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Controllability of Strongly and Weakly Dependent Siphons under Disturbanceless Control*

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Abstract

Li and Zhou propose to add monitors V_s to elementary siphons S only while controlling the rest of dependent siphons—important for large systems but far from being maximally permissive. The control policy for weakly dependent siphons (WDS) is rather conservative due to some negative terms in the controllability. We show that this is no longer true as can be shown that it has the same controllability as that for strongly dependent siphons.

Keywords: Petri Nets, Siphons, Controllability, FMS, S^3PR

1. Introduction

A flexible manufacturing systems (FMS) consists of several *concurrent processes* competing for resources such as machines, robotics, etc. to produce different kinds of parts. Each process performs a sequence of operations to manufacture a part of a product. Mutual waiting for resources can bring the system into a deadlock where no process can proceed.

An FMS can be modeled by a Petri net (PN). System properties such as boundedness, liveness and reversibility are fundamental for an FMS to operate in a sTable, deadlock-free, and periodic fashion.

Deadlock prevention approaches [1-23] create the control policy in a static way by building a Petri net model first and then adding necessary control to it such that the controlled model is deadlock-free. Control places and related arcs are often used to attain such purpose resulting in less states reached, but the system runs quicker as a result of no online computation.

A siphon (trap, respectively) is a set of places [where tokens can leak out (inject in, respectively)] of a PN modeling an FMS. Once the siphon has lost all its tokens, output transitions of places in the siphon can never be executed and the net is not live.

Control places and related arcs are often added upon emptiable siphons to disallow them to become unmarked (no tokens). This disturbs the original model and loses some reachable good states; *i.e.*, less permissive, impacting the performance of the supervisor.

Ezpeleta *et al.* [11] propose adding a monitor upon each problematic siphon for an S^3PR which stands for systems of simple sequential processes with resources. This method generally requires adding too many monitors due to the fact that there are too many emptiable siphons. The iterative control method in [12] reduces the number of monitors by finding all emptiable siphons in each iteration step. The method becomes very difficult and remains complex even for a moderate-size model.

Furthermore, Ezpeleta *et al.* [11] move all output (called Type-2, or source) arcs of each monitor V_S to the output (called source) transition of the entry (called idle place) of input raw materials to limit their rate into the system to avoid generaing new emptiable siphons, called SMSless approach. This may overly constrain the system to reach much fewer reachable states (6287, the same as that by Li *et al*. [13,14] but with a lot more control elements) than the maximal permissive one using the region method by Uzam and Zhou [15].

It is impractical to add a monitor to each emptiable siphon for large systems since the number of emptiable siphons or control elements grows quickly with respect to the size of a Petri net. Li and Zhou [13,14,16,17] tackle this problem by classifying siphons into elementary and dependent ones.

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By adding monitors to only elementary siphons, Li and Zhou [13] greatly reduce the number of control nodes and arcs, essential for large systems. Some of the rest of emptiable (called dependent) siphons may already be controlled depending on the controllability.

Otherwise, the control depth variable may need to be increased to avoid the siphon unmarked and reach fewer states. The control policy for weakly dependent siphons is rather conservative [13] (such that fewer states are reached) by ignoring some negative terms in the controllability.

The control place and arcs for siphon S, similar to resource places, form a number of elementary circuits. Hence, there is an elementary circuit containing adjacent control places, from which we can synthesize new problematic siphons. To avoid such, output arcs of a control place are moved from sink transitions of the siphon S to source transitions of the processes. As a result, the region A (called controller region) covered by control arcs is larger than the region B (called the complementary set of S) to trap tokens from S. The disturbed region becomes larger after the movement of output arcs. This loses more states due to the presence of control places and arcs, which disturbs the markings of the original model.

We [1-4,6,7] show that elementary (resp. strongly dependent) siphons in an S^3PR (systems of simple sequential processes with resources) may be synthesized from elementary (resp. compound) resource circuits. There is no need to compute the basis for the set of elementary siphons from the vector space containing all characteristic T-vectors. Furthermore, we add monitors for different types of siphons in some sequence to avoid redundant monitors and losing live states.

It is unclear whether the same advantage can be extended to weakly dependent siphons. We don't know from what circuits can we synthesize a weakly dependent siphon S, and the condition that S is controlled. This paper shows that weakly dependent siphons have a similar controllability to that for strongly dependent siphons under the disturbanceless control policy even though Li *et al.* prove that the policy for weakly dependent siphon is more conservative than strongly dependent siphons.

The rest of the paper is organized as follows: Section 2 and 3 presents the basis to understand the paper. Section 4 reviews the theory on controllability of strongly dependent siphons in Li and Zhou [13,14]. Section 5 develops the theory to weakly dependent siphons based on Proposition 1. It is interesting to observe that weakly and strongly dependent siphons have the same controllability for compound siphons. Section 6 concludes the paper.

2. Preliminaries

Please refer to [1] for terms related to Petri nets. We now define characteristic T-vectors, elementary and dependent siphons.

Definition 1. [13,14]: *Let* $\Omega \subseteq P$ *be a subset of places of N. P-vector* λ_Ω *is called the characteristic P-vector of* Ω *iff* $\forall p \in \Omega,\ \lambda_\Omega(p) = 1$; *otherwise* $\lambda_\Omega(p) = 0$. η *is called the characteristic T-vector of* Ω, *if* $\eta^T = \lambda_\Omega^T \bullet [N]$, *where* $[N]$ *is the incidence matrix.*

Physically, the firing of a transition t where $\eta(t) > 0$, $\eta(t) = 0$, and $\eta(t) < 0$ increases, maintains and decreases the number of tokens in S, respectively

