Creative Education, 2018, 9, 1525-1535
http://www.scirp.org/journal/ce
ISSN Online: 2151-4771
ISSN Print: 2151-4755

Perspective of "Quantity" between Natural Language and Mathematical Symbols (<, >, =): The Comprehension of Pre-Service and Preschool Teachers

Bat-Sheva Ilany[1], Dina Hassidov[2]

[1]Hemdat Hadarom College, Netivot, Israel
[2]Talpiot College of Education, Holon, Israel
Email: hasidov@netvision.net.il

How to cite this paper: Ilany, B.-S., & Hassidov, D. (2018). Perspective of "Quantity" between Natural Language and Mathematical Symbols (<, >, =): The Comprehension of Pre-Service and Preschool Teachers. *Creative Education, 9,* 1525-1535.
https://doi.org/10.4236/ce.2018.910112

Received: May 14, 2018
Accepted: August 4, 2018
Published: August 7, 2018

Abstract

The goal of the study was to understand how pre-service and preschool teachers understand and use the mathematical symbols <, >, and = when comparing figures and shapes of different sizes and thicknesses. Using both quantitative and qualitative methods, we examined a study population of 71 pre-service teachers attending a course for teaching mathematics to pre-schoolers and 149 preschool teachers. Our results show that the majority of participants did not answer the questions correctly, with a significant difference between how the two groups validated their answers, indicating that the participants do not correctly understand that mathematical symbols should (<, >, and =) only be used in the mathematical context.

Keywords

Preschool Teachers, Pre-Service Teachers, Mathematics Education, Early Childhood, Mathematical Symbol

1. Introduction

1.1. Mathematical Language in Early Childhood

Mathematical language is a language of symbols, concepts, definitions, and theorems. It does not develop naturally like a child's natural language, but needs to be taught (Ilany & Margolin, 2010). In essence, children are engaged in mathematics in daily life from birth, and today's global trend is to introduce "formal" mathematics at a young age. Preschool math practice aims to develop mathematical awareness and cultivate mathematical thinking from an early age, thus shaping the child's future mathematical thinking, general thinking, and cognitive abilities (Baroody, 2000). Studies have shown that the volume and quality of preschool math practice predict a child's success in math in elementary school (Clements & Sarama, 2006, 2015).

According to the accepted Israeli curriculum, first skills include being able to use the concepts (not the actual symbols) of "bigger," "smaller," and "equal to" to recognize differences between objects. Some preschool teachers introduce the mathematical symbols =, <, and > already in preschool and ask the children to use these mathematical relational symbols to compare non-mathematical objects. This may lead children to believe that these symbols are not restricted to mathematical values and, moreover, even when comparing numbers, to use them incorrectly.

A neat illustration of that is described in the literature as "The Stroop Effect", whereby a mismatch between the physical size and the numerical size of an object results in a cognitive dissonance (Algom, Dekel, & Pansky, 1996).

For example, a child in grade one may write "6 < 4" because the four looks bigger and thicker than the six, indicating that he is looking at the numbers as graphical entities and not mathematical ones. Such instances have led to the study of how pre-service and preschool teachers themselves use these mathematical symbols (Ilany & Hassidov, 2012; Hassidov & Ilany, 2017).

This paper deals with a study that relates to the understanding of the concepts that constitute part of symbolic thinking. We will present a mathematical reference to these concepts (Sinitsky & Ilany, 2016).

Different quantities are compared through relations of order using various strategies based on the properties of these relations. According to Cantor's sorting principles, the set of real numbers has an intrinsic linear order. In other words, between any two quantities, one and only one of three following options holds true:

1) the two values are equal to each other;

2) the first is greater than the second;

3) the first is smaller than the second.

If we plot real numbers on a number line, two numbers, a and b, are equal only if the points that represent them coincide. If b is greater than a, the point representing b will be to the right of a. Here, we can also say that a is less than b.

