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Universality Class of the Nonequilibrium Phase Transition in Two-Dimensional Ising Ferromagnet Driven by Propagating Magnetic Field Wave

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Abstract

The purpose of this work is to identify the universality class of the nonequilibrium phase transition in the two-dimensional kinetic Ising ferromagnet driven by propagating magnetic field wave. To address this issue, the finite size analysis of the nonequilibrium phase transition, in two-dimensional Ising ferromagnet driven by plane propagating magnetic wave, is studied by Monte Carlo simulation. It is observed that the system undergoes a nonequilibrium dynamic phase transition from a high temperature dynamically symmetric (*propagating*) phase to a low temperature dynamically symmetry-broken (*pinned*) phase as the system is cooled below the transition temperature. This transition temperature is determined precisely by studying the fourth-order Binder Cumulant of the dynamic order parameter as a function of temperature for different system sizes (L). From the finite size analysis of dynamic order parameter ($Q_L \sim L^{-\frac{\beta}{\nu}}$) and the dynamic susceptibility ($\chi_L^Q \sim L^{\frac{\gamma}{\nu}}$), we have estimated the critical exponents $\beta/\nu = 0.146 \pm 0.025$ and $\gamma/\nu = 1.869 \pm 0.135$ (measured from the data read at the critical temperature obtained from Binder cumulant), and $\gamma/\nu = 1.746 \pm 0.017$ (measured from the peak positions of dynamic susceptibility). Our results indicate that such driven Ising ferromagnet belongs to the same universality class of the two-dimensional equilibrium Ising ferromagnet (where $\beta/\nu = 1/8$ and $\gamma/\nu = 7/4$), within the limits of statistical errors.

Keywords

Ising Model, Dynamic Phase Transition, Monte-Carlo Simulation, Propagating Wave, Finite Size Analysis, Critical Exponents, Universality

1. Introduction

The driven Ising ferromagnet shows interesting nonequilibrium phase transitions [1] [2]. This time dependent drive may be of two kinds: 1) an applied magnetic field which is oscillating in time and uniform over the space at any particular instant, 2) the applied magnetic field has a spatio-temporal variation which may be the type of propagating or standing magnetic field wave. The first kind of driving magnetic field has drawn much attention to the researchers and a considerable volume of studies is done in this direction, in last two decades. Here, a few of those may be mentioned as follows: 1) the critical slowing down and the divergence of the specific heat near the dynamic transition temperature [3], 2) the divergence of the fluctuations of the dynamic order parameter [4], 3) the growth of critical correlation near the dynamic transition temperature [5]. These studies are an integrated effort to establish that the nonequilibrium transition in kinetic Ising ferromagnet driven by oscillating magnetic field is indeed a thermodynamic *phase transition*.

The nonequilibrium phase transitions in other magnetic models (e.g., Blume-Capel, Blume-Emery-Griffiths models etc.) driven by oscillating (in time but uniform over the space) magnetic field have been studied [6] [7] [8] also in last few years to present some interesting nonequilibrium behaviors. The nonequilibrium phase transitions were studied in [9] [10] [11] [12] [13] mixed spin systems driven by oscillating magnetic field, recently.

The another kind of external drive may be the magnetic field with spatio-temporal variation. The prototypes of these spatio-temporal drives are propagating or standing magnetic field waves. In the last few years, a number of investigations, on the nonequilibrium phase transitions in Ising ferromagnet driven by propagating and standing magnetic field wave, are done [14] [15] [16] [17] [18] through Monte Carlo methods. Here, the essential findings are the nonequilibrium phase transitions between two phases, namely, the low temperature ordered pinned phase (where the spins do not flip) and a high temperature disordered phase where a coherent propagation (in the case of propagating magnetic field wave) or coherent oscillation (in the case of standing magnetic field wave) of spin bands are observed. The transitions are marked by the divergences of dynamic susceptibility near the transition point.

However, the detailed finite size analyses were not yet performed to know the *universality class* of this nonequilibrium phase transition observed in Ising ferromagnet driven by propagating magnetic field wave. This is the key issue of the present study.

In this paper, we have investigated the nonequilibrium behaviour and the finite size effect of spin-$\frac{1}{2}$ Ising ferromagnet under the influence of propagating magnetic wave by Monte Carlo methods. The paper is organized as follows: The model and the MC simulation technique are discussed in Section II, the numerical results are reported in Section III and the paper ends with a summary in Section IV.

