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Economics of Decision Making: Exploring Analytical Hierarchical Process (AHP)

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Abstract

Decision making in any field, like all other economic issues, involves allocating scarce resources to meet various needs. Since ages decision making has always intrigued the mankind. A host of research study has been conducted in past few decades on economics of decision making. A number of very effective decision tools have been suggested which falls under the category of multiple criteria decision making. The paper presents one such decision making tool called Analytical Hierarchical Process (AHP), a technique for complex decision making used for large-scale, multi-party, multi-criteria decision analysis. AHP converts comparative evaluations to numerical values that can be processed and weighed over the entire range of the problem. A numerical weight or priority is derived for each element of the hierarchy, allowing diverse and often incommensurable elements to be compared to one another in a rational and consistent way. The objectives of the article are to understand the interrelationship between considered criteria and available alternatives, to grasp the basics of decision making and decision analysis and to use decision making and decision analysis in assessing the scope for cost-reduction (economics of decision making). The outcome of the study is a mathematical solution to the perennial subjective decision making process in the form of a structured methodology culled out from varied disciplines of economics, psychology and mathematics. The applicability of the AHP Model is demonstrated with an illustrative real life example.

Keywords

Decision Making, Multi Criteria, Analytical Hierarchical Process, Pairwise Comparison, Eigen Vectors, Economics, Cost Benefit Analysis

1. Introduction

Analytic hierarchy process (AHP) was developed by Thomas L. Saaty in the

1970s and has been extensively studied, analyzed and refined since its inception [1]. It is a structured technique for organizing and analyzing complex decisions, based on mathematics and psychology [2]. This technique facilitates a logical and comprehensive structured framework for framing a decision dilemma, for representing and quantifying its criteria, for relating those criteria into overall objectives, and for evaluating alternative options and solutions [3].

AHP has particular application in group decision making [4] and is used around the world in a wide variety of decision situations, in fields such as government, business, industry, healthcare, shipbuilding and education [5]. Rather than prescribing a *correct* decision, the AHP helps decision makers find one that best suits their goal and their understanding of the problem [6].

The first step in the analytic hierarchy process is to model the problem as a hierarchy. A hierarchy is a stratified system of ranking and organizing people, things, ideas, etc., where each element of the system, except for the top one, is subordinate to one or more other elements. Though the concept of hierarchy is easily grasped intuitively, it can also be described mathematically [7]. The process of AHP begins by converting an existing decision dilemma into a hierarchy of easily deciphered and understood sub-problems, each of which can then be analyzed independently. The entities in the hierarchy can depict any aspect of the decision situation—it can be implicit or explicit, exactly monitored or approximated guess, thoroughly or partially represented, completely or poorly understood—anything at all that fits into the dilemma on hand.

Once the hierarchy is ready, the users then evaluate its different entities by comparing them to one another two at a time, as to what is the impact of this entity on the entity in higher up hierarchy. It is the essence of the AHP that human judgments, and not just the underlying information, can be used in performing the evaluations [8]. In doing this, participants explore the aspects of the problem at levels from general to detailed, then express it in the multileveled way that the AHP requires. As they work to build the hierarchy, they increase their understanding of the problem, of its context, and of each other's thoughts and feelings about both [9].

2. Literature Review of AHP Use and Application in Recent Times (Post 2000)

A large number of research papers and articles were studied and analyzed specifically in knowing as to how AHP as decision making tool have been used and applied in recent times. As can be seen the literature review focuses on the latest work and application of AHP especially post 2000.

Walailak & McCarthy (2002) propounded an application of the analytical hierarchy process to international location decision-making [10]. Dey & Prasanta Kumar (2003) discussed about how analytic hierarchy process analyzes risk of operating cross-country petroleum pipelines in India [11]. Yasemin (2006) carried out a case study in Turkey for determining key capabilities in technology management using fuzzy analytic hierarchy process [12].