In this context, it is significant to note that the usual way of presenting such problems in textbooks is to phrase it as "Which number is greater?" However, such phrasing ignores the possibility that the numbers might be equal to each other. In fact, there are actually three possible alternatives, and the "greater or lesser?" question, inspired by binary logic, may be misleading. Cognitively, comparing amounts or dimensions should actually begin by asking "Are these quantities equal?" or, alternatively, "Is there any quantitative difference between these two entities?" If the answer is that they are different, then the next question is "Which one is greater/lesser, larger/smaller, etc.?"

It is useful to present the ways that a relationship between two quantities can be described by using three pairs of relations, where each proposition of the pair is the negation of the other:

$$a = b \qquad a \neq b$$

$$a > b \qquad a \leq b$$

$$a < b \qquad a \geq b$$

Recall that the strategies used to compare two quantities are based on the general properties of comparison relations. The relation of equality is an equivalence and maintains the three properties of any equivalence relation: reflexivity, a = b (each value is equal to itself); symmetry, $a = b \Leftrightarrow b = a$; and transitivity, $a = b \,\&\, b = c \Rightarrow a = c$ (two values equal to a third are also equal to each other).

The relations of greater than and less than are strict (a > a is never true) and asymmetric (there is no pair where both a > b and b > a are true), but do hold transitivity. This last property ($a < b, b < c \Rightarrow a < c$) is expressed in the well-known "rule of transition" (similar to equality): if one value is less than another, and that second value is less than a third, then the first is less than the third.

1.2. The Development of Symbolic Understanding in Early Childhood

Symbolic reasoning means the ability to grasp the meaning of a symbol representing an object or idea, without having an expression in the symbol itself (Bialystok, 1992). It is an evolving ability and one of the developing expressions of thought (Thomas, Jolley, Robinson, & Champion, 1999). Its development is characterized by changes that occur in the form of the mental representation of an object. Young children believe that the symbolic representation reflects the nature of the object it represents (Bialystok, 1992), For example, children may write the names of large objects using large letters (Thomas et al., 1999). Nemirovsky and Monk (2000) noted that young children do not distinguish between the symbol and the object that the symbol represents.

Although young children can identify symbols and write them, this does not necessarily reflect an understanding of the symbols' mathematical meaning or their relationship to numbers. The concept of equality is an especially difficult concept to comprehend for children, since this term can be used both relationally and mathematically.

The early development of symbolic reasoning in children should allow them to properly use mathematical symbols later in formal math.

1.3. Teaching Mathematics by Preschool Teachers

Teaching mathematics to pre-schoolers today requires professional knowledge on the part of the teachers (Charalambous, Panaoura, & Philippou, 2009). Unfortunately, studies conducted in recent years indicate that teachers assigned with teaching preschool mathematics do not have adequate knowledge. This may stem from negative personal experiences or a lack of appropriate training in college (Tirosh & Graeber, 1990; Hassidov & Ilany, 2014, 2015). They often use the knowledge and experience they bring from daily life, meaning that they might not always give the correct mathematical importance to the symbol. If teachers incorrectly understand the use of mathematical symbols, it is reasonable to assume that they will subsequently pass this misinformation on to the children, leading to incorrect use in the future. It is thus crucial to teach the proper mathematical use of symbols from the preschool level (Hassidov & Ilany, 2017).

Using the "=" symbol incorrectly with children makes it even harder for them to properly understand its concept.

Many studies have examined how pre-service teachers of various ages comprehend the "equal" sign (e.g.: Mark-Zigdon & Tirosh, 2008). They show that children aged 5 - 12 tend to perceive the equal sign as an operational symbol and not as a sign of comparison. Pre-service teachers translate the symbols as a command to perform a mathematical operation. It is important to grasp that the meaning of a symbol cannot be changed by non-mathematical factors (such as a change in size or other physical factor).