2. Model and Simulation

The *time dependent* Hamiltonian of a two dimensional driven Ising ferromagnet is represented by,

$$H(t) = -J\Sigma\Sigma' s^z(x,y,t)s^z(x',y',t) - \Sigma h^z(x,y,t)s^z(x,y,t). \quad (1)$$

Here $s^z(x,y,t) = \pm 1$, is the Ising *spin variable* at lattice site (x,y) at time *t*. The summation Σ' extends over the nearest neighbour sites (x',y') of a given site (x,y). $J(>0)$ is the *ferromagnetic spin-spin interaction strength* between the nearest neighbour pairs of Ising spins. For simplicity, we have considered the value of *J* to be uniform over the whole lattice. The externally applied driving *magnetic field*, is denoted by $h^z(x,y,t)$, at site (x,y) at time *t*. $h^z(x,y,t)$ has the following form for propagating magnetic wave,

$$h^z(x,y,t) = h_0 \cos\left\{2\pi\left(ft - \frac{x}{\lambda}\right)\right\} \quad (2)$$

Here h_0 and *f* represent *the field amplitude and the frequency* respectively of the propagating magnetic wave, whereas λ represents *the wavelength* of the wave. The wave propagates along the *X*-direction through the lattice.

An $L \times L$ square lattice of Ising spins is taken here as a model system. The boundary conditions applied at both directions are *periodic* which preserve the translational invariances in the system. Using *Monte Carlo Metropolis single spin flip algorithm* with *parallel* updating rule [19], the dynamics of the system are simulated. The initial state of the system is chosen as the high temperature random disordered phase, in which, at any lattice site, both the two states (±1) of the Ising spins have equal probabilities. The system is then slowly cooled down to any lower temperature *T* and the dynamical quantities are calculated. The *Metropolis probability* [19] of single spin flip at temperature *T* is given by,

$$W\left(\left(s^z\right)_i \to \left(s^z\right)_f\right) = \min\left[\exp\left(\frac{-\Delta E}{k_B T}\right), 1\right] \quad (3)$$

where ΔE is the energy change due to spin flip from *i*-th state to *f*-th state and k_B is the Boltzman constant. In a chosen configuration, the probability of flipping of each spin is calculated from the above rule. Then prepared a list of L^2 such values of probability of flipping. On the other hand, a list of L^2 random fraction (collected from a uniformly distributed random numbers) is prepared, keeping in mind that each random fraction is associated to the probability of flipping of each spin. The spins are flipped *simultaneously* where the probability of flipping exceeds (or equal to) the random fraction. This is so called *parallel updating* of spins. Such parallel updating of L^2 spin states in an $L \times L$ square lattice constitute the unit time step and is called *Monte Carlo Step per Spin* (MCSS). The applied magnetic field and the temperature are measured in the units of *J* and J/k_B respectively. The choices of such units of applied magnetic field and the temperatures are very common in the literatures [19] of the simulation of the Statistical Mechanics of Ising ferromagnet in the presence of magnetic field.

3. Results

The nonequilibrium behaviour of the two dimensional Ising ferromagnet is studied here in $L \times L$ square lattices of different sizes (L) where a propagating magnetic wave is passing through the system. The frequency (f) of magnetic field oscillation, wavelength (λ) of the magnetic wave and the amplitude (h_0) of field strength are kept constant throughout the present study. These constant values are respectively $f = 0.01(\text{MCSS})^{-1}$, $\lambda = 16$ lattice units and $h_0 = 0.3J$. For $f = 0.01$, 100 MCSS is required for a complete time cycle.

Since we have chosen the values of L in the multiple of 16, the wavelength $\lambda = 16$ is a reasonable choice. In this case, the smallest system will contain a full wave. The frequency, $f = 0.01$, is chosen to have the adequate number of cycles, of the propagating magnetic field, to get a reasonable average value. The choice of the amplitude $h_0 = 0.3J$ is just to keep the nonequilibrium phase transition in the higher temperature range.