Management Science (2008) reviewed 15 years—1990 to 2005—of progress through various research papers and study in all areas of multi-criteria decision making showed that AHP publications have far outnumbered those in any other area, characterizing their growth as "enormous" [13]. In 2008, the major society for operations research, the Institute for Operations Research and the Management Sciences formally recognized AHP's broad impact on its fields [14]. Wallenius, Dyer, Fishburn, Steuer, Zionts & Deb (2008) discussed recent accomplishments in the field of multiple criteria decision making and multi attribute utility theory and also focussed on what lies ahead [15]. Ammar, Hafsa & Ouni (2011) utilized analytic hierarchical process for multi-criteria decision making in design of flying voltage source multi-level inverters [16]. Hwang & Yoon (2012) carried out an extensive survey about various methods and application of multiple attribute decision making esp. AHP [17]. Locatelli & Mancini (2012) used an AHP framework for the selection of the right nuclear power plant [18]. Dymova, Sevastjanov & Tikhonenko (2013) attempted a direct interval extension of TOPSIS method [19]. Triantaphyllou (2013) carried out an elaborate comparative study on multi-criteria decision making methods [20]. Saracoglu (2013) utilized fuzzy AHP in selecting industrial investment locations in master plans of different countries in Eurasia region [21].

Liu, You, Fan & Lin (2014) used AHP for failure mode and effects analysis using D-Numbers and grey relational projection method [22]. Chaudhury, Mandal & Das (2015) used AHP for selection of appropriate fluid delivery technique for grinding titanium Grade-1 [23]. Salavati, Haghshenas, Ghadirifaraz, Laghaei & Eftekhari (2016) carried out a case study of Isfahan city by applying AHP and clustering approaches for public transportation decision making [24]. Li, Hu, Zhang & Deng (2016) used AHP for novel distance function of D-Numbers and its application in product engineering [25]. Mallick, Sarkar & Das (2017) proposed a unified decision framework for inventory classification through graph theory using AHP [26]. Munasinghe, Hemakumara & Mahanama (2017) used AHP in GIS application for finding the best residential lands in Ratnapura municipal council area of Sri Lanka [27]. Zong & Wang (2017) followed a D-AHP approach in their seminal work for evaluation of university scientific research ability based on the output of science and technology papers [28].

3. AHP—Methodology and Application for Selection of a Mobile Phone

The intent of the study is to introduce and elaborate the procedure of analytic hierarchy process based multi criteria decision making process. The following steps discuss the methodology and its implementation as applied to a certain product (mobile phone), thus elucidating the important issues and aspects in building up AHP Model.

The first step is to structure the situation (problem) as a hierarchy containing the decision objective, the option available for achieving the objective, and the grounds on which the alternatives are evaluated. The next step is to set up and create the prioritization levels among the various criteria in the hierarchy by making a series of calculations incumbent (based) on pair wise comparison of the criteria. Once, pair-wise comparison is over, it's time to synthesize and integrate these conclusions in order to arrive at a set of holistic priority for the hierarchy. This will translate the buyer's decision for all the products under consideration into overall priorities for each individual product. The last step is to have a look at various values and results generated through the process to arrive at a final decision.

The objective of the study: To select the best mobile phone on the basis of chosen parameters (out of the available alternatives). The models of the mobile phones were chosen with certain minimum common features and if they fulfilled all these aspects only then they were considered for study:

- Cost of the mobile: between INR 25,000 to 45,000 (INR—India Rupees: $1 USA = INR 70);
- Operating system—Android;
- 4G;
- Full HD display;
- Dual SIM card;
- GSM + GSM; and
- Minimum 64 GB.

The final four mobile phones were chosen from four different brands/companies, and fulfilled and met all the above expectations. Four alternatives are referred to as: Brand A, Brand B, Brand C and Brand D in the body of the paper (due to confidentiality clause the brand names of the mobile phone could not be divulged and also the intention of the article is not to choose the brand *per se* but to explain the workings and methods of the decision tool - AHP).

Criterion for selection: (five)

a) Qualitative: Performance and Specifications, Applications and Functions, Features and Style Quotient

b) Quantitative: Camera power in Megapixels and Cost of the mobile

Pair wise comparisons of all criteria—one against the other. In order to bring in their judgments about various criteria in the hierarchy, decision makers compare the criteria in a pair wise manner. The need was to decide which one of the criteria was more important than the others in selecting the mobile phone model/brand. The decision had to be taken and priorities had to be set, as to which criterion was more important for the decision maker in achieving its objective, and how much more important it was than the other criteria.

