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Two Contemporary and Efficient Two-Stage Sampling Methods for Estimating the Volume of Forest Stands: A Brief Overview and Unified Mathematical Description

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Abstract

Big Basal Area Factor (Big BAF) and Point-3P are two-stage sampling methods. In the first stage the sampling units, in both methods, are Bitterlich points where the selection of the trees is proportional to their basal area. In the second stage, sampling units are trees which are a subset of the first stage trees. In the Big BAF method, the probability of selecting trees in the second stage is made proportional to the two BAFs' ratio, with a basal area factor larger than that of the first stage. In the Point-3P method the probability of selecting trees, in the second stage, is based on the height prediction and use of a specific random number table. Estimates of the forest stands' volume and their sampling errors are based on the theory of the product of two random variables. The increasing error in the second stage is small, but the total cost of measuring the trees is much smaller than simply using the first stage, with all the trees measured. In general, the two sampling methods are modern and cost-effective approaches that can be applied in forest stand inventories for forest management purposes and are receiving the growing interest of researchers in the current decade.

Keywords

1. Introduction

Forest Inventory (FI) plays a vital role in all the aspects of forest management (Rice, Weiskittel, & Wagner, 2014), providing the necessary high-quality bio-

physical data derived from forest resources. In the management of forest ecosystems, the variable with the greatest interest, usually, is the timber volume.

Sampling is a fundamental pillar of FI (Gregoire & Valentine, 2007b; Iles, 2003; Kershaw Jr., Ducey, Beers, & Husch, 2016a). Operational FI depends directly on efficient sampling design (Yang, Hsu, Kershaw, McGarrigle, & Dan, 2017), which is a balance between standard error objectives (precision) and cost constraints (Yang et al., 2017). Besides the sampling method, precision and cost of sampling estimates, depend from the type of forest, the spatial distribution of the trees and their growth mode (Osman & Idris, 2012).

The sampling designs use either sampling with equal probability (like fixed area plot sampling) or sampling with variable (unequal) probability (like Bitterlich sampling) (Kershaw Jr., Ducey, Beers, & Husch, 2016b; Rice et al., 2014). Fixed-area sample plots are sampling units of small areas of circular (fixed-radius), square or rectangular shape (Kershaw Jr. et al., 2016a). "Fixed-area plot sampling has been widely used in forestry since the 19th Century" (Osman & Idris, 2012), particularly with continuous FIs (Ware & Cunia, 1962) as Scott (1990) refer. Variable probability sampling which is broadly applied in FIs can further be categorized into: 1) sampling with probability proportional to size (PPS sampling), 2) list sampling and 3) sampling with probability proportional to prediction (3P sampling) (Kershaw Jr. et al., 2016b).

Bitterlich sampling, as used extensively in the forest literature, is an application of PPS sampling (Gregoire & Valentine, 2007a; Kershaw Jr. et al., 2016b; Marshall, Iles, & Bell, 2004; Rice et al., 2014). Bitterlich sampling utilizes a prism or Spiegel-Relaskop as an angle gauge instrument to project a horizontal angle at tree diameter at breast height and therefore selects potential samples proportional to the basal area (Yang et al., 2017). Using Bitterlich sampling, some bias is expected to occur in the final estimates due to edge effects, poor visibility, treatment of borderline trees and counting errors (Iles, 1989; Osman & Idris, 2012). But, as Kershaw Jr. et al. (2016a) mentions, "while these measured values will differ slightly (or in some cases greatly), these differences are no more unexpected than differences obtained using fixed-area plots of different sizes". Studies (Rice et al., 2014) indicate the fixed area sampling is still more time-consuming than variable radius methods, even with the usage of time-saving technology (like ultrasonic hypsometer). "In general, we did not find significant accuracy differences between the inventory systems for most of the stand variables and forest types studied, as expected by established angle-count sampling theory" (Piqué, Obon, Condés, & Saura, 2011).

Moving forward to find more effective sampling designs than fixed area or variable (Bitterlich) area plots, two-stages, and two-phases (double) sampling schemes has been proposed. Two-stages, double sampling designs, based on cheaper to acquire auxiliary variables (covariates) such as tree stem basal area (less height), for estimating target variables like stock volume (Burk, 2004; West, 2011). The first stage (or phase) constitutes with measurements of auxiliary variable, taken by a larger sample plots (points of fixed areas), while in the second stage (or

phase) a subsample (a subset of the initial sample) with the variable of interest is measured (Burk, 2004; Dahl, Harding, & Wiant, 2008; West, 2011). The auxiliary variable is closely related (high correlation) to the variable of interest and relatively quickly and easily can be measured (Burk, 2004; West, 2011).

Big Basal Area Factor (Big BAF) and Point-3P sampling methods are relatively new in the history of sampling designs and were developed during the last few decades. Two-stage sampling, with explicit use of Bitterlich sampling at the first stage, is the common characteristic of these methods (Lindemuth, 2007). The sample points can be spread randomly or systematically (Marshall et al., 2004; Rennie, Wood, Schreuder, & Lund, 1991) in the forest area. The measurements of the first stage include the number of trees (of various species) and give the basal area from the Bitterlich sample using a BAF angle gauge (Lindemuth, 2007; Marshall et al., 2004; Rennie et al., 1991; Yang & Burkhart, 2018). Many applications of both methods, Big BAF and Point-3P, involved operational FIs (Marshall et al., 2004; Opalach, 2017; Rice et al., 2014; West, 2011; Yang & Burkhart, 2018).

