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Prediction of Symmetrical and Asymmetrical of Diurnal Global Solar Irradiance Distribution—New Approach

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Abstract

A simple formula to predict the received global solar irradiance $q(t)$, W/m^2 for clear days is suggested on pure theoretical basis. It is expressed in terms of the length of the local day time t_d which is well defined in literatures on meteorological basis. The introduced distribution is also a function of the maximum value of the daily received irradiance $q_{max.}$ which in turn is expressed in term of the solar constant. This renders the trial to be a closed system. Thus the obtained distribution is not a semi empirical one. Both cases of symmetrical and asymmetrical distributions for $q(t)$ are considered. For its simplicity it can be easily integrated along the length of the day to get the daily totals of solar energy received by unit horizontal area. This is important for practical applications. Comparison between computed according to the present model and published experimental meteorological data in Barcelona (Spain), Hong Kong (China), Jeddah and Makkah (Saudi Arabia) is given as illustrative examples. Better fitting relative to the published trials for the same locations are obtained. The introduced model itself gives good fitting for the intermediate intervals points of the local day time which is the more effective region. The estimated relative error is 12% for Hong Kong, and it is 7% for Barcelona, Jeddah and Makah.

Keywords

Global Solar Irradiance, Symmetrical and Asymmetrical Distributions, Prediction Formula, Solar Constant, Comparative Study

1. Introduction

The prediction of the diurnal global solar radiation $q(t)$ W/m^2 is needed as one important input parameter to study theoretically the design and performance estimation of solar systems for solar energy exploitation, for example, the performance and efficiency of a solar cell, flat plate collector, water heating and treatment, pool heating, space heating, solar cookers, (Heating, Ventilation and air conditioning) (HVAC) technological systems. It is also required to study the production of electricity using molten salt technologies in which the liquid salt is pumped through panels in a solar collector for further heating.

Analysis of solar radiation measurements has aroused the interest of many investigators. As an example, different distributions for solar irradiance with different fitting degrees are given [1]-[6]. A lot of experimental meteorological data for many locations are published [7]-[14]. Trials to introduce governing formulae are given [5] [10] [11] [15].

The received solar energy is a function of several variables [2] such as the nature and extent of cloud cover, the aerosol and other atmospheric constituents such as O_2, N_2, CO_2, O_3, dust etc.

Such a function depends also on other parameters such as the sunshine hours, the solar declination angle, the latitude, the altitude and the relative humidity [2]. As a result of these challenges, it is not always possible to predict theoretically the actual shape of such a function to get accurate values of the received irradiance for a given location. Different trials are given by different authors with different degrees of fitting accuracy [1] [2] [3] [7] [16] [17] [18]. Most of such trials are either semi empirical or incomplete to form a closed system or they are difficult to be integrated.

The need for more accurate trials with better fitting degrees is still required. El-Adawi *et al.* [2] introduce a power expression for such a function, the parameters of which were determined through the least fitting technique. The given expression is not easy to be integrated.

Good fitting with published experimental meteorological data is obtained with maximum relative error 11%. Other trials expressed the required distribution in the form of polynomial [in($t - t_{max}$)] [16] with relative maximum error 16%, or in $\left(\frac{t-t_{max}}{t_{max}}\right)^2$ [18] with maximum relative error 15% or as polynomial in (t/t_d) with a correction factor $\left[\sin\left(\pi t/t_d\right)\right]$ [4] with maximum relative error 15%.

The present trial represents a new approach to introduce a suggested formula based on well-established solar data such as the length of the solar day "t_d" in hours, which is well defined in [15], and is also expressed through the maximum value of the daily solar irradiance q_{max} W/m^2. The expression for (t_d) is well defined in literatures on meteorological basis [15].

To get a closed system, the value of q_{max} is suggested in terms of the extraterrestrial solar constant adjusted for the variation of the distance between the sun and the earth along the time of year [2] [3]. Thus the introduced distribution is not a semi-empirical one. This is an advantage of the present trial. Moreover, it can be easily integrated and thus it is feasible for practical applications.

A comparative study between the experimental meteorological published data of the received global solar irradiance in different locations [7] [9] [12] [13] and that computed using the present suggested model is given. The relative errors are indicated.