Scale of pair-wise comparison: 1—3—5—7—9

- 1—Equal importance—two element contribute equally to the objective
- 3—Moderate importance—experience and judgment slightly favor one element over the other
- 5—Strong importance—experience and judgment strongly favor one element

over the other

- 7—Very strong importance—one element is favored strongly over the other; its dominance is demonstrated in practice
- 9—Extreme importance—evidence favoring one element over the other is of the highest possible order of affirmation

4. AHP—Implemented Model for Selection of a Mobile Phone

In order to take a final decision regarding the selection of mobile brand/model, various criteria and their rankings of different models/brands in reckoning are considered and a final judgment is achieved through Analytical Hierarchy Process. To bring in judgments about various criteria in the hierarchy, decision makers compare the criteria in a pair wise manner, this is the first step in AHP. The decision has to be taken and priorities have to be set as to which criterion is more important in achieving the objective, and how much more important it is than the other criteria. The next step is to convert these judgments into priorities for each of these criteria. The detailed procedure and computations are show through various tables, and a detailed explanation is given below:

The first Table is the data available to us through the open source/market regarding the cost of each mobile phone and the camera's power/size in megapixels, and is shown in Table 1.

The very first step in AHP Process is to the comparison of the criteria considered for the study. This is carried out through a subjective assessment of importance of one criteria over the other referred to as 'pair-wise comparison' and the result of which is given in Table 2. Further we convert these values in fractional form to decimal form for the sake of ease and calculations. The result of the same is given in Table 2(a). The next step in AHP is to square the matrix of decimal values (*i.e.* do matrix multiplication of Table 2(b) and Table 2(b)). The aim here is to calculate the first Eigen Vectors by summing up the rows and normalizing each row value by dividing it by the sum total of the column, we keep using a number of iterations for this step until we don't see much of a difference between the Eigen vectors. In our case, we use three more iteration to get the accuracy needed. The values of Eigen vectors tell us the relative ranking of the criteria in terms of their importance. Refer to Tables 2(c)-(e). Finally after three iterations we got the values of our Eigen Vectors. In this case, the most important criterion is features and style quotient followed by applications and functions, performance and specifications and then camera power in megapixels. Refer Table 2(f).

On the similar lines, the next step in AHP is to focus on all the qualitative criteria under considerations for our study and make a pair-wise matrix for all these criteria—how different brands rate in comparison to one another in terms of fulfilling these criteria. The result obtained is matrices with pair-wise values for different brands/models of mobile phones under study, and they are shown in Table 3, Table 4 and Table 5.

Table 1. Data available for the Mobile Phones considered for the purpose of study. (Source: secondary sources—local market/internet).

BRANDS	CAMERA POWERIN MEGAPIXELS	COST IN INR (INDIA RUPEES)
BRAND A	16	28,700
BRAND B	12	36,900
BRAND C	14	38,900
BRAND D	20	42,000

Table 2. Pair-wise comparison of Criteria considered for the purpose of study. (a) Converting Table 2 Values—Fraction to Decimals; (b) Square of the above matrix; (c) First Iteration for computing row sums and normalized values; (d) Second Iteration for computing row sums and normalized values; (e) Third and Final Iteration—Eigen Vectors representing Rankings of the Criteria; (f) Eigen Vector Values representing Rankings of the Criteria considered for Mobile Phone.

CRITERIA	PERFORMANCE & SPECIFICATIONS	APPLICATIONS & FUNCTIONS	FEATURES & STYLE QUOTIENT	CAMERA (MEGAPIXELS)
PERFORMANCE & SPECIFICATIONS	1	3/1	1/5	7/1
APPLICATIONS & FUNCTIONS	1/3	1	3/1	5/1
FEATURES & STYLE QUOTIENT	5/1	1/3	1	7/1
CAMERA (MEGAPIXELS)	1/7	1/5	1/7	1

(a)

CRITERIA	PERFORMANCE & SPECIFICATIONS	APPLICATIONS & FUNCTIONS	FEATURES & STYLE QUOTIENT	CAMERA (MEGAPIXELS)
PERFORMANCE & SPECIFICATIONS	1.0000	3.0000	0.2000	7.0000
APPLICATIONS & FUNCTIONS	0.3333	1.0000	3.0000	5.0000
FEATURES & STYLE QUOTIENT	5.0000	0.3333	1.0000	7.0000
CAMERA (MEGAPIXELS)	0.1429	0.2000	0.1429	1.0000