Definition 2. [13,14]: *Let* $N = (P, T, F)$ *be a net with* $|P| = m$, *which has k siphons* $S_1, S_2, \cdots, S_k$, m, $k \in \boldsymbol{IN}$, *where* $\boldsymbol{IN} = \{0, 1, 2, \cdots\}$. *Define* $[\lambda]_{k\times m} = [\lambda_1|\lambda_2|\cdots|\lambda_k]^T$ *and* $[\eta]_{k\times m} = [\eta_1|\eta_2|\cdots|\eta_k]^T$. $[\lambda]$ (*resp.* $[\eta]$) *is called the characteristic P*(*resp. T*)*-vector matrix* $[\lambda]$ (*resp.* $[\eta]$) *of the siphons in N. Let* η_{S_α}, η_{S_β}, $\cdots$, *and* η_{S_γ} (α, β, $\gamma \subseteq$ 1, 2, $\cdots$, *k*) *be a linear independent maximal set of matrix* $[\eta]$. *Then* $\Pi_E = \{S_\alpha, S_\beta, \cdots, S_\gamma\}$ *is called a set of elementary siphons.* $S \notin \Pi_E$ *is called a strongly dependent siphon if* $\eta_S = \sum_{S_i \in \Pi_E} a_i \eta_{S_i}$ *where* $a_i \geq 0$. $S \notin \Pi_E$ *is called a weakly dependent siphon if* $\exists$ *non-empty A*, $B \subset \Pi_E$, *such that* $A \cap B = \emptyset$ *and* $\eta_S = \sum_{S_i \in A} a_i \eta_{S_i} - \sum_{S_i \in B} a_i \eta_{S_i}$ *where* $a_i > 0$.

In [13,14], a strongly dependent siphon is also called a strict redundant one. Li and Zhou propose to find elementary siphons by constructing the characteristic P-vector (resp. T-vector)-vector matrix $[\lambda]$ (resp. $[\eta]$) of the siphons in N followed by finding linearly independent vectors in $[\lambda]$ (resp. $[\eta]$) The siphons corresponding to these independent vectors are the elementary siphons in the net system.

Note that Def. 2 and the above calculation of linearly independent vectors do not assume N to be an S^3PR and are applicable to arbitrary nets.

Figure 1(a) shows an example of weakly dependent siphon. **Table 1** below lists the four strict minimal siphons and their η, where $\eta_4 = \eta_1 + \eta_2 - \eta_3$.

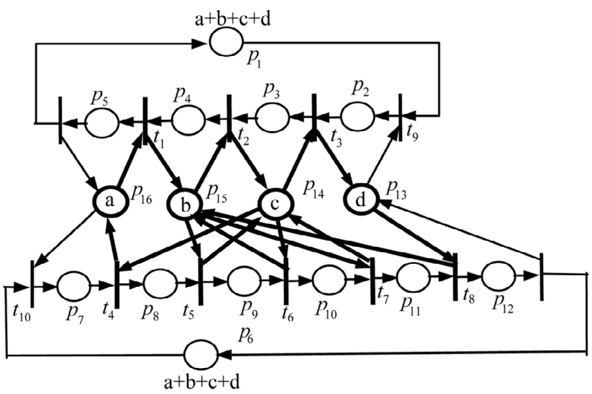
3. Types of SMS

In [2,3,6], we show that SMS can be synthesized from resource or core subnets. New types (such as control siphons) of SMS can be synthesized from control subnets formed by control places. If we add monitors to these different types of siphons in a certain order, then some siphons may be redundant.

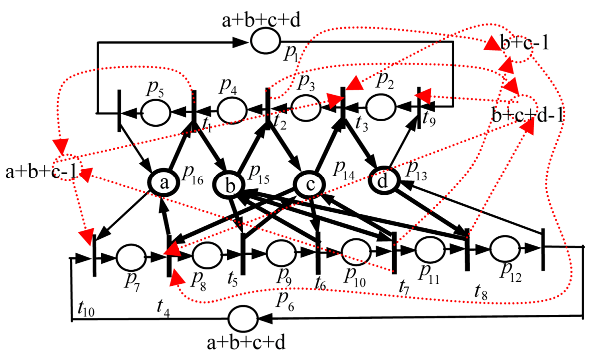
We construct an SMS based on the concept of handles. Roughly speaking, a "handle" is an alternate disjoint path

Table 1. Four SMS in Figure 1(a) and their η, where $\eta_4 = \eta_1 + \eta_2 - \eta_3$.

S	η	Set of places	[S]
S_1	$t_2 - t_4 + t_8 - t_9$	$p_4, p_{12}, p_{13}, p_{14}, p_{15}$	$p_2, p_3, p_8, p_9, p_{10}, p_{11}$
S_2	$t_1 - t_3 + t_7 - t_{10}$	$p_5, p_{11}, p_{14}, p_{15}, p_{16}$	$p_3, p_4, p_7, p_8, p_9, p_{10}$
S_3	$t_2 - t_3 - t_4 + t_7$	$p_4, p_{11}, p_{14}, p_{15}$	p_3, p_8, p_9, p_{10}
S_4	$t_1 + t_8 - t_9 - t_{10}$	$p_5, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}$	$p_2, p_3, p_4, p_7, p_8, p_9, p_{10}, p_{11}$



(a)



(b)

Figure 1. (a) Example weakly dependent siphon [14]. $r_a = p_{16}$, $r_b = p_{15}$, $r_c = p_{14}$, $r_d = p_{13}$, $t_a = t_1$, $t_b = t_2$, $t_c = t_3$, $t_d = t_8$, $t'_c = t_4$; (b) controlled model of that in Figure 1(a).

between two nodes. A *PT-handle* starts with a *place* and ends with a *transition* while a *TP-handle* starts with a *transition* and ends with a *place* A core subnet can be obtained from an elementary circuit, called *core circuit*, by repeatedly adding handles.

The control place and arcs for siphon *S*, similar to resource places, form a number of elementary circuits. Hence, there is an elementary circuit containing adjacent control places, from which we can synthesize new problematic siphons.

Definition 3. *An elementary resource circuit is called a basic circuit, denoted by* c_b. *The siphon constructed from* c_b *is called a basic siphon. A compound circuit* $c = c_1 \circ c_2 \circ \cdots \circ c_{n-1} \circ c_n$ *is a circuit consisting of multiply interconnected elementary circuits* c_1, c_2, $\cdots$, c_n *such that* $c_i \cap c_{i+1} = \{r_{p_i}\}, r_{p_i} \in R$ *(i.e.,* c_i *and* c_{i+1} *intersects at a resource place* r_i*). The SMS synthesized from compound circuit c using the Handle-Construction Procedure in* [9] *is called an n-compound* (*resp. control, mixture*) *siphon S, denoted by* $S = S_1 \circ S_2 \circ \cdots \circ S_{n-1} \circ S_n$.