In a study dealing with the knowledge of pre-service teachers and preschool teachers regarding their understanding of the significance and use of mathematical symbols between numbers, Hassidov and Ilany (2017) found that pre-serviceand preschool teachers do not fully understand that mathematical symbols should relate only to the mathematical nature of the object. If one number was written in a larger, smaller, or thicker format than another, they often regarded the physical qualities and not the mathematical (i.e., the values of the numbers). Furthermore, even when they used the symbol correctly, the reasoning behind its use was often flawed. For example, the participants were asked for the symbol that should be put between "6" and "4." 91.6% of the pre-service teachers answered correctly compared with 77.9% of teachers ($p < 0.01$). 63.4% of the pre-service teachers and 24.8% of the teachers correctly explained that it was due to the sequence of numbers. Some participants (8.5% of pre-service teachers, 5.4% of teachers) incorrectly based their answer on the number of items on each side and not their numerical value.

1.4. Research Questions

This study, as a continuation of the previous studies by the authors, examines how pre-service and veteran preschool teachers understand the concepts of >, <, and =. Its objectives were twofold:

1) to understand how pre-service and preschool teachers comprehend and use the relational symbols>, <, and =, and

2) to compare how pre-service and preschool teachers comprehends and uses these symbols?

2. Method

2.1. Population

The study population comprised 71 second- or third-year pre-service teachers participating in a year-long course dedicated to the teaching and learning of mathematics in early childhood and 149 veteran preschool teachers (Table 1).

2.2. Research Tools

Data were collected via semi-structured interviews and a 25-item questionnaire designed by the authors. Of the 25 questions in the questionnaire, four (questions 1, 2, 3, and 17) addressed the use of mathematical symbols between shapes that had some graphical difference (size, thickness, placement) (Table 2). Respondents were asked to either place a relational symbol between two figures or indicate "X" if they believed there was no appropriate answer, and then justify their answers. The correct answer in each case was "X," since none of the questions compared mathematical values. Analysis was both qualitative and quantitative.

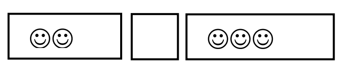
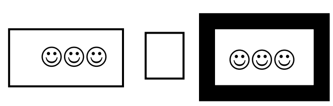
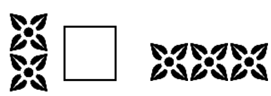
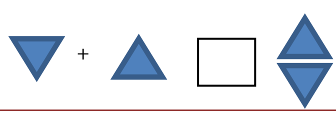
2.3. Procedure

1) Testing pre-service teachers:

Table 1. Description of the preschool teachers.

Work experience	N	Education	N
Up to 10 years	60	BA	78
More than 10 years	89	MA	23
		Another	27

Table 2. Analysis of the responses of pre-service (PST) and preschool (PT) teachers to questions 1, 2, 3, and 17. (All values represent percentages.)

Question		Possible Answers							
		<		>		=		X*	
		PT	PST	PT	PST	PT	PST	PT	PST
1		98	100	0	0	0	0	2	0
2		5	0	16	6	74	94	4	0
3		95.4	96	1.3	1	1.3	3	3	0
17		1.3	0	2.7	4	92	90	4	6

*correct answer.

Questionnaires were filled out by the pre-service teachers before any formal study of the subject. The researchers interviewed a random sampling of 30 pre-service teachers. This was followed by a class discussion on the use and meaning of mathematical symbols, and the subject's place in the preschool curriculum.

2) Testing preschool teachers:

Each preschool teacher filled out a questionnaire. Then a random sample of 25 preschool teachers were interviewed by the researchers to ascertain the preschool teachers' reasoning for their answers.

3. Results

1) Overall Difference

Overall, not one of the participants gave the correct answer and justification for any of the questions. Even the very few who gave the correct answer ("X") gave flawed justifications, the correct one being that these symbols cannot be used for graphical objects and only for numerical entities.

A significant difference was found between the two groups: a large number of preschool teachers did not supply any justification for their reasoning (58.4% for question 1, and 57.7%, 60.4%, and 66.4% for questions 2, 3, and 17, respectively) compared to the number of pre-service teachers who did not (19.8%, 14.1%, 16.9%, 28.2%, respectively).