(b)

CRITERIA	PERFORMANCE & SPECIFICATIONS	APPLICATIONS & FUNCTIONS	FEATURES & STYLE QUOTIENT	CAMERA (MEGAPIXELS)
PERFORMANCE & SPECIFICATIONS	4.0000	7.4667	10.4000	30.4000
APPLICATIONS & FUNCTIONS	16.3810	4.0000	6.7810	33.3333
FEATURES & STYLE QUOTIENT	11.1111	17.0667	4.0000	50.6667
CAMERA (MEGAPIXELS)	1.0667	0.8762	0.9143	4.0000

(c)

CRITERIA	PERFORMANCE & SPECIFICATIONS	APPLICATIONS & FUNCTIONS	FEATURES & STYLE QUOTIENT	CAMERA (MEGAPIXELS)	ROW SUM	NORMAL
PERFORMANCE & SPECIFICATIONS	4.0000	7.4667	10.4000	30.4000	52.2667	0.2582
APPLICATIONS & FUNCTIONS	16.3810	4.0000	6.7810	33.3333	60.4952	0.2988
FEATURES & STYLE QUOTIENT	11.1111	17.0667	4.0000	50.6667	82.8444	0.4092
CAMERA (MEGAPIXELS)	1.06670	0.87620	0.9143	4.0000	6.8571	0.0339
					202.4635	1.0000

(d)

CRITERIA	PERFORMANCE & SPECIFICATIONS	APPLICATIONS & FUNCTIONS	FEATURES& STYLE QUOTIENT	CAMERA (MEGAPIXELS)	ROW SUM	NORMAL
PERFORMANCE & SPECIFICATIONS	286.2933	263.8629	161.6254	1019.0222	1730.8038	0.2827
APPLICATIONS & FUNCTIONS	241.9471	283.2457	255.0857	1108.2159	1888.4944	0.3084
FEATURES & STYLE QUOTIENT	422.5016	263.8899	293.6076	1312.0000	2291.9992	0.3743
CAMERA (MEGAPIXELS)	33.04490	30.57780	24.34900	123.95680	211.9285	0.0346
					6123.2259	1.0000

(e)

CRITERIA	PERFORMANCE & SPECIFICATIONS	APPLICATIONS & FUNCTIONS	FEATURES & STYLE QUOTIENT	CAMERA (MEGAPIXELS)	ROW SUM	NORMAL
PERFORMANCE & SPECIFICATIONS	346,109.9921	293,955.49	281,055.7545	1,228,063.9	2,149,185.12	0.2743
APPLICATIONS & FUNCTIONS	387,308.9272	317,331.85	314,891.1939	1,351,988.5	2,371,520.48	0.3027
FEATURES & STYLE QUOTIENT	486,107.8519	397,347.95	381,309.8408	1,681,453.9	2,946,219.52	0.3761
CAMERA (MEGAPIXELS)	59,936.18319	49,451.839	48,501.07873	209,668.77	367,557.86	0.0469
					6,123.2259	1.0000

(f)

CRITERIA	FINAL EIGEN VECTOR VALUES	RANKS
PERFORMANCE & SPECIFICATIONS	**0.2743**	**#3**
APPLICATIONS & FUNCTIONS	**0.3027**	**#2**
FEATURES & STYLE QUOTIENT	**0.3761**	**#1**
CAMERA POWER (MEGAPIXELS)	**0.0469**	**#4**

Repeating the steps which get mentioned above to find the relative rankings (Eigen Vectors) for each of these criteria. First we convert all these fraction tables into decimal tables—refer Table 3(a), Table 4(a) and Table 5(a). These tables/matrices helps us in computing different Eigen vectors which determines the relative rankings of mobile phones alternatives (brands/models) under each criterion, viz. considering performance and specifications, applications and functions, & features and style quotient.

For Performance and Specifications: [Tables 3(a)-(f)]

From Series of Table 3, we can conclude that as far as performance and specifications as a criterion is concerned, the models/brands can be ranked as 1) Brand D, followed by 2) Brand B, 3) Brand A and 4) Brand C.