2. Theory

The experimental measured meteorological values of the global solar irradiance $q(t)$, W/m^2 received on a horizontal surface as measured by different authors [1] [2] [7] revealed a symmetrical distribution about a maximum average value q_{max} acquired at midday time (t_{max} between sunrise t_r and sunset (t_s) *i.e.* $t_{\max} = \frac{t_s - t_r}{2} = \frac{t_d}{2}$.

This symmetrical behavior is shown to be true for the whole solar year [7].

Moreover, the behavior of this function for different locations reveals its universal character [1] [2] [8] [9] [17] [19].

However, some authors [4] discussed the case of asymmetrical distribution for which q_{max} occurs at "t_{max}" shifted from the midday times

$$i.e.\quad t_{\max} \neq \frac{t_d}{2}$$

This case will be considered in the present trial.

In the present trial the suggested model to predict the function $q(t)$ W/m^2 is given in the form:

$$q(t) = a_0 \left(\frac{t}{t_{\max}}\right)^2 \left(\frac{t_d - t}{t_d - t_{\max}}\right)^m \tag{1}$$

Shifted time scale is considered for which $t_r = 0$. This distribution satisfies the following conditions:

i) At $\quad t = t_r = 0 \quad q(t_r) = 0 \quad$ (2)

ii) At $\quad t = t_d \quad q(t_d) = 0 \quad$ (3)

iii) At $\quad t = t_{\max} \quad q(t) = q_{\max} \quad$ (4)

This gives: $\quad a_0 = q_{\max} \quad$ (5)

iv) At $\quad t = t_{\max} \quad \left.\frac{\partial q(t)}{\partial t}\right|_{t=t_{\max}} = 0 \quad$ (6)

This gives:

$$m = 2\left(\frac{t_d - t_{\max}}{t_{\max}}\right) \tag{7}$$

For symmetrical distribution, $t_{\max} = \frac{t_d}{2}$.

This gives:

$$m = 2 \tag{8}$$

Finally, one gets for symmetrical distribution the following expression:

$$q(t) = q_{\max} \left(\frac{t}{t_{\max}}\right)^2 \left(\frac{t_d - t}{t_d - t_{\max}}\right)^2 \tag{9}$$

For asymmetrical distribution:

$$q(t) = q_{\max}\left(\frac{t}{t_{\max}}\right)^2 \left(\frac{t_d - t}{t_d - t_{\max}}\right)^{\left(\frac{2(t_d - t_{\max})}{t_{\max}}\right)} \tag{10}$$

For symmetrical distribution the total daily solar energy received per unit area of a horizontal surface is given as:

$$\int_0^{t_d} q(t)\,\mathrm{d}t = \left(\frac{q_{\max}}{t_{\max}^2}\right)\frac{1}{t_d - t_{\max}}\int_0^{t_d} t^2 \left(t_d - t\right)^2 \mathrm{d}t = 0.533 q_{\max} t_d \tag{11}$$

Authors of different trials obtained for the same integral the following values:

$0.565 q_{\max} t_d$ [1]

$0.517 q_{\max} t_d$ [4]

$0.533 q_{\max} t_d$ [16]

$0.557 q_{\max} t_d$ [17]

While for asymmetrical distribution the obtained value is:

$0.4715 q_{\max} t_d$ [4].

This shows that the daily totals of the global solar irradiance on a horizontal surface depends on the degree of symmetry about the point $t = t_{\max}$, at which the received solar irradiance attains its maximum value [4]

It is worth to note that the length of the day "t_d" can be expressed in terms of the latitude L and the solar declination "δ" [15] as follows:

$$t_d = \frac{24h}{180^\circ}\cos^{-1}\left(\tan\delta \tan L\right) \tag{12}$$

where,

$$\delta = 23.45 \sin 360\left(\frac{284 + n}{365}\right) \tag{13}$$

and "n" is the day number of the year starting from 1 January *i.e.*, ($1 \le n \le 365$).

To get a closed system of equations, the physical quantity $q_{\max}$ is suggested on theoretical basis to be in the form [2] [3] [5] [10] [11]:

$$q_{\max} = \alpha \overline{s} \tag{14}$$

where, $\overline{s}$ is the extraterrestrial solar constant adjusted for the variation of the distance between the sun and the earth and along the time of the year [11]:

$$\overline{s} = s\left(1 + 0.033\cos\left(\frac{360 + n}{365}\right)\right) \tag{15}$$

And, $s = 1353\ \mathrm{W/m^2}$ [10] is the solar constant. It is worth to note that $q_{\max}$ is computed [3] according to Equation (14) for Jeddah and Makkah.