The finite size effect is studied by taking into account four different system sizes (within the limited computational facilities available) such as L = 16, 32, 48 and 128. The system (for any fixed value of L) has been cooled down in small steps ($\Delta T = 0.005 J/k_B$) from high temperature phase, *i.e.* the dynamical disordered phase, to reach any dynamical steady state at temperature T. The dynamical quantities are calculated when the system has achieved the nonequilibrium steady state. For this we have kept the system in constant temperature for a sufficiently long time; 12,000 (for $L = 128$) to 32,000 (for $L = 16$) cycles of magnetic oscillations and discarding the initial (or transient) 1000 cycles and taking average over the remaining cycles. We have detected a dynamical phase transition from high temperature symmetric propagating (spin bands) phase to low temperature symmetry-broken pinned phase. The dynamic *Order parameter* for the phase transition is defined as the average magnetisation per spin over a full cycle of external magnetic field, *i.e.*

$$Q = f \times \oint M(t) \mathrm{d}t, \tag{4}$$

where $M(t)$ is the value of instantaneous magnetisation per spin at time t which can be obtained as

$$M(t) = \frac{1}{L^2} \sum_{i=1}^{L^2} s^z(x, y, t) \tag{5}$$

At very high temperature, the flipping probability of the spin, is quite high alongwith the oscillation of the magnetic field. As a result the value of the instantaneous magnetisation is almost close to zero. Consequently, by definition, the value of the dynamic order parameter is very small, thus identifying the dynamically disordered propagating phase ($Q = 0$) (see **Figure 1(b)**). It may be noted here, that the instantaneous magnetisation fluctuates symmetrically about zero (see **Figure 1(d)**). Hence, this may be characterised as a dynamically symmetric phase. As the system is cooled down below the critical temperature, which depends on the value of magnetic field strength, the flipping probability of

