c^2\right)c^2\cos^2\gamma}{2\cdot 3^{2n+2}}+\frac{\beta^2\left(4-c^2\right)\cos\sigma}{3^{2n+2}}+\frac{c^2\left(4-c^2\right)\cos^2\gamma}{2^{3n+5}}\\&-\frac{c\left(4-c^2\right)\cos^2\gamma}{2^{3n+4}}+\frac{\left(4-c^2\right)^2\cos^2\gamma}{2^2\cdot 3^{2n+2}}=G(c)\end{aligned} \tag{3.19}$$

$$G'(c)=\frac{(3+\beta^2)\cos^2\gamma}{2^{3n+2}}c-\frac{\cos^2\gamma}{2^{3n+2}}c^3$$

Now, the critical points occur at

$$c_0=0,\ c_1=\sqrt{\alpha^2+3}\ \text{and}\ c_2=-\sqrt{\alpha^2+3}$$

but the maximum point occurring at $\sqrt{\alpha^2+3}$ [3.19] becomes

$$G(c)=\frac{\beta^4+4\beta^2\cos\gamma+4\cos^2\gamma}{3^{2n+2}}+\frac{\beta^4\cos^2\gamma}{2^{3n+4}}+\frac{6\beta^2\cos^2\gamma+9\cos^2\gamma}{2^{3n+4}} \tag{3.20}$$

Therefore,

$$\left|a_2a_4-a_3^2\right|\le\frac{\beta^4+4\beta^2\cos\gamma+4\cos^2\gamma}{3^{2n+2}}+\frac{(\beta^4+6\beta^2+9)\cos^2\gamma}{2^{3n+4}} \tag{3.21}$$

4. Conclusion

A subclass of analytic functions which generalize some well known subclasses of analytic and univalent functions was defined. The initial coefficients upper bounds, upper estimates for the Fekete-Szegö functional and the second Hankel determinants for the class were obtained. The study unifies existing results and obtains new results in geometric function theory. Future researches can be done to obtain the geometric properties by using Chebyshev polynomials.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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