The estimated value $q_{\max}$ (Jeddah) = 856.8 W/m^2 while the experimental value is 915 W/m^2 and $q_{\max}$ (Makkah) = 878 W/m^2, while the experimental value is 938 W/m^2. The obtained relative error is 6% [3].

And "α" is a correction factor. It is well defined in the literatures [2] [10] Its value is given by the relation:

$$\alpha \le 1 \tag{16}$$

Its value is estimated [3] to be 0.65 and 0.63 for Jeddah and for Makkah respectively.

This value depends on the optical thickness, the reflectivity of the underlying terrain and depends also on the solar zenith angle [2] [3].

3. Computations

The function $q(t)$ is computed according to Equation (9) for different locations. The obtained results are compared with the corresponding meteorological published data as illustrative examples.

The measure of fitting is taken as $\varepsilon = \frac{q_{exp} - q_{cal}}{q_{exp}}$.

This step is summarized as follows:

1) The considered data for Barcelona (Spain) (41˚23'N, 2˚7'E) January 1973 [9] are given in **Table 1** and are illustrated graphically (as shown in **Figure 1**).

2) The data for Hong Kong (China) (22˚19'N, 114˚10'E) December 1978 [7] are given in **Table 2** and are illustrated graphically (as shown in **Figure 2**).

Table 1. Comparison between experimental [9] and computed values (Equation (9)) for the incident solar irradiance (W/m^2) for Barcelona (Spain) (Latitude 41˚23'N, Longitude 2˚7'E), (June 1973).

Local time, hr	Shifted time, hr	$q_{exp}(t)$, W/m^2	$q_{cal}(t)$, W/m^2	ε %	ε % [4]
4.00	0	0	0	0	0
5.5	1.5	75.8	81.84	7.9	7.56
6.50	2.5	198.3	197.06	0.63	11
7.50	3.5	338	331.13	2.03	8.87
8.50	4.5	471.5	463.30	1.74	5.85
9.50	5.5	576.1	576.97	0.15	2.07
10.50	6.5	684.7	659.66	3.7	3.82
11.50	7.5	707.8	703.08	0.67	0.56
12	8	710	710	0	0
12.50	8.5	700.8	703.08	0.33	0.42
13.50	9.5	661.8	659.66	0.32	0.96
14.50	10.5	580.3	576.97	0.57	2.68
15.50	11.5	456.6	463.30	1.47	2.88
16.50	12.5	324.4	331.13	2.38	5.20
17.50	13.5	184.1	197.06	7.03	4.32
18.5	14.5	60.7	81.84	34.82	14.95
20.00	16	0	0	0	0

* t_{max} = 8 hr, t_d = 16 hr, q_{max} = 710 W/m^2.

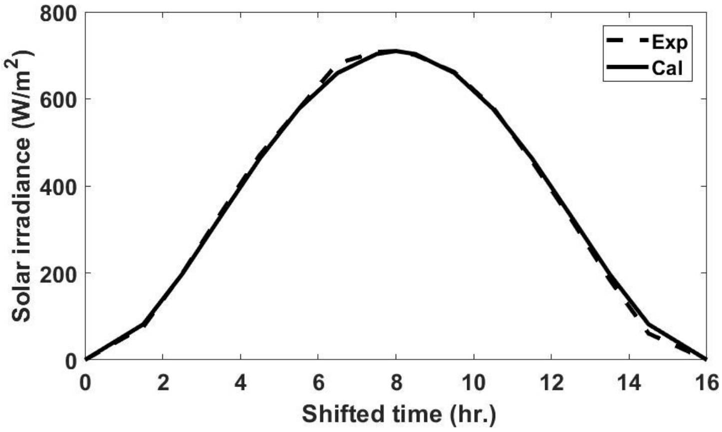

Figure 1. Comparison between experimental [9] and computed values (Equation (9)) for the incident solar irradiance (W/m^2) for Barcelona (Spain).