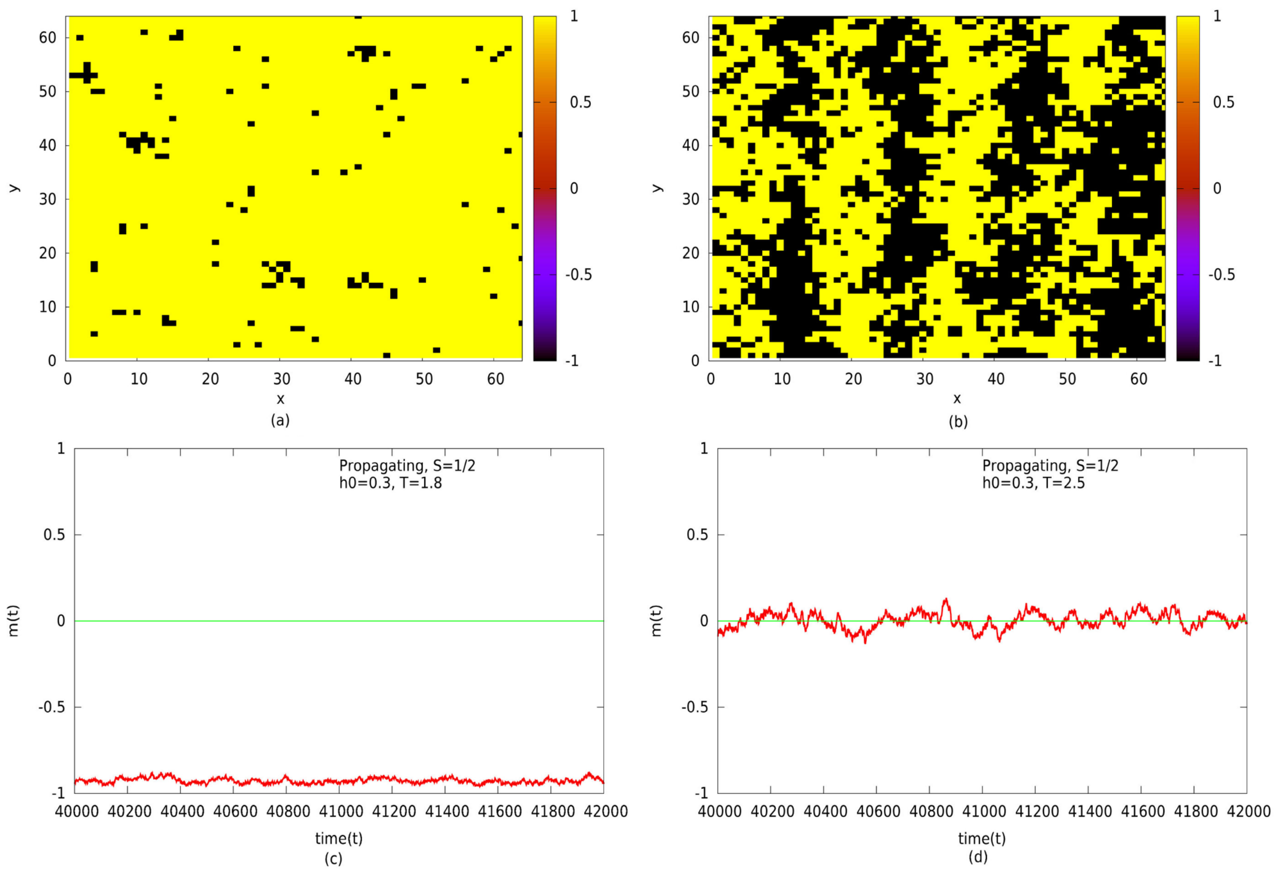

Figure 1. The lattice morphologies of the (a) pinned phase and (b) propagating phase respectively at time $t = 39937$ MCSS for $L = 64$. The dynamical symmetry breaking (change in the value of average magnetisation per spin from non-zero to nearly zero value); (c) at temperature $T = 1.8$ and (d) at temperature $T = 2.5$. The value of the amplitude of the field is $h_0 = 0.3$ in all cases.

the spin gets reduced; also the magnetic field strength may not be adequate to flip the spins and the spins are locked or pinned in a particular orientation giving rise to a large and nearly steady value of average magnetisation. This phase is identified as the dynamically ordered or pinned phase ($Q \neq 0$) (see Figure 1(a)). Unlike, the dynamically symmetric phase (mentioned above), here the instantaneous magnetisation varies asymmetrically about zero (see Figure 1(c)). So, this may be called a dynamically symmetry broken phase. The variation of the order parameter for the dynamic phase transition (DPT) for four different system sizes are shown in Figure 2(a).

The dynamical critical point is determined with high precision by studying the thermal variation of fourth order Binder cumulant ($U_L(T) = 1 - \frac{\langle Q^4 \rangle_L}{3\langle Q^2 \rangle_L^2}$) of dynamic order parameter Q for different system sizes (L). Figure 2(b) shows the variation of the Binder Cumulant ($U_L(T)$) with temperature (T) for different values of L. From this figure we have determined the value of critical temperature as $T_d = 2.011 J/k_B$, which is the value of temperature where the