2) Question 1 asked for which mathematical symbol, if any, should be placed between two rectangles of different sizes. There are two smileys in the smaller rectangle and three in the larger (all smileys are the same size, see **Table 2**).

Table 2 shows that 98% of the preschool teachers thought that "<" was the correct answer. Most of those who justified their answer said it was due to the number of smileys (and not the size of the rectangle). Only a small number in each group based their answer on the size or area of the rectangles. Some of the preschool teachers said they based their answer on both qualities (see **Table 3**).

Table 3. Analysis (value and percent) of the justifications given by pre-service and preschool teachers to question 1.

		Justification				
		Graphic properties*	Numerical properties*	Both size and quantity*	No Answer*	Number who gave correct justification
Pre-service teachers	N	1	56	0	14	0
N = 71	%	1.4	78.8	0	19.8	0
Preschool teachers	N	6	53	3	87	0
N = 149	%	4	35.6	2	58.4	0

$^*p < 0.001$.

3) Question 2 asked for which mathematical symbol, if any, should be placed between two rectangles of different size and thickness, each of which contained three smileys. The results were similar to question 1: most did not answer "X," and those who did, justified it incorrectly. Similarly, there was a significant difference ($p < 0.001$) between the groups (see Table 4).

The vast majority of both pre-service teachers and preschool teachers answered "=," indicating that they focused on the number of smileys (numerical properties). However, one preschool teacher said: "There are the same number of smileys, but the area is different." That is, her answer was based on quantity, but her justification also considered the shape. Another wrote "I counted the smileys." One wrote: "Based on my experience, I would teach that the second is larger. But there can be different levels," indicating that she feels that different criteria can be used under different circumstances. One pre-service teacher wrote: "I looked at the number of smileys. There is no importance to the length of the rectangle, only the number." One pre-service teacher indicated "=" but wrote "The same quantity in each rectangle, although the left rectangle has a greater area."

Those who indicated "<" justified their answer by indicating either the size or thickness of the rectangles. One preschool teacher answered, "They look to me to be the same, except that one rectangle is longer." A pre-service teacher who marked "<" wrote "the rectangle on the right is thicker and coloured."

Again, although 4% of the preschool teachers gave the correct answer ("X") their justifications were incorrect. For example: "They cannot be compared because the shapes are not the same."

4) Question 3 asked for which mathematical symbol, if any, should be placed between graphics of flowers. On the left were two flowers one above the other, on the right were three flowers side by side (Table 2). Again, the vast majority (100% of pre-service teachers and 97% of preschool teachers) answered incorrectly. All the participants did not give any correct justification at all. Also, as previously, significantly more preschool teachers did not give any justification at all (Table 5).

Table 4. Analysis (value and percent) of the justifications given by pre-service and preschool teachers to question 2.

		Justification				
		Graphic properties*	Numerical properties*	Both size and quantity*	No Answer*	Number who gave correct justification
Pre-service teachers	N	2	52	7	10	0
N = 71	%	2.8	73.2	9.9	14.1	0
Preschool teachers	N	18	41	4	86	0
N = 149	%	12.1	27.5	2.7	57.7	0

*$p < 0.001$.

Table 5. Analysis (value and percent) of the justifications given by pre-service and preschool teachers to question 3.

		Justification				
		Graphic properties	Numerical properties*	Both size and quantity*	No Answer*	Number who answered correctly
Pre-service teachers	N	1	57	1	12	0
N = 71	%	1.4	80.3	1.4	16.9	0
Preschool teachers	N	3	54	2	90	0
N = 149	%	2.1	36.2	1.3	60.4	0

*$p < 0.001$.

The vast majority of each group indicated “<” and stated that they focused on the quantity of flowers. However, some of the justifications seemed slightly confusing. One pre-service teacher said: “One group has a larger quantity of items, but the other one is higher.” Another also referred to quantity, but then added: “There are fewer flowers on the right, but the arrangement is different.” One preschool teacher gave a very “non-mathematical” reason: “Two friends are preferable to three, since two together are very happy. A third friend may come between them.”