Table 2. Comparison between experimental [7] and computed values (Equation (9)) for solar irradiance (W/m^2) Hong Kong. China. December (1978) [22˚19'N, 114˚10'E].

t, h Shifled	q_{exp}, W/m^2	q_{cal} (Equation (9)) W/m^2	∈ %	∈ [17] %
0	0	--	--	--
1	100.00	64.97	35.00	25.0
3	422.26	374.22	11.40	11.0
4	536.15	509.35	4.90	10.4
5	590.50	584.71	1	3.0
5.5	594.50	594.5	0.00	0.0
6	583.38	584.71	0.23	3.0
7	541.71	509.35	6	11.3
8	425.03	374.22	12.0	12.0
9	277.80	210.50	24.2	8.0
10	122.23	64.97	47	2.0
11	0	0	0	0

$t_r = 6.5$, $t_s = 17.5$, $t_{\max} = 12$, $t_d = 11$, $q_{\max} = 594.5$ W/m^2.

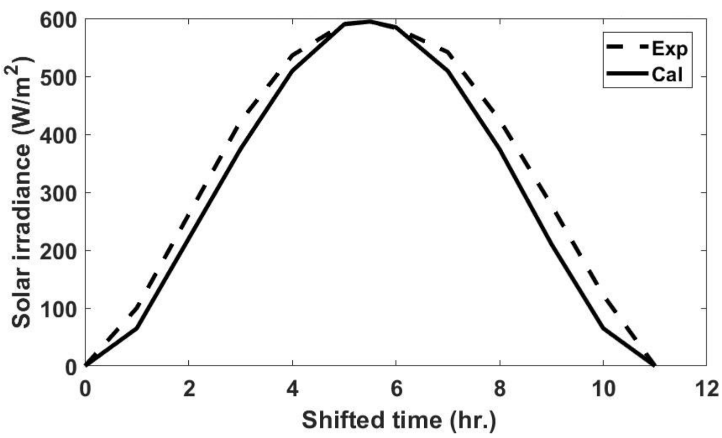

Figure 2. Comparison between experimental [7] and computed values (Equation (9)) for solar irradiance (W/m^2) Hong Kong (China).

3) The data for Jeddah (Saudi Arabia)

(21˚37'N 40˚25'E) April 1982 [12] are given in **Table 3** and are illustrated graphically (as shown in **Figure 3**).

4) The data for Makkah (Saudi Arabia) (38.5˚E, 21.5˚N) March 1983 [13] are given in **Table 4** and are illustrated graphically (as shown in **Figure 4**).

The relative errors obtained according to our model are compared with the corresponding published meteorological values obtained for the same locations and at the same local day time as shown in the corresponding tables.

It is revealed that our model is promising and gives better fitting relative to some other trials as [4] [17] irrespective of the extreme points. The model itself gives good fitting for the intermediate points. This is the more effective one. The

Table 3. Comparison between experimental [12] and computed values (Equation (9)) for solar irradiance (W/m^2) Jeddah Located at [21˚37'N, 40˚25'E] (April 1982).

Local time t, h	Shifted time t, h	q_{exp} [16] W/m^2	q_{cal} (Equation (9)) W/m^2	∈ %	∈ [17] %
6.08	0	0	0	0.00	0.00
8.25	2.17	325.0	298.29	8.20	15.00
9.25	3.17	535.0	520.13	2.80	0.50
10.25	4.17	715.0	718.92	0.55	4.20
11.25	5.17	850.0	857.92	0.93	4.60
12.25	6.17	910.0	914.33	0.48	0.50
12.37	6.29	915.0	915.00	0.00	0.00
13.25	7.17	887.5	879.53	0.89	5.70
14.25	8.17	802.5	758.82	5.44	10.58
15.25	9.17	640.0	571.56	10.69	10.00
16.25	10.17	3760.8	351.15	6.80	10.70
17.25	11.17	148.3	145.00	2.2	67.30
18.66	12.58	0	0	0	0.00

[t_r = 6.8, t_s = 18.66, t_{max} = 12.37, t_d = 12.58, q_{max} = 915 W/m^2].