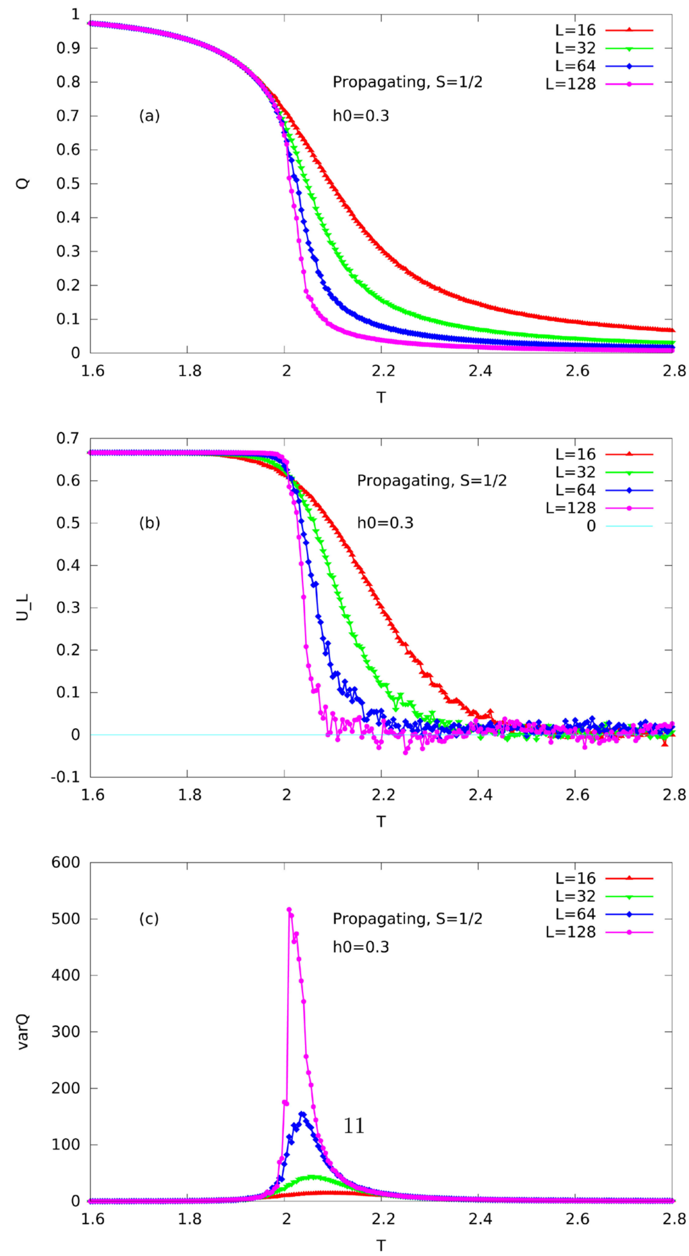

Figure 2. Temperature variation of different quantities for different values of linear system size L: (a) Order parameter Q; (b) Binder cumulant U_L; (c) scaled variance of order parameter $varQ$ or susceptibility χ_L^Q.

Binder cumulants for different lattice sizes have a common intersection. Now it is known from the behaviour of the kinetic Ising model that the scaled variance of the dynamical order parameter may be regarded as the susceptibility of the system, which can be defined as follows:

$$\chi_L^Q = L^2\left(\left\langle Q^2\right\rangle - \left\langle Q\right\rangle^2\right). \tag{6}$$

Figure 2(c) shows the variation of the scaled variance with the temperature. As we see from the figure that the susceptibility gets peaked near the dynamical transition temperature showing the tendency of divergence near T_d, as the system size increases. Now we adopt the *finite-size scaling* analysis to determine the critical exponents for the two dimensional kinetic Ising ferromagnet driven by magnetic wave. For this reason we use the usual technique of expressing the measured quantities as a function of the system size. We assume the following scaling forms for the order parameter Q and susceptibility χ^Q at the critical temperature:

$$\langle Q \rangle_L \propto L^{-\beta/\nu} \tag{7}$$

$$\chi_L^Q \propto L^{\gamma/\nu}. \tag{8}$$

It has to be noted here that though we do not have any value measured at the critical temperature which has been determined (as common intersection) from the Binder cumulant versus temperature curves for different L, the values of Q & χ^Q have been read out from the respective graphs which represent the average values at any temperature. Moreover, the detailed investigations done previously [8], show that the above scaling forms are also applicable to classify the universality classes of the driven magnetic systems. **Figure 3(a)** shows the log-log plot of the dynamic order parameter $\langle Q \rangle_L$ as a function of the linear system size L at the dynamic transition temperature. The value of the critical exponent, as estimated from this simulational study, is $\beta/\nu = 0.146 \pm 0.025$ for the dynamic order parameter. From the log-log plot **Figure 3(b)**. of the susceptibility χ_L^Q or the scaled variance of the order parameter χ_L^Q as a function of linear system size L we obtained the estimate of the value of the critical exponent γ/ν. The values are $\gamma/\nu = 1.869 \pm 0.135$ (using the data obtained at $T_d = 2.011 J/k_B$ from the respective graphs) and $\gamma/\nu = 1.746 \pm 0.017$ (using the data obtained at the peak position of susceptibility). It is interesting to note that these estimated values of the critical exponents, for the two dimensional driven Ising ferromagnet, are very close to those of the two dimensional equilibrium Ising ferromagnet, which are $\beta/\nu = 1/8 = 0.125$ and $\gamma/\nu = 7/4 = 1.75$ [20].