A small number of each group indicated “=” as the correct answer, one preschool teacher explaining that “they are equal because there are flowers on both sides.” An even smaller number of each group indicated “>,” evidently referring to the height of the graphics. Of the small number of preschool teachers who answered “correctly,” one justified this by saying “If you rotate one of the figures and place it next to the other, you can see that one is larger than the other.”

5) Question 17 asked which mathematical symbol, if any, should be placed between figures of triangles (see Table 2). Each side had two triangles, one being “upside down.” On the left, they were in a single row with a plus sign (“+”) between them. On the right, they were one on top of the other. Once again, the vast majority (94% of pre-service teachers and 96% of preschool teachers) answered incorrectly and there was a significant difference ($p < 0.001$) between the justifications they gave (Table 6).

One preschool teacher who indicated “>” said: “There are two triangles and the addition operation, so that side is larger than the right side.” One who indicated “=” wrote, “We haven’t learned this yet.” Another gave an answer that seemed confused, “They are equal from two standpoints. One is that on each side one triangle goes up, and one goes down. So, they make the shape of an equilateral diamond.” A teacher who indicated “=” said, “The placement of the triangles is not important. What is important is their quantity.”

One pre-service teacher who answered “X” justified it with “There is no answer because I didn’t know which symbol to use. There are two triangles on each side, but they are not arranged the same.” Some preschool teachers answered “X” because they did not know which of the others to use.

Table 6. Analysis (value and percent) of the justifications given by pre-service and preschool teachers to question 17.

		Justification					
		Graphic properties*	Numerical properties*	Both size and quantity*	No Answer*	Another answer X	Number who answered correctly
Pre-service teachers	N	10	36	4	20	1	0
N = 71	%	14.1	50.7	5.6	28.2	1.4	0
Preschool teachers	N	4	44	1	99	1	0
N = 149	%	2.7	29.5	0.7	66.4	0.7	0

*$p < 0.001$.

4. Discussion and Conclusion

This study, similar to a previous study by Hassidov and Ilany (2017) found that most of the participants failed to answer the questions correctly. The justifications given to the four questions show a significant difference between the pre-service and preschool teachers with respect to how many justified their answers, yet it is clear that all participants did not appreciate the significance of the mathematical symbols and how to use them, specifically, that mathematical symbols should be used only for mathematical symbols. This was clear since even when the answer given was correct ("X"), the justification was generally incorrect. These findings support the study regarding how pre-service teachers and preschool teachers understand relational symbols (<, >, =) with respect to numbers (Hassidov & Ilany, 2017).

The results of this study show that preschool teachers feel that mathematical symbols may be used in different ways, depending on context: sometimes with respect to the quantity and sometimes to the shape or size of graphical images and they did not restrict them only to their mathematical significance.

The conclusion is that the participants do not properly understand the significance of the symbols =, <, and > nor how to use them. This will, in all probability, mean that they will not teach the concepts properly to preschoolers.

Indeed, studies have shown that teachers believe the signs can be used in many ways. Using the same words in everyday life and in mathematics leads to misconceptions regarding the meaning of the mathematical signs (Ilany & Margolin, 2010). Some of the teachers thus do not see any problem if a child writes "5 > 5," and have stated that they teach the child to use the symbol ">" between two objects, as "in this case the size is important; in another case the length may be important. It depends on the context." Teachers may even believe it is correct to use two different signs at the same time; however, they must understand the cognitive conflict that this gives children and must understand that it is never possible to use two different signs of (<, >, and =) between two numbers at the same time (Ilany & Hassidov, 2012).

Teachers have to be aware that the signs “<, >, and =” ought to be used only in the mathematical sense. Preschool teachers who incorrectly see quantity as a graphical concept and do not see the mathematical significance are likely to pass on this misconception to the children. This might leads the children to think that the size of the number or graphical object is what determines the relationship and which symbol to use.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

Algom, D., Dekel, A., & Pansky, A. (1996). The Perception of Number from the Separability of the Stimulus: The Stroop Effect Revisited. *Memory & Cognition, 24,* 557-572. https://doi.org/10.3758/BF03201083

Baroody, A. J. (2000). Does Mathematics Instruction for 3-to-5-Year Olds Really Make Sense? *Young Children, 55,* 61-67.