4. Controllability for Strongly Dependent Siphons

We review the controllability for strongly dependent siphons to compare with that for weakly ones to be derived in Section 5. We first present the theory below to decide whether a monitor to a compound siphon is redundant.

To disturb the controller region the least, we should allow $M([S_1])$ to reach its maximum; thus setting $M_0(V_{S_1}) = M_0(S_1) - 1$; *S* is said to be limit controlled. In general, $M_0(V_{S_1}) = M_0(S_1) - \xi_{S_1}$, where $\xi_{S_1} \geq 1$ is the control depth variable. ξ_{S_1} is adjusted to be greater than 1 if some dependent siphons are not controlled. As a result, max $M([S_1])$ is less than $M_0(S_1) - 1$ and the controller region is more disturbed causing more states lost.

Definition 4. *Let* $M_0(V_S) = M_0(S) - \xi_S$ *where* $\xi_S \geq 1$ *is called the control depth variable. S is said to reach its limit state when M*(*S*) = 1; *it is limit-controlled iff it is able to reach its limit state but not able to reach unmarked state*; *i.e.,* $\xi_S = 1$ *or* min*M*(*S*) = 1.

Theorem 1. [21]: *Let* (N_0, M_0) *be a net system and* S_0 *be a strongly dependent SMS w.r.t. elementary siphons* S_1, S_2, $\cdots$, *and* S_n *such that where* $\eta_0 = \sum_{i=1}^{n}(\eta_i)$, *and* $\forall i \in 1,2,\cdots,n$, $S_i \cap S_j \neq \varnothing$ *iff* $|i-j| = 1$. N_0 *is extended by n control places* V_{S_1}, V_{S_2}, $\cdots$, *and* V_{S_n} *such that* S_1, S_2, $\cdots$, *and* S_n *are limit-controlled.* S_0 *can never be emptied iff* $b_i = M_0(S_i \cap S_{i+1}) = 1$, $\forall i \in \{1,2,\cdots,n-1\}$.

Note that for strongly dependent siphon $S_0, S_i \cap S_{i+1}$ is a single resource r, $M_0(S_i \cap S_{i+1}) = 1$ implies that there is only one token in the initial marking of r.

5. Controllability for Weakly Dependent Siphons

This section shows that if S_0 weakly depends on S_1 and S_2, then there exists a third siphon S_3 —synthesized from core circuits c_1 and c_2, respectively, such that $\eta_0 = \eta_1 + \eta_2 - \eta_3$. Other properties will also be derived, from which we will derive the controllability of a weakly 2-compound siphon.

Chao [2,3,6], show that in an S^{3}PR, an SMS can be synthesized from a strongly connected resource subnet and any strongly dependent siphon corresponds to a compound circuit where the intersection between any two elementary circuits is at most a resource place.

Let S_0 be a strongly dependent siphon, S_1, S_2, ⋯, *and* S_n be elementary siphons, with $\eta_{S_0} = \eta_{S_1} + \eta_{S_2} + \cdots + \eta_{S_n}$. It is shown in Chao [2,3,6] that c_0 (the core circuit from which to synthesize S_0) is a compound resource circuit containing $c_1, c_2, \cdots, c_n$, and the intersection between any two c_i and c_j, $i = j - 1 > 0$, is exactly a resource place, where c_i (i =0, 1, 2, ⋯, n) is the core circuit from which to synthesize S_i. Thus, if S_0 is a WDS (weakly dependent siphon), the intersection between any two c_i and c_j, $i = j - 1 > 0$, must contain more than one resource place.

Let S_1 and S_2 be the SMS synthesized from basic circuits c_1 and c_2, respectively, where $c_1 \cap c_2 \neq \varnothing$. One can synthesize a third SMS, denoted by S_0, from the strongly connected resource subnet $c_1 \cup c_2$. For S_0 to be a WDS, $c_1 \cup c_2$ must contain more than one resource place.

To simplify the presentation, $\Gamma = c_1 \cap c_2$ is assumed to be $[r_b\ t_b\ r_c]$ on Process 1, where r_b, r_c are two re-source places, $R(c_1) = \{r_a, r_b, r_c\}$ and $R(c_2) = \{r_b, r_c, r_d\}$ as shown in **Figure 1(a)**. The more complicated case can be treated similarly as in [9]. For instance, in **Figure 2**, $R(c_1) = \{p_{14}, p_{15}, p_{16}\}$ and $R(c_2) = \{p_{12}, p_{13}, p_{14}, p_{15}\}$ (one more place than the above one).

It is assumed, that a core circuit spans between process 1 (WP1) and process 2 (WP2) and ($r_1\ r_2 \cdots r_k$) denotes path $[r_1\ t_1\ r_2\ t_2 \cdots r_{k-1}\ t_{k-1}\ r_k]$. Let $c_1 = [r_a\ t_a\ r_b\ t_b\ r_c\ t_c\ r_a]$ and $c_2 = [r_b\ t_b\ r_c\ t_c'\ r_d\ t_d\ r_b]$ span between process 1 and process 2 [see **Figure 1(a)**, where $r_a = p_{16}$, $r_b = p_{15}$, $r_c = p_{14}$, $r_d = p_{13}$, $t_a = t_1$, $t_b = t_2$, $t_c = t_3$, $t_d = t_8$, $t_c' = t_4$]. Note that t_a, t_b, t_c, t_c' and t_d may not be in the same process; some are in process 1 and the rest are in process 2. Consider the resource path on process 1. There are only 2 possible cases satisfying the above conditions are as follows: 1) $(r_a\ r_b\ r_c\ r_d)$ 2) $(r_d\ r_b\ r_c\ r_a)$. Case 2 is equivalent to Case 1 by relabeling r_a by r_d and r_d by r_a, respectively.