The object of this paper is to present the two sampling methods of Big BAF and Point-3P in a systematic way. These methods constitute a form of two-stage Bitterlich sampling for the estimation of timber volume. In the next two sections, Big BAF and Point-3P sampling methods are defined and referenced to the literature. A brief mathematical description of the two methods is given in the fourth section, and the last section links the two sampling methods to forest inventories and forest management.

2. The Big BAF Sampling Method

In Big BAF sampling two angle gauges are used: 1) in the first stage a small BAF angle gauge counts a sufficient number of "in" trees relatively quickly for estimating basal area and 2) in the second stage a larger BAF angle gauge subsamples measure trees (a subset of the first stage trees counted with the small-angle gauge) that will provide the volume/basal area ratio (VBAR) (Bell, Iles, & Marshall, 1983; Brooks, 2006; Kershaw Jr. et al., 2016b; Marshall et al., 2004; Yang et al., 2017). Big BAF sampling design exploits the common observation that estimates of the basal area have a higher variability than estimates of VBAR (Bruce, 1961). Because these two parts have different variability, they could be represented in the sample with varying sample size intensity (Samiotis & Stamatellos, 2011). As a result, in the first stage, a larger sample of trees is used to estimate basal area per unit area (basal area/ha or acre) and a smaller subsample of trees is used to estimate mean VBAR (volume/m2 or /ft2) (Iles, 2003; Marshall et al., 2004). "In general, the large-factor BAF is four to five times that of the small-factor BAF" (Yang & Burkhart, 2018) and corresponds to a proportion of approximately 1 VBAR tree would be chosen out of 5 counted trees (Iles, 2012).

The separation of basal area (tree counts) and VBAR as separate issues, led to the invention of Big BAF method, in which VBARs with lower variability than

counting trees are subsampled (Iles, 2017). Some alternative methods before the proposed Big BAF are: 1) the double sampling, instead of measuring all "in" trees, we record the count trees on every Bitterlich point (first phase) and then we subsample the VBARs from every nth sample point (second phase) (Desmarais, 2001; Iles, 1989, 2012); usually we measure all of the trees on every fourth (less third) sample point cluster of trees (Brooks, 2006; Iles, 1989; Yang & Burkhart, 2018), 2) the two-stage sampling in which measurements are taken in systematic manner, every 10th (nth) tree "in" on the second stage (Iles, 1989) or 3) randomly selecting two trees on every point of the second stage (Iles, 1989; Yang & Burkhart, 2018). It is not acceptable the subjective selection of closest trees to the sample point, because in that case smaller trees oversampled and usually the stand volume underestimated; all "in" trees must have an equal probability of being sampled (Iles, 1989).

Big BAF sampling has been applied or studied in various forest types with different species. References can be found for softwood forests like fir (*Abies × borisii-regis* Mattf) (Samiotis, 2008), redwood/Douglas-fir (Corrin, 1998; Marshall et al., 2004; Opalach, 2017), white pine in New Hampshire (Desmarais, 2003), spruce-fir forests in Maine (Lindemuth, 2007), hemlock (Desmarais, 2002), Jeffery pine (Marshall et al., 2004), in hardwoods, like Appalachian of West Virginia (Brooks, 2006), Northern red oak (Desmarais, 2002) and Aspen in Minnesota (Deegan, 2011). Big BAF has been applied also in mixed species forest structures (Yang et al., 2017), with uneven-age and multistory oak-pine stands (Lindemuth, 2007) and in plantations, like loblolly pine with spatial heterogeneity (Yang & Burkhart, 2018). Geographically, the Big BAF applied in Canada and the western United States (Brooks, 2006; Chen, Yang, Hsu, Kershaw, & Prest, 2019; Corrin, 1998; Desmarais, 2002; Marshall et al., 2004), while studies have been done for potential applicability in northeastern North America (Brooks, 2006; Burk, 2004; Deegan, 2011; Lindemuth, 2007; Yang et al., 2017) and in Europe (Samiotis & Stamatellos, 2011).

The main applications of Big BAF sampling concerns estimations of *stand volume* (Lindemuth, 2007; Marshall et al., 2004; Opalach, 2017; Yang & Burkhart, 2018), including total volume, merchantable volume, sawlog volume (saw timber) and pulpwood volume (Kershaw Jr. et al., 2016b; Yang et al., 2017), generally for timber cruising (Corrin, 1998). Current applications of Big BAF turns from estimations of volume to estimations of the average *forest biomass* (Ellis et al., 2019; Griscom, Ellis, & Putz, 2014) and *carbon*, demonstrating the efficiency and overall inventory costs reduction of this method (Chen et al., 2019). Additionally, Big BAF is suitable for landowners that seeking a cost-effective method for estimating carbon (Chen et al., 2019). Currently, Big BAF becomes an option for *collecting field data* among other sampling methods (Waterman, 2016b). Big BAF was utilized also for *overstory estimates* (DiNardo, 2015). We can consider the Big BAF as one of the subsampling methods for the *selection of VBAR trees* (Deegan, 2011) and "*measure*" trees for detailed measurements (tree height, bearing form the plot center, distance from the plot center, crown dimensions,

and number of silvicultural logs) (Fraser & Congalton, 2019). Big BAF has been conducted for collecting attributes on the *overstory species composition and understory* of inventory plots (Eisenhaure & Belair, 2016; Waterman, 2016a).