For Applications and Functions: [Tables 4(a)-(f)]

From Series of Table 4, we can conclude that as far as applications and functions as a criterion is concerned, the models/brands can be ranked as—1) Brand A, followed by 2) Brand B, 3) Brand D and 4) Brand C.

Table 3. Pair-wise comparison for Criteria—Performance and Specifications for all Brands considered. (a) Converting **Table 3** Values from Fraction to Decimals; (b) Square of the above matrix; (c) First Iteration for computing row sums and normalized values; (d) Second Iteration for computing row sums and normalized values; (e) Third &Final Iteration—Rankings of the Brands on the criterion of Performance and Specifications; (f) Eigen Vector Values representing Rankings of the Mobile Phone Brands for Performance and Specifications.

P & S	BRAND A	BRAND B	BRAND C	BRAND D
BRAND A	1/1	1/3	5/1	1/9
BRAND B	3/1	1/1	5/1	1/7
BRAND C	1/5	1/5	1/1	1/7
BRAND D	9/1	7/1	7/1	1/1

(a)

P & S	BRAND A	BRAND B	BRAND C	BRAND D
BRAND A	1.0000	0.3333	5.0000	0.1111
BRAND B	3.0000	1.0000	5.0000	0.1429
BRAND C	0.2000	0.2000	1.0000	0.1429
BRAND D	9.0000	7.0000	7.0000	1.0000

(b)

P & S	BRAND A	BRAND B	BRAND C	BRAND D
BRAND A	4.0000	2.4444	12.4444	0.9841
BRAND B	8.2857	4.0000	26.0000	1.3333
BRAND C	2.2857	1.4667	4.0000	0.3365
BRAND D	40.4000	18.4000	94.0000	4.0000

(c)

P & S	BRAND A	BRAND B	BRAND C	BRAND D	ROW SUM	NORMAL
BRAND A	4.0000	2.4444	12.4444	0.9841	19.8730	0.0886
BRAND B	8.2857	4.0000	26.0000	1.3333	39.6190	0.1766
BRAND C	2.2857	1.4667	4.0000	0.3365	8.0889	0.0360
BRAND D	40.4000	18.4000	94.0000	4.0000	156.8000	0.6988
					224.3810	1.0000

(d)

P & S	BRAND A	BRAND B	BRAND C	BRAND D	ROW SUM	NORMAL
BRAND A	104.4571	55.9153	255.6190	15.3199	431.3115	0.1007
BRAND B	179.5810	98.9206	436.4444	27.5701	742.5161	0.1734
BRAND C	44.0330	23.5124	114.2095	6.8971	188.6520	0.0441
BRAND D	690.5143	383.8222	1733.1556	111.9238	2919.4159	0.6818
					4281.8954	1.0000

(e)

P & S	BRAND A	BRAND B	BRAND C	BRAND D	ROW SUM	NORMAL
BRAND A	42,786.933	23,262.28	106,851.13	6619.5487	179,519.88	0.0993
BRAND B	74,778.266	40,670.476	186707	11,574.343	313,730.08	0.1735
BRAND C	18,613.442	10,120.561	46,515.005	2882.474	78,131.48	0.0432
BRAND D	294,657.37	160,287.74	735,949.92	45,641.238	1,236,536.2	0.6840
					1,807,917.7	1.0000

(f)

PERFORMANCE & SPECIFICATION	FINAL EIGEN VECTOR VALUES
BRAND A	**0.0993**
BRAND B	**0.1735**
BRAND C	**0.0432**
BRAND D	**0.6840**

Table 4. Pair-wise comparison for Criteria - Applications and Functions for all Brands considered. (a) Converting Table 4 Values from Fraction to Decimals; (b) Square of the above matrix; (c) First Iteration for computing row sums and normalized values; (d) Second Iteration for computing row sums and normalized values; (e) Third &Final Iteration—Rankings of the Brands on the criterion of Applications and Functions; (f) Eigen Vector Values representing Rankings of the Mobile Phone Brands for Applications and Functions.