Note that it is possible that $(r_a\ r_b\ r_c\ r_b\ r_c \cdots r_b\ r_c\ r_d)$, which can be treated similarly as in Chao [10]. For Case 1, c_1 can be broken two paths: one, $[r_a\ t_a\ r_b\ t_b\ r_c]$, in process 1, denoted by $[r_a\ t_a\ r_b\ t_b\ r_c]_1$; another, $[r_c\ t_c\ r_a]$, in process 2, denoted by $[r_c\ t_c\ r_a]_2$; *i.e.*, $c_1 = [r_a\ t_a\ r_b\ t_b\ r_c]_1 [r_c\ t_c\ r_a]_2$. Similarly, c_2 can be broken two paths: one, $[r_b\ t_b\ r_c\ t_c'\ r_d]$, in process 1, denoted by $[r_b\ t_b\ r_c\ t_c'\ r_d]_1$; another, $[r_d\ t_d\ r_b]$, in process 2, denoted by $[r_d\ t_d\ r_b]_2$; *i.e.*, $c_2 = [r_b\ t_b\ r_c\ t_d']_1 [r_d\ t_d\ r_b]_2$. In the sequel, η_i refers to the T-characteristic vector of siphon S_i synthesized from c_i.

Theorem 2. [9]: *Let* $S = S_1 \oplus S_2$. *Then* $\eta_0 = \eta_1 + \eta_2 - \eta_3$.

On the Process 2 side in **Figure 1(a)**, there should be no PP'-path of the form [$r_d\ t'\ r_c$] (resp. [$r_b\ t_a^*$]) to form basic circuit c_4 (resp. c_4) consisting of only two resource places of r_d and r_c (resp. r_a and r_b), since S_0 is no longer a weakly dependent siphon as derived below. First, $c_1 = c_4 \circ c_3$ and $c_2 = c_5 \circ c_3$. This leads to

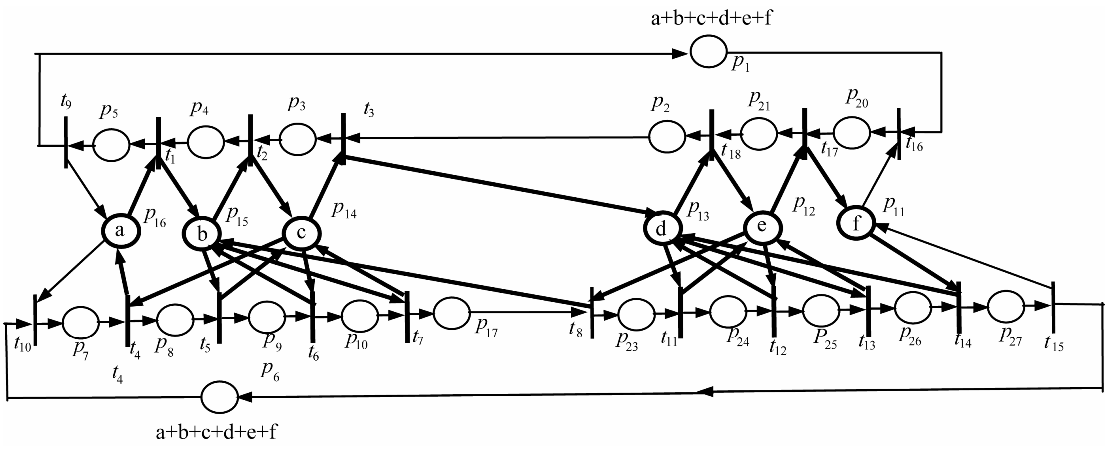


Figure 2. Example a 3-dependent weakly dependent siphon. $S_0 = S_6 = S_1 \oplus S_2 \oplus S_3$, $S_4 = S_{1,2}$, $S_5 = S_{2,3}$ **and** $\eta_0 = \eta_1 + \eta_2 + \eta_3 - \eta_4 - \eta_5$.

$\eta_1 = \eta_3 + \eta_4$, $\eta_2 = \eta_3 + \eta_5$, and $\eta_0 = \eta_1 + \eta_2 - \eta_3 = \eta_3 + \eta_4 + \eta_5$; S_0 strongly depends on S_3, S_4, and S_5 and S_0 is no longer a weakly dependent siphon.

Lemma 1. [9]: *Let* (N_0, M_0) *be a net system and* S_0 *be a weakly dependent SMS w.r.t. elementary siphons* S_1, S_2, *and* S_3 *where* $\eta_0 = \eta_1 + \eta_2 - \eta_3$. *Then*

1) $(A \cup B) = (S_3 \cap P) \subset H(r_b)$, where $A = S_1 \cap [S_2]$, $B = S_2 \cap [S_1]$, $r_b \in S_3$.

2) $H(r_c) \subset [S_3]$ and $H(r_c) \subset [S_0]$.

3) $minM([S_3]) = M_0(r_c) = M_0(S|\{r_b\})$.

Consider the S[3]PR in **Figure 1(a)**, $A = S_1 \cap [S_2] = \{p_4\}$, $B = S_2 \cap [S_1] = \{p_{11}\}$, $A \cup B \subset H(r_b = p_{15}) \subset S_3$. Furthermore, $r_c = p_{14}$, $H(r_c) = \{p_3, p_8, p_{10}\} \subset [S_3] = [p_3, p_8, p_9, p_{10}]$ and $H(r_c) \subset [S_0] = \{p_2, p_3, p_7, p_8, p_9, p_{10}\}$.

Define $\boldsymbol{S_{12}} = \boldsymbol{S_3}$ and $\boldsymbol{S_0} = \boldsymbol{S_1} \oplus \boldsymbol{S_2}$ since $R(S_1 \cap S_2) = R(S_3)$. $S_1 \oplus S_2$ is similar to $S_1 \circ S_2$ in that S_0 can never be emptied if $b = 1$ for both cases. $S_1 \oplus S_2$ is different than $S_1 \circ S_2$ in that $R(S_1 \cap S_2)$ for the former contains more than one resource place while the latter contains only one resource place.