The main advantages of Big BAF sampling that greatly improve sample efficiency are: 1) the simple and easily applicable method of selecting the measured trees (Corrin, 1998; Marshall et al., 2004), 2) the improved distribution (dispersal) of the measured trees throughout the area of interest over the two-phase sampling (Brooks, 2006; Corrin, 1998; Marshall et al., 2004), 3) allowing cruisers (forest specialists) to optimize their samples, selecting the effective analogy (balancing the sampling effort)of "in" trees and the VBAR trees by choosing small and large angle gauge correspondingly, that subsequently will lead to large cost savings (Corrin, 1998; Marshall et al., 2004), 4) the Big BAF method does not require any special computational techniques (Corrin, 1998; Marshall et al., 2004), 5) because "in" trees of the second stage (VBAR trees) are very close to point center (Desmarais, 2002), simultaneously the travel distance between the sampling point and measurement trees decreasing and avoiding potential bias in tree selection (Rice et al., 2014), and 6) lastly because in Big BAF the basal area is more intensively sampled, this lead to better in estimations of species composition (Corrin, 1998).

Disadvantages of Big BAF (Brooks, 2006; Marshall et al., 2004; Osman & Idris, 2012) can be: 1) the selection of the appropriate sized BAF angle gauge. By choosing small BAF many trees measured for volume (not economically efficient) and subject to personal error increasing from missing trees and edge effects (increased sampling error); by choosing a large BAF few trees being measured for volume estimation but can result excessive variability (increased sampling error), 2) more Bitterlich plots, in comparison to classical Bitterlich sampling, will be received for measuring VBAR trees, 3) may be difficult to find large BAF angle gauges for the second stage in the case of very big trees with large diameter at breast height.

In the last few years, Big BAF has a steadily increasing number of research studies in the literature, for example (Chen et al., 2019; Iles, 2012; Kershaw Jr. et al., 2016b; Lei et al., 2019; McTague, 2010; Rice et al., 2014; Samiotis & Stamatellos, 2011; Yang & Burkhart, 2018; Yang, Kershaw, Weiskittel, Lam, & McGarrigle, 2019).

3. The Point-3P Sampling Method

Grosenbaugh (1963) introduced a sampling method in which trees are selected for measurement with a probability proportional to some predicted tree value (e.g. diameter at breast height, basal area, height, volume). This method is known as sampling with Probability Proportional to Prediction (3P, 3-P, PPP) and its advantage is that it does not require a list of trees in a forest area. The 3P method provides high accuracy and precision but is time-consuming and expensive when every tree must be visited for ocular prediction (Osman & Idris, 2012). This drawback led Grosenbaugh to invent an efficient sampling scheme for large

areas, a two-stage Point-3P sampling design (Grosenbaugh, 1971, 1974, 1979), almost a decade later based on the 3P method.

Point-3P or P3P sampling is a two-stage sample (also known as Point-Poisson sampling). In the first stage, we take Bitterlich point samples. In the second stage, we predict (visual estimation) the total height for each Bitterlich sample tree (as an estimate of VBAR) and we apply the 3P sample technique to those individual trees (Kershaw Jr. et al., 2016b; Rennie, 1976; Wood & Wiant, 1992). A subsample of the first stage trees is selected with a probability proportional to the prediction of the tree height (Matis, 2004). In order to select the 3P trees each estimated height is paired with a random number from a list that we produced (Rennie, 1976), if the predicted value is larger than the generated number, the tree is selected for detailed/accurately measurement for volume (Kershaw Jr. et al., 2016b; Rennie, 1976).

Point-3P applied mainly in North America (Rennie, 1976; Rennie et al., 1991; West, 2011) and research studies have been conducted in Chile-South America (Harris-Pascal, 2015), in Canada (Williams & Wiant, 1998), in Australia (Wood & Schreuder, 1986; Wood & Wiant, 1992; Wood & Wiant Jr., 1992), in New Zealand (Lee & Goulding, 2002), in Switzerland (Mandallaz & Massey, 2012), in Greece (Stamatellos, 1995) and in Eastern Africa (Tanzania) (Osman & Idris, 2012). Applications of Point-3P have been conducted in various forest types: from softwoods (Stamatellos, 1995; Williams & Wiant, 1998), to hardwoods of *Eucalyptus sp.* (Wood & Wiant Jr., 1992), Miombo savanna woodlands (Osman & Idris, 2012) and plantations of hardwood (Harris-Pascal, 2015) and softwood (Lee & Goulding, 2002) trees.

The vast majority of Point-3P applications refer to timber volume estimations (Gregoire & Valentine, 2007a; Harris-Pascal, 2015; Rennie et al., 1991; Stamatellos, 1995; West, 2011; Williams & Wiant, 1998; Wood & Schreuder, 1986) and merchantable log products for timber sales (saw-log, pulp-millable wood volume) (Lee & Goulding, 2002; Osman & Idris, 2012; Rennie, 1976; Schreuder, Ouyang, & Williams, 1992; Wood & Wiant, 1992; Wood & Wiant Jr., 1992). Apart from volume estimation, Point-3P sampling can be implemented for any stand attribute estimations (Kershaw Jr. et al., 2016b).