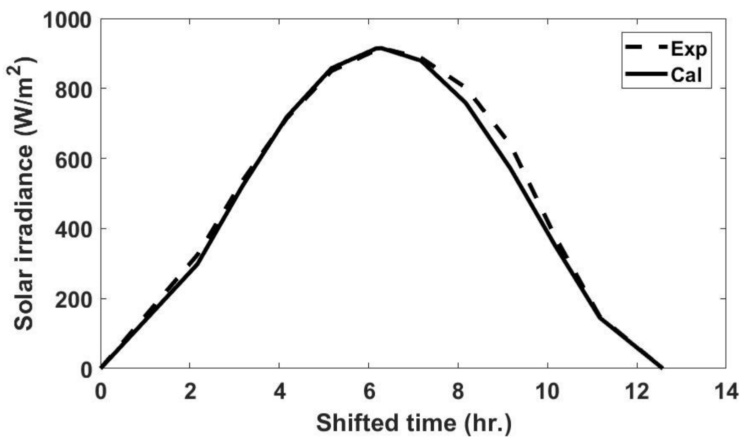

Figure 3. Comparison between experimental [12] and computed values (Equation (9)) for solar irradiance (W/m^2) Jeddah (Saudi Arabia).

Table 4. Comparison between experimental [13] and computed values (Equation (9)) for the incident solar irradiance (W/m^2) for Makkah (38.5˚E, 21.5˚N), (March 1983).

Local time, hr	Shifted time, hr	$q_{exp}(t)$, W/m^2	$q_{cal.}(t)$, W/m^2	ε %	ε% [17]
6.00	0	0	0	0	0
6.50	0.5	15	20	27.00	-
7.50	1.5	168	156.37	6.90	12.4
8.50	2.5	393	362.10	7.80	5.0
9.50	3.5	600	581.00	3.20	8.7
10.50	4.5	767	768.84	0.24	8.0
11.50	5.5	890	894.17	0.47	5.7
12.50	6.5	938	938	0	0
13.50	7.5	902	894.17	0.87	7.0
14.50	8.5	760	768.84	1.10	7.10
15.50	9.5	586	581.00	0.85	6.50
16.50	10.5	367	362.10	1.34	1.70
17.50	11.5	133	156.37	17.57	42.00
19.00	13.0	0	0	0	0

t_{max} = 6.5 hr, t_d = 13 hr, q_{max} = 938.

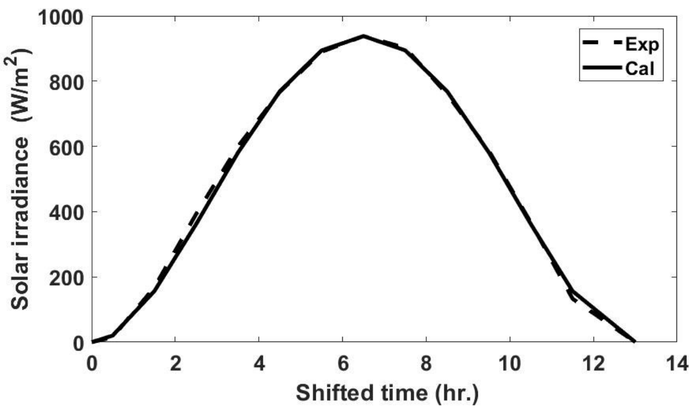

Figure 4. Comparison between experimental [13] and computed values (Equation (9)) for the incident solar irradiance (W/m^2) for Makkah (Saudi Arabia).

obtained relative errors are: 12% for Hong Kong and 7% for Barcelona, Jeddah and Makkah respectively.

4. Conclusions

1) The introduced trial to predict the daily global solar irradiance for clear days is promising. It gives good fitting $(\cong 12\%)$ when compared with the corresponding measured data.

2) The introduced formula can easily be integrated along the local day time to get the total energy received per day per unit area.

This is of vital importance for technological applications.

3) The symmetrical and asymmetrical distributions are considered.

4) The given distribution is expressed in terms of a well-defined parameter which is the length of the solar day t_d [15]. It is also expressed in term of the maximum value of the solar irradiance $q_{\max}$ attained during the considered day.

5) The latter is suggested in the present study to be expressed in terms of a modified solar constant. Thus the formula is based totally on pure theoretical arguments.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Nomenclatures

t, Time variable (hr.).

t_r, sunrise time (hr.).

t_s, sunset time (hr.).

$t_d = (t_s - t_r)$, The length of the day (hr.).

$t_{\max}$, The midtime between sunrise and sunset.

$q(t)$, W/m^2 Solar irradiance.

L, latitude.

Greek Symbols

δ , Solar declination (defined in the text).