Consider the S[3]PR in **Figure 2**. **Table 2** lists eight SMS S, and core circuits in **Figure 2**. Note that $c_1 \cap c_2 = [p_{15}\ t_2\ p_{14}]$ is not a single resource place and hence $S_7 = S_1 \oplus S_2$ cannot be a strongly dependent siphon and is a weakly dependent siphon. Similarly, $c_2 \cap c_3 = [p_{13}\ t_{18}\ p_{12}]$ is not a single resource place and hence $S_8 = S_2 \oplus S_3$ cannot be a strongly dependent siphon and is a weakly dependent siphon. Note that c_2 contains 4 rather than 3 resource places assumed above. Yet, all relevant theory remains true.

Theorem 3. *Let* (N_0, M_0) *be a net system and* S_0 *be a weakly dependent SMS w.r.t. elementary siphons* S_1, S_2, *and* S_3 *where* $\eta_0 = \eta_1 + \eta_2 - \eta_3$. N_0 *is extended by control places* V_{S_1}, V_{S_2}, *and* V_{S_3}, *such that* S_1, S_2, *and* S_3 *are limit-controlled. Let* $A = S_1 \cap [S_0]$, $B = S_2 \cap [S_0]$, *and* $A \cup B = (S_3 \cap P) \subset H(r)$, *(by Proposition* 1) $r \in S_3$, S_0 *can never be emptied iff* $b = M_0(r) = 1$; 2) S_0 *is limit controlled iff* $b = M_0(r) = 1$.

Proof. 1) ($\leftarrow$) Assume S_0 is unmarked (hence $M(r) = 0$, $\forall r \in S_0$), while each of S_1, S_2, and S_3 is marked; *i.e.*, $M(S_1) > 0$, $M(S_2) > 0$, and $M(S_3) > 0$. By Proposition 1, it holds that $A \cup B = (S_3 \cap P) \subset H(r)$. Let $M(S_1 \cap [S_0]) = M(S_2 \cap [S_0]) \ge 1$ since S_1 and S_2 are marked. $M_0(r) = M(H(r)) = 1$ since $M(r) = 0$ and $M_0(r) = 1$. Now $M(H(r)) \ge M(S_3 \cap P) = M(A \cup B) \ge 2$. However, $1 = M(H(r)) \ge M(S_3) \ge 2$, which is impossible. Thus, it is impossible that $M(S_1 \cap [S_0]) = M(S_2 \cap [S_0]) \ge 1$. This leads to that either $M(S_1 \cap S_0) > 0$ or $M(S_2 \cap S_0) > 0$, which implies that S_0 can never be emptied. ($\rightarrow$) Assume contrary and $b = M_0(r) > 0$. Then it is possible that $M(A) > 0$ and $M(B) > 0$ such that each of S_1, S_2, and S_3 is marked, while $M(S_1 \cap S_0) = M(S_2 \cap S_0) = 0$ or S_0 is unmarked against the assumption that S_0 is marked. 2) ($\leftarrow$) If $b = M_0(r) = 1$, then there is a reachable marking such that $M(A) = 1$, $M(B) = 0$ or $M(B) = 1$, $M(A) = 1$. Either one implies that $M(S_0) = 1$. ($\rightarrow$) Assume contrary and $b = M_0(r) > 1$. The proof of part 1 of this theorem indicates that S_0 is unmarked. Hence S_0 cannot be limit controlled against the assumption. ◊

Thus, it is similar to a strongly dependent siphon S synthesized from a compound circuit $c_1 \circ c_2$ where S is also controlled if $b_1 = 1$ and both S_1 and S_2 are limit controlled. For the S[3]PR in **Figure 2** where each elementary siphon is limit-controlled and $S_0 = S_4$ is controlled as well. Assume otherwise and S_4 is empty. Then $M(S_1 \cap [S_0]) = M(S_2 \cap [S_0]) = 1$. Let $A = S_1 \cap [S_0] = \{p_4\}$, $B = S_2 \cap [S_0] = \{p_{11}\}$, $A \cup B = (S_3 \cap P) \subset H(r = p_{11})$. If $b = 1 = M_0(r)$, then it is impossible that $M(S_1 \cap [S_0]) = M(S_2 \cap [S_0]) = 1$. Thus, S_0 can never be emptied.

The condition (*i.e.*, $A \cup B = (S_3 \cap P) \subset H(r)$, $r \in S_3$) in Theorem 3 is generally true for vertically stacked S[3]PR (in most literatures); that is sink transitions of S_1 and S_2 are in the same processes. The condition may not hold for horizontally stacked S[3]PR; that is, sink transitions of S_1 and S_2 are in different processes. In this case, $A = S_1 \cap [S_0] \subset H(r_1)$ and $B = S_2 \cap [S_0] \subset H(r_2)$, $r_1 \ne r_2$. However, it remains true that $A \cup B = (S_3 \cap P)$. S_0 can become unmarked when $M(S_1 \cap [S_0]) = M_0(r_1) > 0$, $M(S_2 \cap [S_0]) = M_0(r_2) > 0$, $M(S_3) > M(A \cup B) > 0$.

Define $\boldsymbol{S_{12}} = \boldsymbol{S_3}$ and $\boldsymbol{S_0} = \boldsymbol{S_1} \oplus \boldsymbol{S_2}$ since $R(S_1 \cap S_2) = R(S_3)$. $S_1 \oplus S_2$ is similar to $S_1 \circ S_2$ in that S_0 can never be emptied if $b = 1$ for both cases. $S_1 \oplus S_2$ is different than $S_1 \circ S_2$ in that $R(S_1 \cap S_2)$ for the former contains more than one resource place while the latter contains only one resource place.

Theorem 4. *Let* (N_0, M_0) *be a net system and* S_0 *be a weakly dependent SMS such that* $S_0 = S_1 \oplus S_2 \oplus \cdots \oplus S_n$ *denoting the fact that* $\forall i \in 1, 2, \cdots, n$, $S_i \oplus S_j$ *holds iff* $|i - j| = 1$. *Let* $A_i = S_i \cap [S_0]$, $B_i = S_{i+1} \cap [S_0]$, $A_i \cup B_i = (S_{i,i+1} \cap P) \subset H(r_i)$, *where* $r_i \in S_{i,i+1}$. N_0 *is extended by control places* V_{S_1}, V_{S_2}, $V_{S_{12}}$, V_{S_3}, $V_{S_{32}}$, ⋯, *and* V_{S_n}, $V_{S_{n-1,n}}$ *such that* S_1, S_2, S_{12}, S_3, S_{23}, ⋯, *and* S_n, $S_{n-1,n}$ *are limit-controlled*, 1) S_0 *can never be emptied iff* $b_i = M_0(r_i) = 1$, $\forall i \in \{1, 2, \cdots, n-1\}$; 2) S_0 *is limit controlled iff*

Table 2. Eight SMS S, and core circuits in Figure 2.