The main advantages that characterize Point-3P are: 1) the use of the power of 3P sampling can be applied for large areas (greater than stand-level) more efficiently (Iles, 1995; Lee & Goulding, 2002; Stamatellos, 1995), 2) the time and cost efficiency in comparison with fixed-area plots (Osman & Idris, 2012) or Bitterlich plots (Stamatellos, 1995), 3) the sampling efficiency comparing to the "classical" 3P (Basic-3P) or "ordinary" 3P sampling, due to its use of variable radius (Bitterlich) plot sampling in the first stage (West, 2011), 4) is cheaper than either Fixed-area plot or Basic-3P designs (Osman & Idris, 2012) and 5) in Point-3P we expect higher sampling accuracy than Big BAF, because the selection of trees (sampling units) in the second stage, use the height as additional variable to basal area (height multiplied by basal area), which is more relevant and commonly proportional to the volume of interest (Rennie, 1976).

Disadvantages can attribute to Point-3P: 1) in the case of inventorying small areas with large variation between plots and desiring high precision, in that case, 3P sampling has better cost efficiency (Lee & Goulding, 2002), 2) from the selection of proper sample size on the second-stage; in the case of small sample size the precision of the final estimate will be less than planned and in the case of large sample size the total cost will be greater than anticipated (Wood & Wiant Jr., 1992) and 3) from the program acquisition/creation that will "produce an appropriate random number list for selecting the second-stage sample (trees)" (Wood & Schreuder, 1986). Comparing the disadvantages of Point-3P & Big BAF we understand that Big BAF method is easier and quicker applicable, because there is no need for height estimations of all "in" trees in the second stage, and there is no need to create and use random number lists for further measurements.

Recent publications since 2010 for Point-3P sampling are presented in these sources: (Harris-Pascal, 2015; Mandallaz & Massey, 2012; Osman & Idris, 2012; West, 2011, 2017; Yih Lam, Hsu, Yang, Kershaw, & Su, 2017). From the current bibliography, Big BAF tends to be used more frequently than Point-3P sampling method.

4. Mathematical Description of Two Sampling Methods

Big BAF and Point-3P sampling are two-stage sampling methods. Both methods have primary Bitterlich sampling units (points) and secondary units, are a subset of trees selected from the first stage. These methods can be considered as extensions of Bitterlich sampling. In the first stage of Bitterlich sampling, the probability of selecting trees $\left(\pi_1 \right)$ is proportional to their basal area (De Vries, 1986; Gregoire & Valentine, 2007b; Kershaw Jr. et al., 2016a; Overton & Stehman, 1995; Schreuder, Gregoire, & Wood, 1993), $\pi_1 \propto g$ where $g = d^2 \left(\pi/4 \right)$ the basal area of the tree with a diameter at breast height d. In the second sampling stage, for the Big BAF method, the probability of selecting trees from those selected in the first stage $\left(\pi_{2|1} \right)$, is a constant proportional to the ratio of basal area factors $\left(F_1/F_2 \right)$ that are used in the two sampling stages, $\pi_{2|1} \propto F_1/F_2$. In the second stage of the Point-3P method, the probability of the trees' selection is proportional to a prediction of their height (Gregoire & Valentine, 2007b; Kershaw Jr. et al., 2016a; Schreuder et al., 1993), $\pi_{2|1} \propto h$ where h is a prediction of the trees' height. The variables that determine the probabilities of selecting trees in the second stage are well correlated with the variable of interest, which is the VBAR. In Big BAF method, the ratio of the probabilities of selecting two trees in the sample equals to the ratio of their basal areas, while in the Point-3P sampling the same ratio equals the ratio of the products of basal areas on their heights. In the Point-3P sampling, the total probability of selecting trees correlates more strongly with the tree volume and is expected to be more effective than the Big BAF method (De Vries, 1986; Rennie, 1976).

From the previous description, it appears that the Big BAF and Point-3P sampling methods are similar in their probability basis and therefore it is possible

for the estimators and variances to be given in the same way. Let X and Y to be two random variables (not necessarily independent), the variance $V(XY)$ of the product XY (Bohrnstedt & Goldberger, 1969: p. 1439; Goodman, 1960: p. 712; Mood, Graybill, & Boes, 1974: p. 180) is

$$V(XY) = E^2(X)V(Y) + E^2(Y)V(X) + 2E(X)E(Y)Cov\left[E(X),E(Y)\right] \\ + 2E(X)E_{12} + 2E(Y)E_{21} + E_{22} - E_{11}^2 \tag{1}$$

where $E(.)$ is the expected value, $V(.)$ is the variance, $Cov(,)$ is the covariance, $\Delta x = X - \mathrm{E}(X)$, $\Delta y = Y - \mathrm{E}(Y)$, $E_{12} = E\left[(\Delta x)^1(\Delta y)^2\right]$, $E_{21} = E\left[(\Delta x)^2(\Delta y)^1\right]$ and $E_{22} - E_{11}^2 = V(\Delta x \Delta y)$.