4. Summary

In this study, we have mainly focused our attention on the finite size analysis and the critical aspects of the dynamic phase transition near the dynamic transition temperature of an $L \times L$ square type Ising ferromagnet driven by propagating magnetic wave. We have taken four different sizes of square lattice (L = 16, 32, 64 and 128). We have simulated the results using Monte Carlo methods using the Metropolis single spin flip algorithm with parallel updating rules. Our findings suggest that, within the limits of statistical errors obtained in this study, the estimated values of the critical exponents near the dynamic transition temperature are very close to those for the two-dimensional equilibrium Ising ferromagnet. As concluding remarks, we state that the nonequilibrium

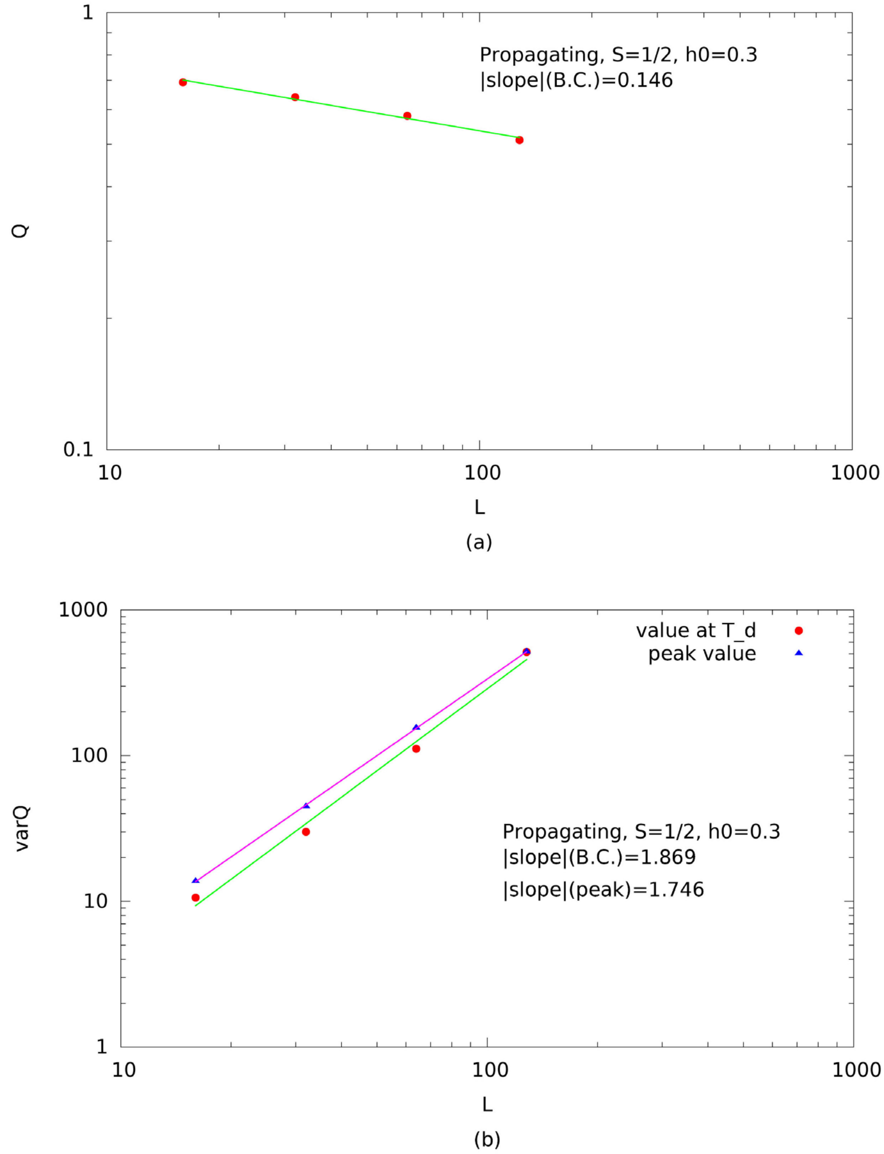

Figure 3. Log-log plot of (a) order parameter Q and (b) scaled variance *varQ* or susceptibility χ_L^Q as a function of linear system size L. In (b) red dots represent the value of susceptibility at T_d whereas blue triangles represent the same at peak positions.

phase transition, observed in the two-dimensional Ising ferromagnet driven by magnetic field wave, belongs to the same universality class of equilibrium two-dimensional Ising equilibrium ferro-para phase transition. Recently, the nonequilibrium phase transition in the kinetic Ising model via the violation of principle of detailed balance was studied (Manoj Kumar and ChandanDasgupta, IISc, Bangalore) and estimated the exponents in close agreement with the present observations.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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