S	Set of places	c
S_1	$p_5, p_{17}, p_{14}, p_{15}, p_{16}$	$c_1^e = c_1\ [p_{14}\ t_4\ p_{16}\ t_1\ p_{15}\ t_2\ p_{14}] + H^{PP'}\ [p_{15}\ t_5\ p_{14}], [p_{14}\ t_6\ p_{15}]$ and $[p_{15}\ t_7\ p_{14}]$.
S_2	$p_4, p_{26}, p_{12}, p_{13}, p_{14}, p_{15}$	$c_2^e = c_2\ [p_{15}\ t_2\ p_{14}\ t_3\ p_{13}\ t_{18}\ p_{12}\ t_8\ p_{15}] + H^{PP'}\ [p_{13}\ t_{11}\ p_{12}], [p_{12}\ t_{12}\ p_{13}], [p_{15}\ t_5\ p_{14}], [p_{14}\ t_6\ p_{15}]$ and $[p_{15}\ t_7\ p_{14}]$
S_3	$p_2, p_{27}, p_{11}, p_{12}, p_{13}$	$c_3^e = c_3\ [p_{13}\ t_{18}\ p_{12}\ t_{17}\ p_{11}\ t_{14}\ p_{13}] + H^{PP'}\ [p_{13}\ t_{11}\ p_{12}], [p_{12}\ t_{12}\ p_{13}]$, and $[p_{13}\ t_{13}\ p_{12}]$
S_4	$p_4, p_{17}, p_{14}, p_{15}$	$c_4^e = c_4\ [p_{15}\ t_2\ p_{14}\ t_6\ p_{15}] + H^{PP'}\ [p_{15}\ t_5\ p_{14}] + H^{PP}\ [p_{15}\ t_7\ p_{14}]$
S_5	$p_2, p_{26}, p_{12}, p_{13}$	$c_5^e = c_5\ [p_{13}\ t_{18} p_{12}\ t_{12}\ p_{13}] + H^{PP'}\ [p_{13}\ t_{11}\ p_{12}]$, and $[p_{13}\ t_{13}\ p_{12}]$
S_6	$p_5, p_{27}, p_{11}, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}$	$c_6^e = c_1^e \oplus c_2^e \oplus c_3^e$
S_7	$p_5, p_{26}, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}$	$c_7^e = c_1^e \oplus c_2^e$
S_8	$p_4, p_{27}, p_{11}, p_{12}, p_{13}, p_{14}, p_{15}$	$c_8^e = c_2^e \oplus c_3^e$

$b_i = M_0(r_i) = 1$, $\forall i \in \{1, 2, \cdots, n-1\}$.

Proof. 1) Prove by induction. By Theorem 3, the theorem holds for $n = 2$. Assume it holds for $k = n-1$. We need to prove that it also holds for $k = n$. Let $S^* = S_{n-1} \oplus S_n$, $S_u = S_1 \oplus S_2 \oplus \cdots \oplus S_{n-1}$, $A = S_u \cap [S_0]$, $A^* = S_{n-1} \cap [S^*]$ and $B = S_n \cap [S_0]$. By Theorem 3.2, S_u is limit controlled. It is easy to see that $A^* = A$ and $A^* \cup B = (S_{n-1,n} \cap P) \subset H(r_{n-1})$. Hence $A \cup B(S_{n-1,n} \cap P) \subset H(r_{n-1})$ and $A \cup B \subset S_{u,n} H(r_{n-1})$. $\subset S_0$ can never be emptied by Theorem 3. 2. ($\rightarrow$) If S_0 is limit controlled, then it can never be emptied. By part 1 of this theorem, $b_i = M_0(r_i) = 1$, $\forall i \in \{1, 2, \cdots, n\}$. ($\leftarrow$) If $b_i = M_0(r_i) = 1$, $\forall i \in \{1, 2, \cdots, n\}$. Consider S^*, S_u, A, A^* and B defined in the proof of part 1 of this theorem. Prove by induction. Assume it holds for $k = n-1$. Then S_u is limit controlled since $b_i = M_0(r_i) = 1$, $\forall i \in \{1, 2, \cdots, n-2\}$. We need to prove that it also holds for $k = n$. By Theorem 3, it holds for $S' = S_u \oplus S_n = S_0$ since both S_u and S_n are limit controlled and $b_{n-1} = M_0(r_{n-1}) = 1$. Thus, S_0 is limit controlled. ◊

This theorem implies that if the condition in the theorem is satisfied, then the compound siphon is already controlled. Thus, Theorem 4 shows that the controllability for $S_0 = S_1 \oplus S_2 \oplus \cdots \oplus S_n$ for a weakly dependent siphon S_0 is similar to that for a strongly dependent siphon. As a result, the control for WDS needs no longer be that conservative as by Li and Zhou.

Table 3 lists eight SMS S and their [S]. $S_0 = S_6 = S_1 \oplus S_2 \oplus S_3$, $S_4 = S_{1,2}$, $S_5 = S_{2,3}$ and $\eta_0 = \eta_1 + \eta_2 + \eta_3 - \eta_4 - \eta_5$. It can be verified that

$A_1 = S_1 \cap [S_2] = \{p_{17}\}$, $B_1 = S_2 \cap [S_1] = \{p_4\}$, $A_1 \cup B_1 \subset H(p_{15}) \subset S_{1,2}$.

$A_2 = S_2 \cap [S_3] = p_{26}$, $B_2 = S_3 \cap [S_2] = p_2$, $A_2 \cup B_2 \subset H(p_{13}) \subset S_{2,3}$.