An approximate estimation of $V(XY)$, considering the first three terms of "Equation (1)" is given as

$$V(XY) = E^2(X)V(Y) + E^2(Y)V(X) + 2E(X)E(Y)Cov\left[E((X),E(Y)\right]. \tag{2}$$

In our work, let $\bar{Q}$ to be the mean volume per ha of a forest area at n points of sampling (Bitterlich) and $\bar{X}$ the mean basal area per ha in the Big BAF method and the product of the BAF and the sum of heights predictions in the Point-3P method. $\bar{Y}$ is the mean per tree volume ratio to their basal area, in the Big BAF method and $\bar{Y}$ is the mean per tree ratio of measured VBAR to estimated tree height in the Point-3P sampling. Then the mean volume per ha, $\bar{Q}$, is given as the product of $\bar{X}$ and $\bar{Y}$ (Kershaw Jr. et al., 2016a; Marshall et al., 2004)

$$\bar{Q} = \bar{X}\bar{Y}, \tag{3}$$

with approximate variance $V(\bar{Q})$,

$$V(\bar{Q}) = \bar{X}^2 V(\bar{Y}) + \bar{Y}^2 V(\bar{X}) + 2\bar{X}\bar{Y}Cov(\bar{X},\bar{Y}), \tag{4}$$

and estimated % sampling error, *SE%* to be given as

$$SE(\bar{Q})\% = \sqrt{\frac{V(\bar{Q})}{\bar{Q}^2}}100. \tag{5}$$

In similar applications of timber stock estimations in forest ecosystems, the covariance term can be ignored because its size is small (Marshall et al., 2004). In this case, the variance and sampling error is further simplified to

$$SE(\bar{Q})\% = \sqrt{\left[SE(\bar{X})\%\right]^2 + \left[SE(\bar{Y})\%\right]^2}. \tag{6}$$

Then confidence intervals for $\bar{Q}$ can be constructed on the assumption that $\bar{Q}$ is normally distributed. This assumption is difficult to satisfy because the distribution of the product of two normal variables is generally not normal (Lomnicki, 1967; Springer & Thompson, 1966), when the variables are additionally correlated, skewness is usual and normality does not apply (Oliveira, Oliveira, & Seijas-Macias, 2016). In this case, confidence intervals are created with resampling methods such as bootstrap and jackknife methods (Buonaccorsi & Liebhold, 1988; Efron, 1982; Efron & Tibshirani, 1986).

5. The Two Sampling Methods in Forest Inventories and Forest Management

The two sampling methods presented, Big BAF and Point-3P, are more effective than the commonly used methods, fixed-plot, and Bitterlich sampling. Generally, their efficiency relies on the combination of Bitterlich method and two-stage sampling advantages. Greater efficiency is mainly due to reducing costs because many fewer trees are measured, although there is a slight increase in sampling error. The applicability of these methods in the management of forest ecosystems depends on the acceptance and adoption of Bitterlich point sampling from practitioners. These methods are proposed in the management of forest ecosystems at a local and regional level. In the broader and national forest inventories, remote sensing data can be used in the early stages and then the two proposed methods for collecting data in the field can be applied.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

Bell, J., Iles, K., & Marshall, D. (1983). Balancing the Ratio of Tree Count-Only Sample Points and VBAR Measurements in Variable Plot Sampling. In *Renewable Resource Inventories for Monitoring Changes and Trends: Proceedings of an International Conference* (pp. 699-702). Corvallis, OR: Oregon State University.

Bohrnstedt, G. W., & Goldberger, A. S. (1969). On the Exact Covariance of Products of Random Variables. *Journal of the American Statistical Association, 64,* 1439-1442. https://doi.org/10.1080/01621459.1969.10501069

Brooks, J. R. (2006). An Evaluation of Big Basal Area Factor Sampling in Appalachian Hardwoods. *Northern Journal of Applied Forestry, 23,* 62-65.

Bruce, D. (1961). *Prism Cruising in the Western United States and Volume Tables for Use Therewith.* Portland: Mason, Bruce, and Girard.

Buonaccorsi, J. P., & Liebhold, A. M. (1988). Statistical Methods for Estimating Ratios and Products in Ecological Studies. *Environmental Entomology, 17,* 572-580. https://doi.org/10.1093/ee/17.3.572

Burk, T. E. (2004). *Quicker/Cheaper Stand Assessments.* Minneapolis, MN: Department of Forest Resources, College of Natural Resources and Minnesota Agricultural Experi-

ment Station, University of Minnesota.

Chen, Y., Yang, T.-R., Hsu, Y.-H., Kershaw, J. A., & Prest, D. (2019). Application of Big BAF Sampling for Estimating Carbon on Small Woodlots. *Forest Ecosystems, 6,* 13. https://doi.org/10.1186/s40663-019-0172-4

Corrin, D. (1998). *BIG BAF: A Very Efficient Sampling Methods for Cruising Timber.* http://john-bell-associates.com/guest/guest43a.htm

Dahl, C. A., Harding, B. A., & Wiant, H. V. (2008). Comparing Double-Sampling Efficiency Using Various Estimators with Fixed-Area and Point Sampling. *Northern Journal of Applied Forestry, 25,* 99-102.