A & F	BRAND A	BRAND B	BRAND C	BRAND D
BRAND A	1/1	3/1	9/1	1/1
BRAND B	1/3	1/1	5/1	3/1
BRAND C	1/9	1/5	1/1	1/7
BRAND D	1/1	3/1	7/1	1/1

(a)

A & F	BRAND A	BRAND B	BRAND C	BRAND D
BRAND A	1.0000	3.0000	9.0000	1.0000
BRAND B	0.3333	1.0000	5.0000	3.0000
BRAND C	0.1111	0.2000	1.0000	0.1429
BRAND D	1.0000	0.3333	7.0000	1.0000

(b)

A & F	BRAND A	BRAND B	BRAND C	BRAND D
BRAND A	4.0000	8.1333	40.0000	12.2857
BRAND B	4.2222	4.0000	34.0000	7.0476
BRAND C	0.4317	0.7810	4.0000	0.9968
BRAND D	2.8889	5.0667	24.6667	4.0000

(c)

A & F	BRAND A	BRAND B	BRAND C	BRAND D	ROW SUM	NORMAL
BRAND A	4.0000	8.1333	40.0000	12.2857	64.4190	0.4116
BRAND B	4.2222	4.0000	34.0000	7.0476	49.2698	0.3148
BRAND C	0.4317	0.7810	4.0000	0.9968	6.2095	0.0397
BRAND D	2.8889	5.0667	24.6667	4.0000	36.6222	0.2340
					156.5206	1.0000

(d)

A & F	BRAND A	BRAND B	BRAND C	BRAND D	ROW SUM	NORMAL
BRAND A	103.1026	158.5524	899.5810	195.4794	1356.7153	0.4296
BRAND B	68.8169	112.6011	614.7302	142.1460	938.2942	0.2971
BRAND C	9.6310	14.8097	84.4106	18.7828	127.6341	0.0404
BRAND D	55.1534	83.2931	485.1556	111.7884	735.3905	0.2329
					3158.0341	1.0000

(e)

A & F	BRAND A	BRAND B	BRAND C	BRAND D	ROW SUM	NORMAL
BRAND A	40,986.5037	63,804.9783	360,988.158	81,440.968	547,220.608	0.4282
BRAND B	28,604.4005	44,533.8643	251,978.334	56,894.6883	382,011.287	0.2989
BRAND C	3861.04221	6009.18994	34,005.6015	7672.97372	51,548.8074	0.0403
BRAND D	22,256.5078	34,619.8296	196,004.784	44,230.3469	297,111.468	0.2325
					1,277,892.17	1.0000

(f)

APPLICATIONS AND FUNCTIONS	FINAL EIGEN VECTOR VALUES
BRAND A	**0.4282**
BRAND B	**0.2989**
BRAND C	**0.0403**
BRAND D	**0.2325**

Table 5. Pair-wise comparison for Criteria—Features and Style Quotient for all Brands considered. (a) Fraction to Decimals; (b) Square of the above matrix; (c) First Iteration for computing row sums and normalized values; (d) Second Iteration for computing row sums and normalized values; (e) Third & Final Iteration—Eigen Vectors representing Rankings of Brands for criterion of Features & Style Quotient; (f) Eigen Vector Values representing Rankings of the Mobile Phone Brands for Features & Style Quotient.

F & SQ	BRAND A	BRAND B	BRAND C	BRAND D
BRAND A	1/1	5/1	9/1	1/7
BRAND B	1/5	1/1	3/1	5/1
BRAND C	1/9	1/3	1/1	1/5
BRAND D	7/1	1/5	5/1	1/1

(a)

F & SQ	BRAND A	BRAND B	BRAND C	BRAND D
BRAND A	1.0000	5.0000	9.0000	0.1429
BRAND B	0.2000	1.0000	3.0000	5.0000
BRAND C	0.1111	0.3333	1.0000	0.2000
BRAND D	7.0000	0.2000	5.0000	1.0000

(b)

F & SQ	BRAND A	BRAND B	BRAND C	BRAND D
BRAND A	57.0000	14.5286	33.7143	28.2857
BRAND B	53.4000	4.5000	32.8000	11.0286
BRAND C	7.5778	1.4288	3.9998	2.2157
BRAND D	44.0400	37.9000	73.6000	4.6667