By Theorem 3, S_6 can never be emptied *iff* $b_1 = M_0(p_{15}) = b_2 = M_0(p_{13}) = 1$. This has been confirmed using the INA (Integrated Net Analyzer). The resulting controlled net (see **Table 4**) reaches 32298 states out of the total 45135 states of the uncontrolled model, where $M_0(p_{16}) = M_0(p_{14}) = M_0(p_{12}) = M_0(p_{11}) = 2$ and $M_0(p_1) = M_0(p_6) = 2$. Note that even though some new siphons (such as control siphons) are generated by the presence of monitor places, the controlled net is live without adding monitors for these new siphons. Why this is so is a subject for future research.

Physically, for S_6 to become empty under M, $M(r) = 0$, for every resource place in S_6. Thus, all tokens in $M_0(r)$ must stay in $H(r)$. If $M_0(p_{13}) > 1$, it is possible that both p_2 and p_{26} are marked, which implies both S_2 and S_3 are marked. Even if $M_0(p_{15}) = 1$, this token may go to p_{17} such that S_1 is also marked. S_6 may become unmarked when all tokens in p_{16} and p_{11} go to p_7 and p_{20}, respectively. Thus, even though each of S_1, S_2, and S_3 is marked by adding a monitor, S_6 may still become unmarked and needs a monitor.

On the other hand, if $b_1 = M_0(p_{15}) = b_2 = M_0(p_{13}) = 1$, we show that if S_6 becomes unmarked, at least one of S_1, S_2, and S_3 is also unmarked contradicting the fact that each of S_1, S_2, and S_3 is controlled by adding a monitor. For S_6 to become unmarked, all tokens in p_{16} and p_{11} go to p_7 and p_{20}, respectively.

Hence, for S_1 and S_3 to be marked, p_{17} and p_2 must be both marked since $S_1 \cap [S_6] = \{p_{17}\}$ and $S_3 \cap [S_6] = \{p_2\}$. Now, both p_4 and p_{26} are unmarked since $\{p_4, p_{17}\} \subset H(p_{15}), \{p_2, p_{26}\} \subset H(p_{13})$, and $M_0(p_{15}) = M_0(p_{13}) = 1$. But then S_2 is unmarked contradicting the fact that each of S_2 is controlled by adding a monitor. Thus, the assumption that S_6 becomes unmarked is incorrect and we prove that S_0 is con-

Table 3. Eight SMS *S*, [*S*] and η in Figure 2.

S	$[S]$	η
S_1	$p_3, p_4, p_7, p_8, p_9, p_{10}$	$t_1 - t_3 + t_7 - t_{10}$
S_2	$p_2, p_3, p_8, p_9, p_{10}, p_{17}, p_{21}, p_{23}, p_{24}, p_{25}$	$t_2 - t_4 + t_{13} - t_{17}$
S_3	$p_{20}, p_{21}, p_{23}, p_{24}, p_{25}, p_{26}$	$-t_8 + t_{14} - t_{16} + t_{18}$
S_4	p_3, p_8, p_9, p_{10}	$t_2 - t_3 - t_4 + t_7$
S_5	$p_{21}, p_{23}, p_{24}, p_{25}$	$-t_8 + t_{13} - t_{17} + t_{18}$
S_6	$p_2, p_3, p_4, p_7, p_8, p_9, p_{10}, p_{17}, p_{20}, p_{21}, p_{23}, p_{24}, p_{25}, p_{26}$	$t_1 - t_{10} + t_{14} - t_{16}$
S_7	$p_2, p_3, p_4, p_7, p_8, p_9, p_{10}, p_{17}, p_{21}, p_{23}, p_{24}, p_{25}$	$t_1 - t_{10} + t_{13} - t_{17}$
S_8	$p_2, p_3, p_8, p_9, p_{10}, p_{17}, p_{20}, p_{21}, p_{23}, p_{24}, p_{25}, p_{26}$	$t_2 - t_4 + t_{14} - t_{16}$

Table 4. Disturbanceless control model of the net in Figure 2.

S	Monitor	$V_S^\bullet$	${}^\bullet V_S$	M_0
S_1	V_1	t_3, t_{10}	t_1, t_7	$a + b + c - 1$
S_2	V_2	t_4, t_{17}	t_2, t_{13}	$b + c + d + e - 1$
S_3	V_3	t_8, t_{16}	t_{14}, t_{18}	$d + e + f - 1$
S_4	V_4	t_3, t_4	t_2, t_7	$b + c - 1$
S_5	V_5	t_8, t_{17}	t_{13}, t_{18}	$d + e - 1$
S_6	V_6	t_{10}, t_{16}	t_1, t_{14}	$a + b + c + d + e + f - 1$
S_7	V_7	t_{10}, t_{17}	t_1, t_{13}	$a + b + c + d + e - 1$
S_8	V_8	t_4, t_{16}	t_2, t_{14}	$b + c + d + e + f - 1$

trolled.

Alternatively, we will prove based on the following observations from **Table 3** that:

$[S_6] = [S_1] + [S_2] + [S_3] - [S_4] - [S_5]$,

$[S_7] = [S_1] + [S_2] - [S_4]$,

and $[S_8] = [S_2] + [S_3] - [S_5]$.

In general, if $\eta_{S_0} = \sum_{i=1}^{n}\left(a_i \eta_{S_i}\right) - \sum_{j=1}^{m}\left(b_{n+j}\eta_{S_{n+j}}\right)$, then $[S_0] = \sum_{i=1}^{n} a_i [S_i] - \sum_{j=1}^{m} b_{n+j}[S_{n+j}]$ or $\lambda_{[S_0]} = \sum_{i=1}^{n}\left(a_i \lambda_{[S_i]}\right) - \sum_{j=1}^{m}\left(b_{n+j}\lambda_{[S_{n+j}]}\right)$ as shown in the following theorem.