De Vries, P. G. (1986). *Sampling Theory for Forest Inventory: A Teach-Yourself Course.* Berlin: Springer Science & Business Media. https://doi.org/10.1016/0378-1127(89)90042-X

Deegan, G. (2011). *The Camp Riley Cruise Experiment.* http://john-bell-associates.com/regular/regular_online7.htm

Desmarais, K. (2003). Cruising White Pine Stands Efficiently. In *Managing White Pine in a New Millennium: 2003 Workshop Proceedings* (pp. 16-19). Hillsborough, NH: Agricultural Extension Service, University of New Hampshire, .

Desmarais, K. M. (2001). *What Do Your Cruise Results Mean*? Paper Presented at the Forest Measurements for Natural Resource Professionals, Caroline A. Fox Research and Demonstration Forest Hillsborough, NH.

Desmarais, K. M. (2002). *Using BigBAF Sampling in a New England Mixed Wood Forest.* Inventory & Cruising Newsletter. http://john-bell-associates.com/guest/guest58b.htm

DiNardo, K. (2015). *Silviculture Prescription Erving Red Pine.* Boston, MA: Massachusetts Department of Conservation and Recreation. https://www.mass.gov/files/documents/2016/12/nc/erving-state-forest-red-pine-final.pdf

Efron, B. (1982). *The Jackknife, the Bootstrap, and Other Resampling Plans.* CBMS-NSF Regional Conference Series in Applied Mathematics Vol. 38, SIAM. https://doi.org/10.1137/1.9781611970319

Efron, B., & Tibshirani, R. (1986). Bootstrap Methods for Standard Errors, Confidence Intervals, and Other Measures of Statistical Accuracy. *Statistical Science, 1,* 54-75. https://doi.org/10.1214/ss/1177013815

Eisenhaure, S., & Belair, E. (2016). *2016 Management Plan, Davis Park.* Durham: University of New Hampshire. https://colsa.unh.edu/sites/default/files/davis-park-plan.pdf

Ellis, P. W., Gopalakrishna, T., Goodman, R. C., Putz, F. E., Roopsind, A., Umunay, P. M., Griscom, B. W. et al. (2019). Reduced-Impact Logging for Climate Change Mitigation (RIL-C) Can Halve Selective Logging Emissions from Tropical Forests. *Forest Ecology and Management, 438,* 255-266. https://doi.org/10.1016/j.foreco.2019.02.004

Fraser, B. T., & Congalton, R. G. (2019). Evaluating the Effectiveness of Unmanned Aerial Systems (UAS) for Collecting Thematic Map Accuracy Assessment Reference Data in New England Forests. *Forests, 10,* 24. https://doi.org/10.3390/f10010024

Goodman, L. A. (1960). On the Exact Variance of Products. *Journal of the American Statistical Association, 55,* 708-713. https://doi.org/10.1080/01621459.1960.10483369

Gregoire, T. G., & Valentine, H. T. (2007a). Bitterlich Sampling. In T. G. Gregoire, & H. T. Valentine (Eds.), *Sampling Strategies for Natural Resources and the Environment* (Chapter 8, 32 p.). Boca Raton, FL: Chapman and Hall/CRC. https://doi.org/10.1201/9780203498880

Gregoire, T. G., & Valentine, H. T. (2007b). *Sampling Strategies for Natural Resources*

and the Environment. Boca Raton, FL: Chapman and Hall/CRC. https://doi.org/10.1201/9780203498880

Griscom, B., Ellis, P., & Putz, F. E. (2014). Carbon Emissions Performance of Commercial Logging in East Kalimantan, Indonesia. *Global Change Biology, 20,* 923-937. https://doi.org/10.1111/gcb.12386

Grosenbaugh, L. (1963). *Some Suggestion for Better Sample-Tree-Measurement.* In *Proceedings of Society of American Foresters* (pp. 36-42). Boston, MA: Society of American Foresters.

Grosenbaugh, L. R. (1971). *STX 1-11-71 for Dendrometry of Multistage 3P Samples.* Estacada, OR: United States Department of Agriculture, Forest Service.

Grosenbaugh, L. R. (1974). *STX 3-3-73: Tree Content and Value Estimation Using Various Sample Designs, Dendrometry Methods, and VSL Conversion Coefficients.*

Grosenbaugh, L. R. (1979). *3P Sampling Theory, Examples, and Rationale* (Vol. 331). Washington DC: US Dept. of the Interior, Bureau of Land Management. https://doi.org/10.5962/bhl.title.59383

Harris-Pascal, D. (2015). *Measure Plantations and Designing Thinnings Using Smartphone Mensuration Tools in Chile.* https://www.academia.edu/20414532/Measure_Plantations_and_Designing_Thinnings_Using_Smartphone_Mensuration_Tools_in_Chile

Iles, K. (1989). Subsampling for VBAR. *Inventory Cruising News, 8,* 2-3.

Iles, K. (1995). *Point-3P Sampling: Extending 3P Sampling to Large Areas.* Inventory & Cruising. http://www.john-bell-associates.com/3p/point3p.htm

Iles, K. (2003). *A Sampler of Inventory Topics: A Practical Discussion for Resource Samplers, Concentrating on Forest Inventory Techniques.* British Columbia: Kim Iles & Associates Ltd.