(c)

F & SQ	BRAND A	BRAND B	BRAND C	BRAND D	ROW SUM	NORMAL
BRAND A	57.0000	14.5286	33.7143	28.2857	133.5286	0.3251
BRAND B	53.4000	4.5000	32.8000	11.0286	101.7286	0.2477
BRAND C	7.5778	1.4288	3.9998	2.2157	15.2221	0.0371
BRAND D	44.0400	37.9000	73.6000	4.6667	160.2067	0.3901
					410.6859	1.0000

(d)

F & SQ	BRAND A	BRAND B	BRAND C	BRAND D	ROW SUM	NORMAL
BRAND A	5526.0073	2013.7067	4614.9304	1979.2160	14,133.8604	0.3803
BRAND B	4018.3488	1260.9232	2890.8392	1684.2275	9854.3387	0.2651
BRAND C	636.1194	206.2138	481.4177	249.3023	1573.0531	0.0423
BRAND D	5297.3831	1092.4146	3365.7491	1848.5394	11,604.0862	0.3122
					37,165.3385	1.0000

(e)

F & SQ	BRAND A	BRAND B	BRAND C	BRAND D	ROW SUM	NORMAL
BRAND A	80,299,811	45,625,717	99,543,529	27,506,557	252,975,614.6052	0.2872
BRAND B	60,348,798	34,256,372	74,774,605	20,690,113	190,069,887.8920	0.2158
BRAND C	53,901,661	30,955,178	67,576,225	18,356,552	170,789,616.0983	0.1939
BRAND D	84,512,950	48,255,728	105,344,296	28,851,403	266,964,377.4070	0.3031
					880,799,496.0025	1.0000

(f)

FEATURES & STYLE QUOTIENT	FINAL EIGEN VECTOR VALUES
BRAND A	**0.2872**
BRAND B	**0.2158**
BRAND C	**0.1939**
BRAND D	**0.3031**

For Features and Style Quotient: [Tables 5(a)-(f)]

From Series of Table 5, we can conclude that as far as features and style quotient as a criterion is concerned, the models/brands can be ranked as—1) Brand D, followed by 2) Brand A, 3) Brand B and 4) Brand C.

For Camera's Power in Megapixels:

This can be used directly as the quantitative data is available (Table 1). As mentioned earlier AHP can both use qualitative and quantitative data; in this case, we simply normalize the camera power information, which makes it simple to use the normalized data with other rankings.

From Table 6, we can conclude that as far as the camera power as a criterion is concerned—the models/brands can be ranked as—1) Brand D, followed by 2) Brand A, 3) Brand C and 4) Brand B. Now that we have all the values of rankings/Eigen vector values for all the brands under different criteria [Table 3(f), Table 4(f), Table 5(f) and Table 6] along with rankings/Eigen vector values the criteria preference [Table 2(f)]. We have to find the product of these two matrices as shown in Table 7. The product of these two matrices gives us the final ranking for these mobile phone brand/models, and this is shown in Table 8. So, as we note the best mobile phone brand/model is 1) Brand D, followed by 2) Brand A, 3) Brand B and 4) Brand C. Refer to Table 8.

Considering Cost Information:

But all said and done, we are still missing out on one important piece of information and that is Cost. Let's now consider the last of the given data in the form of cost. Cost could have been included, but as you are very well aware, in many complex decision-making situation it is always preferable to put cost aside until the relative benefits and priority rankings of the alternatives are evaluated. It has been seen and the research proves it that lumping the cost of the unit under consideration together with the relative benefits and priorities more often than not is a potent mixture for many politically and emotionally charged debates.

This can be used directly as the quantitative data is available (from Table 1). As we did in the case of camera power, we simply normalize the cost information. Refer to Table 9. And lastly we use, the cost to benefit ratio—the benefits are the same as what we derived in the form of the final rankings (Eigenvectors) for different transformers. Numerical priorities, derived from decision makers'

Table 6. Eigen vector values representing rankings of the mobile phone brands for camera power in megapixels.

CAMERA POWER	MEGAPIXELS	NORMAL
BRAND A	16	**0.2580**
BRAND B	12	**0.1935**
BRAND C	14	**0.2258**
BRAND D	20	**0.3225**
	62	1.00

Table 7. Matrix multiplication—rank values for brands/models on each criterion and rank values for criteria.