Bialystok, E. (1992). Symbolic Representation of Letters and Numbers. *Cognitive Development, 7,* 301-316. https://doi.org/10.1016/0885-2014(92)90018-M

Charalambous, C. Y., Panaoura, A., & Philippou, G. (2009). Using the History of Mathematics to Induce Changes in Pre-Service Teachers’ Beliefs and Attitudes: Insights from Evaluating a Teacher Education Program. *Educational Studies in Mathematics, 71,* 161-180. https://doi.org/10.1007/s10649-008-9170-0

Clements, D. H., & Sarama, J. (2006). Young Children Mathematical Mind. *Scholastic Parent & Child, October 2006,* 30-37.

Clements, D. H., & Sarama, J. (2015). Developing Young Children’s Mathematical Thinking and Understanding. In S. Robson, & S. Flannery (Eds.), *The Routledge International Handbook of Young Children’s Thinking and Understanding* (pp. 331-344). New York: Routledge.

Hassidov, D., & Ilany, B. (2014). A Unique Program (“Senso-Math”) for Teaching Mathematics in Preschool: Evaluating Facilitator Training. *Creative Education (CE), 5,* 976-988. https://doi.org/10.4236/ce.2014.511112

Hassidov, D., & Ilany, B. (2015). The “Senso-Math” Preschool Program: Successful Cooperation between Mathematics Facilitators and Preschool Teachers. In *Proceedings of the 39th Conference of the International Group for the Psychology of Mathematics* (Vol. 3, pp. 41-48). Hobart: PME.

Hassidov, D., & Ilany, B. (2017). Between Natural Language and Mathematical Symbols (<, >, =): The Comprehension of Pre-Service and Preschool Teachers-Perspective of Numbers. *Creative Education, 8,* 1903-1911. https://doi.org/10.4236/ce.2017.812130

Ilany, B., & Hassidov, D. (2012). The Image of <, >, = by Pre-School Teachers. In *36th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, p. 243).Taipei: PME

Ilany, B., & Margolin, B. (2010). Language and Mathematics: Bridging between Natural Language and Mathematical Language in Solving Problems in Mathematics. *Creative Education (CE), 1,* 138-148. https://doi.org/10.4236/ce.2010.13022

Mark-Zigdon, N., & Tirosh, D. (2008). What Counts and What Does Not Count as Legitimate Arithmetic Number Sentences: The Case of Kindergarten and First Grade

Children. In J. J. Kaput, M. Blanton, & D. Carraher (Eds.), *Algebra in the Early Grades* (pp. 201-210). Hillsdale, NY: Lawrence Erlbaum Associates.

Nemirovsky, R., & Monk, S. (2000). 'If You Look at It the Other Way…' An Exploration into the Nature of Symbolizing. In P. Cobb, E. Yackel, & K. McClain (Eds.), *Symbolizing and Communicating in Mathematics Classrooms: Perspectives on Discourse, Tools, and Instructional Design*. Hillsdale, NJ: Lawrence Erlbaum.

Sinitsky, I., & Ilany, B., (2016). *Change and Invariance-Algebraic Insight into Numbers and Shapes.* Rotterdam: Sense Publishers.

Thomas, G. V., Jolley, R. P., Robinson, E. J., & Champion, H. (1999). Realist Errors in Children's Responses to Pictures and Words as Representations. *Journal of Experimental Child Psychology, 74,* 1-20. https://doi.org/10.1006/jecp.1999.2506

Tirosh, D., & Graeber, O. A. (1990). Evoking Cognitive Conflict to Explore Pre-Service Teachers' Thinking. *Journal for Research in Mathematics Education, 21,* 98-108. https://doi.org/10.2307/749137