Iles, K. (2012). Some Current Subsampling Techniques in Forestry. *Mathematical and Computational Forestry & Natural Resource Sciences, 4,* 77.

Iles, K. (2017). *"Double Sampling"—Some History and a Practical Extension.* http://john-bell-associates.com/regular/regular_online17.htm

Kershaw Jr., J. A., Ducey, M. J., Beers, T. W., & Husch, B. (2016a). *Forest Mensuration* (5th ed.). Hoboken, NJ: John Wiley & Sons. https://doi.org/10.1002/9781118902028

Kershaw Jr., J. A., Ducey, M. J., Beers, T. W., & Husch, B. (2016b). Inventory of Standing Trees Using Sampling with Varying Probability. In B. Husch et al. (Eds.), *Forest Mensuration* (5th ed., Chapter 11). Hoboken, NJ: John Wiley & Sons. https://doi.org/10.1002/9781118902028.ch11

Lee, K., & Goulding, C. (2002). Practicality of 3P Sampling with Accurate Dendrometry for the Pre-Harvest Inventory of Plantations. *New Zealand Journal of Forestry Science, 32,* 279-296. https://doi.org/10.1109/TGRS.2018.2860590

Lei, Y., Siqueira, P., Torbick, N., Ducey, M., Chowdhury, D., & Salas, W. (2019). Generation of Large-Scale Moderate-Resolution Forest Height Mosaic with Spaceborne Repeat-Pass SAR Interferometry and Lidar. *IEEE Transactions on Geoscience and Remote Sensing, 57,* 770-787. https://digitalcommons.library.umaine.edu/etd/810 https://doi.org/10.1109/TGRS.2018.2860590

Lindemuth, R. M. (2007). *A Field Trial Comparison of Sampling Methods for Estimating Basal Area and Volume in Partially Harvested Stands in Maine.* Master Thesis, Orono, ME: University of Maine. https://digitalcommons.library.umaine.edu/etd/810

Lomnicki, Z. A. (1967). On the Distribution of Products of Random Variables. *Journal of the Royal Statistical Society. Series B (Methodological), 29,* 513-524.

https://doi.org/10.1111/j.2517-6161.1967.tb00713.x

Mandallaz, D., & Massey, A. (2012). Comparison of Estimators in One-Phase Two-Stage Poisson Sampling in Forest Inventories. *Canadian Journal of Forest Research, 42,* 1865-1871. https://doi.org/10.1139/x2012-110

Marshall, D. D., Iles, K., & Bell, J. F. (2004). Using a Large-Angle Gauge to Select Trees for Measurement in Variable Plot Sampling. *Canadian Journal of Forest Research, 34,* 840-845. https://doi.org/10.1139/x03-240

Matis, K. (2004). *Sampling of Natural Resources.* Thessaloniki: Pegasus.

McTague, J. P. (2010). New and Composite Point Sampling Estimates. *Canadian Journal of Forest Research, 40,* 2234-2242. https://doi.org/10.1139/X10-158

Mood, A. M., Graybill, F. A., & Boes, D. C. (1974). *Introduction to the Theory of Statistics: McGraw-Hill Series in Probability and Statistics* (3rd ed.). https://doi.org/10.2307/2286195

Oliveira, A., Oliveira, T. A., & Seijas-Macias, A. (2016). Skewness into the Product of Two Normally Distributed Variables and the Risk Consequences. *REVSTAT: Statistical Journal, 14,* 119-138. https://doi.org/10.1063/1.4952232

Opalach, D. (2017). On the Variation of Inventory Estimates for Redwood Stands. In R. B. Standiford, et al. (Eds.), *Coast Redwood Science Symposium—2016: Past Successes and Future Direction* (Vol. 258, pp. 47-54). Albany, CA: U.S. Department of Agriculture, Forest Service, Pacific Southwest Research Station.

Osman, E., & Idris, E. (2012). Comparative Study on the Efficiency of Some Selected Sampling Designs in Assessing Miombo Woodland. *Journal of Forest Products Industries, 1,* 4-10.

Overton, W. S., & Stehman, S. V. (1995). The Horvitz-Thompson Theorem as a Unifying Perspective for Probability Sampling: With Examples from Natural Resource Sampling. *The American Statistician, 49,* 261-268. https://doi.org/10.1080/00031305.1995.10476160

Piqué, M., Obon, B., Condés, S., & Saura, S. (2011). Comparison of Relascope and Fixed-Radius Plots for the Estimation of Forest Stand Variables in Northeast Spain: An Inventory Simulation Approach. *European Journal of Forest Research, 130,* 851-859. https://doi.org/10.1007/s10342-010-0477-x

Rennie, J. C. (1976). Point-3P Sampling: A Useful Timber Inventory Design. *The Forestry Chronicle, 52,* 145-146. https://doi.org/10.5558/tfc52145-3

Rennie, J. C., Wood, G. B., Schreuder, H. T., & Lund, H. G. (1991). Point-Model Based Sampling in Forestry: Principles and Practices. *Southern Journal of Applied Forestry, 15,* 109-113.