FINAL EIGEN VECTOR VALUES	PERFORMANCE & SPECIFICATIONS [Table 3(f)]	APPLICATIONS & FUNCTIONS [Table 4(f)]	FEATURES & STYLE QUOTIENT [Table 5(f)]	CAMERA'S POWER MEGAPIXELS [Table 6]	CRITERIA	EIGEN VECTORS [Table 2(f)]
BRAND A	0.0993	0.4282	0.2872	0.2580	PERFORMANCE & SPECIFICATIONS	0.2743
BRAND B	0.1735	0.2989	0.2158	0.1935	APPLICATIONS & FUNCTIONS	0.3027
BRAND C	0.0432	0.0403	0.1939	0.2258	FEATURES & STYLE QUOTIENT	0.3761
BRAND D	0.6840	0.2325	0.3031	0.3225	CAMERA'S POWER MEGAPIXELS	0.0469

Table 8. Rankings of mobile phone brands/models—result of matrix multiplication.

MOBILE BRANDS	EIGEN VECTORS	RANKS
BRAND A	0.2769	#2
BRAND B	0.2283	#3
BRAND C	0.1075	#4
BRAND D	0.3871	#1

Table 9. Normalized values/Eigen vectors for cost of each mobile phone.

BRANDS	COST	EIGEN VECTORS
BRAND A	28,700	0.1959
BRAND B	36,900	0.2518
BRAND C	38,900	0.2655
BRAND D	42,000	0.2866
	146,500	1

input, are shown in Table 10. The decision to select "the best mobile phone brand/model" out of four given choices, on the basis of five important criteria. It seems even after consideration of cost vs. benefit analysis that—Brand A is the best of the lot with first priority, followed by Brand D, 3) Brand B and 4) Brand C. Features and style quotient are the most important criterion in making the decision, closely followed by applications and functions, performance and specifications and then camera power in megapixels. Refer to Table 11.

5. Conclusions

It can be said that with the article's focus on only four mobile phone brands/models it was missing out on opportunities to explore other options. The author is also aware that these were by no means the only feasible ones, but to keep decision making to a less complex level, it decided to compare only these four. The decision making though quantitative in nature, involved a lot of qualitative options and hence a lot of subjectivity.

Table 10. Cost (Eigen vectors from Table 9) vs. benefit (benefits are the rank values/ Eigen vectors from Table 8).

BRANDS	COST	NORMAL	EIGEN VECTORS	BENEFITS V/s COST
BRAND A	28,700	0.1959	0.2769	1.4138
BRAND B	36,900	0.2518	0.2283	0.9065
BRAND C	38,900	0.2655	0.1075	0.4051
BRAND D	42,000	0.2866	0.3871	1.3503

Table 11. Final ranks for mobile phone brands/models by analytical hierarchy process (AHP).

FINAL RANKS	BENEFITS VIS A VIS COST OF THE MOBILE PHONE	BRAND/MODEL
#1	1.4065	BRAND A
#2	1.3503	BRAND D
#3	0.9065	BRAND B
#4	0.4051	BRAND C

(Source: From **Tables 2-11**—Computed and Tabulated by Author using AHP).

In this paper an attempt has been made to explain and apply an AHP Model. The techniques and concepts which can be used to address the issue fall under: Analytical Hierarchy Process (AHP)—including hierarchical trees, pair wise comparison, criteria weights, matrix algebra, eigenvector values, and cost-benefit analysis for decision making purposes. The paper focuses on "the use of a decision making technique which decomposes the problem into a hierarchy of more easily comprehended sub-problems, each of which can be analyzed independently". Although the process is fairly straight forward and the alternative options are easy to comprehend, there are other relevant factors to be kept in mind while making decisions. Also, as to how AHP can incorporate both quantitative and qualitative data in its stride, the study provides a very good foundation for quantitative decision making. The paper attempts to inculcate readers to take a holistic view of complex decision making involving a large number many options and criteria. The paper also makes practitioners think critically about utilizing the AHP tool in various scenarios like production planning, scheduling and allocation of scarce resources, prioritization and ranking.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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