Rice, B., Weiskittel, A. R., & Wagner, R. G. (2014). Efficiency of Alternative Forest Inventory Methods in Partially Harvested Stands. *European Journal of Forest Research, 133,* 261-272. https://doi.org/10.1007/s10342-013-0756-4

Samiotis, K. (2008). *Estimation of Stand Volume with the Use of Big Baf Method.* Master Thesis, Thessaloniki: Aristotle University of Thessaloniki.

Samiotis, K., & Stamatellos, G. (2011). Estimation of Stand Volume with the Use of the Big Baf Method. *Geotechnical Scientific Issues, 21,* 73-79.

Schreuder, H. T., Gregoire, T. G., & Wood, G. B. (1993). *Sampling Methods for Multiresource Forest Inventory.* Hoboken, NJ: John Wiley & Sons. https://doi.org/10.2307/2533478

Schreuder, H. T., Ouyang, Z., & Williams, M. (1992). Point-Poisson, Point-PPS, and Modified Point-PPS Sampling: Efficiency and Variance Estimation. *Canadian Journal*

of *Forest Research, 22,* 1071-1078. https://doi.org/10.1139/x92-142

Scott, C. T. (1990). An Overview of Fixed versus Variable-Radius Plots for Successive Inventories. In *Proceedings of Symposium, State-of-the-Art Methodology of Forest Inventory* (pp. 97-104). Syracuse, NY: USDA Forest Service, Pacific Northwest Research Station.

Springer, M. D., & Thompson, W. E. (1966). The Distribution of Products of Independent Random Variables. *SIAM Journal on Applied Mathematics, 14,* 511-526. https://doi.org/10.1137/0114046

Stamatellos, G. S. (1995). Comparison of Point and Point-3P Sampling for Forest Volume Estimation with Cost Analysis. *Forest Ecology and Management, 74,* 75-79. https://doi.org/10.1016/0378-1127(94)03511-T

Waterman, M. (2016a). *Silviculture Prescription Brookline Road.* Boston, MA: Massachusetts Department of Conservation and Recreation. https://www.mass.gov/files/documents/2017/01/xe/brookline-road-prescription.pdf

Waterman, M. (2016b). *Silviculture Prescription Hansen Lot.* Boston, MA: Massachusetts Department of Conservation and Recreation. https://www.mass.gov/files/documents/2016/08/tt/hansen-prescription.pdf

West, P. W. (2011). Potential for Wider Application of 3P Sampling in Forest Inventory. *Canadian Journal of Forest Research, 41,* 1500-1508. https://doi.org/10.1139/x11-062

West, P. W. (2017). Population Structure and Correlation between Auxiliary and Target Variables May Affect Precision of Estimates in Forest Inventory. *Communications in Statistics: Simulation and Computation, 46,* 4951-4965. https://doi.org/10.1080/03610918.2016.1139128

Williams, M. S., & Wiant, H. V. (1998). Double Sampling with Importance Sampling to Eliminate Bias in Tree Volume Estimation of the Centroid Method. *Forest Ecology and Management, 104,* 77-88. https://doi.org/10.1016/S0378-1127(97)00257-0

Wood, G. B., & Schreuder, H. T. (1986). Implementing Point-Poisson and Point-Model Based Sampling in Forest Inventory. *Forest Ecology and Management, 14,* 141-156. https://doi.org/10.1016/0378-1127(86)90099-X

Wood, G. B., & Wiant Jr., H. V. (1992). Comparison of Point-3P and Modified Point-List Sampling for Inventory of Mature Native Hardwood Forest in Southeastern New South Wales. *Canadian Journal of Forest Research, 22,* 725-728. https://doi.org/10.1139/x92-097

Wood, G. B., & Wiant, H. V. (1992). Test of Application of Centroid and Importance Sampling in a Point-3P Forest Inventory. *Forest Ecology and Management, 53,* 107-115. https://doi.org/10.1016/0378-1127(92)90038-B

Yang, S.-I., & Burkhart, H. E. (2018). Comparison of Volume and Stand Table Estimates with Alternate Methods for Selecting Measurement Trees in Point Samples. *Forestry: An International Journal of Forest Research, 92,* 42-51. https://doi.org/10.1093/forestry/cpy029

Yang, T.-R., Hsu, Y.-H., Kershaw, J. A., McGarrigle, E., & Dan, K. (2017). Big BAF Sampling in Mixed Species Forest Structures of Northeastern North America: Influence of Count and Measure BAF under Cost Constraints. *Forestry: An International Journal of Forest Research, 90,* 649-660. https://doi.org/10.1093/forestry/cpx020

Yang, T.-R., Kershaw, J. A., Jr., Weiskittel, A. R., Lam, T. Y., & McGarrigle, E. (2019). Influence of Sample Selection Method and Estimation Technique on Sample Size Requirements for Wall-to-Wall Estimation of Volume Using Airborne LiDAR. *Forestry: An International Journal of Forest Research, 92,* 311-323. https://doi.org/10.1093/forestry/cpz014

Yih Lam, T., Hsu, Y.-H., Yang, T.-R., Kershaw, J. A., & Su, S.-H. (2017). Sampling with Probability Proportional to Prediction: Rethinking Rapid Plant Diversity Assessment. *Forestry: An International Journal of Forest Research, 91,* 17-26. https://doi.org/10.1093/forestry/cpx044