Theorem 5. *Let* (N_0, M_0) *be a net system and* S_0 *be a dependent SMS w.r.t. elementary siphons* S_1, $S_2, \cdots, S_n$, S_{n+1}, $S_{n+2}, \cdots,$ *and* S_{n+m} *where*

$$\eta_{S_0} = \sum_{i=1}^{n}\left(a_i \eta_{S_i}\right) - \sum_{j=1}^{m}\left(b_{n+j}\eta_{S_{n+j}}\right) = \sigma_a - \sigma_b,$$

$$\sigma_a = \sum_{i=1}^{n}\left(a_i \eta_{S_i}\right), \text{ and } \sigma_b = \sum_{j=1}^{m}\left(b_{n+j}\eta_{S_j}\right).$$

Then

1) $\forall S \in \{S_0, S_1, S_2, \cdots, S_n, S_{n+1}, S_{n+2}, \cdots, S_{n+m}\}$, $\eta_S = -\eta_{[S]}$ (characteristic T-vector of the complementary set of siphon S equals the negative of that of S).

2) $\lambda_{[S_0]} = \sum_{i=1}^{n}\left(a_i \lambda_{[S_i]}\right) - \sum_{j=1}^{m}\left(b_{n+j}\lambda_{[S_{n+j}]}\right)$, where a_i, $b_j \in R$ (set of real numbers), $i \in \{1, 2, \cdots, n\}$ and $j \in [1, 2, \cdots, m]$ (characteristic P-vectors of the complementary sets of siphon S, S_1, S_2, $\cdots$, S_n, S_{n+1}, S_{n+2}, $\cdots$, S_{n+m} follow the same equation as that of the corresponding characteristic T-vectors).

3) The Marking Equality (ME) holds:,

$$M([S_0]) = \sum_{i=1}^{n}\left(a_i M([S_i])\right) - \sum_{j=1}^{m}\left(b_{n+j} M([S_{n+j}])\right),$$
$$M \in R(N, M_0) \qquad (1)$$

(total tokens in the complementary sets of siphon S, S_1, S_2, $\cdots$, S_n, S_{n+1}, S_{n+2}, $\cdots$, S_{n+m} follow the same equation as that of the corresponding characteristic

T-vectors).

Proof. 1) $S \cup [S] = S_R \cup \left(\cup_{r \in S_R} H(r) \right)$ is the support of a P-invariant I based on Property 1 and $S \cap [S] = \varnothing$. Note that $S_R = S \cap P_R$. $\forall p \in S \cup [S]$, $I(p) = 1$ (valid for OPN); otherwise, $I(p) = 0$. Thus, $I = \lambda_S + \lambda_{[S]}$. $I^T \bullet [N] = \lambda_S^T \bullet [N] + \lambda_{[S]}^T \bullet [N] = 0$ (By the definition of P-invariant), where 0 is a vector with all components being 0.

$$\Rightarrow \eta_S = -\eta_{[S]}.$$

2) Based on equation $\eta_{S_0} = \sigma_a - \sigma_b$, the fact that $\eta_S = -\eta_{[S]}$ and $\eta_S^T = \lambda_S^T \bullet [N]$, we have

$$\eta_{[S_0]} = \sum_{i=1}^{n} \left(a_i \eta_{[S_i]} \right) - \sum_{j=1}^{m} \left(b_{n+j} \eta_{[S_{n+j}]} \right) \Rightarrow$$

$$\lambda_{[S_0]}^T \bullet [N] = \sum_{i=1}^{n} \left(a_i \lambda_{[S_i]}^T \bullet [N] \right) - \sum_{j=1}^{m} \left(b_{n+j} \lambda_{[S_{n+j}]}^T \bullet [N] \right) \Rightarrow$$

$$\left(\lambda_{[S_0]} - \sum_{i=1}^{n} a_i \lambda_{[S_i]} + \sum_{j=1}^{m} b_{n+j} \lambda_{[S_{n+j}]} \right)^T \bullet [N] = 0$$

If $\zeta = \lambda_{[S_0]} - \sum_{i=1}^{n} a_i \lambda_{[S_i]} + \sum_{j=1}^{m} b_{n+j} \lambda_{[S_{n+j}]} \neq 0$, then ζ is a P-invariant. However, all places in $[S_0]$, $[S_1]$, $[S_2]$, ⋯, and $[S_{n+m}]$ are not marked in the initial marking of N and hence the union of $[S_0]$, $[S_1]$, $[S_2]$, ⋯, and $[S_{n+m}]$ cannot be the support of a P-invariant. This implies that $\zeta = 0 \Rightarrow$

$$\lambda_{[S_0]} = \sum_{i=1}^{n} a_i \lambda_{[S_i]} - \sum_{j=1}^{m} b_{n+j} \lambda_{[S_{n+j}]}.$$

3) Multiplying both sides of the equation in Equation (1) by M^T, we have

$$\lambda_{[S_0]} \bullet M^T = \sum_{i=1}^{n} a_i \lambda_{[S_i]} \bullet M^T - \sum_{j=1}^{m} b_{n+j} \lambda_{[S_{n+j}]} \bullet M^T \Rightarrow$$

$$M([S_0]) = \sum_{i=1}^{n} a_i M([S_i]) - \sum_{j=1}^{m} b_{n+j} M([S_{n+j}]). \quad \square$$

This theorem holds for FMS modeled by OPN [not General PN (GPN)] such as an S^3PMR since we have assumed $\forall p \in S \cup [S]$, $Y(p) = 1$. However, it can be extended to FMS modeled by GPN such as S^4PR and S^3PGR^2 by replacing M with *W*((*M*(*A*))), the weighted sum of tokens in $A = S$ or [*S*].

This *ME* says that the total number of tokens trapped in [S_0] and [S_i], follow the same linear algebraic relationship between η_{S_0} and η_{S_i}, *i* = 1, 2, ⋯, *n*, *n* + 1, ⋯, *n* + *m*. This is because physically, $-\eta_S(t)$ is the number of tokens removed from S by firing t once. Now, $\max tM([S_i]) = M_0(S_i) - 1$ (S_i is said to be *limit controlled*) for S_i to have tokens. In order for S_0 to be controlled, we have $M(S_0) > \max M([S_0])$ or

$$M(S_0) > \max\left(\sum_{i=1}^{n} a_i M([S_i]) \right) - \min\left(\sum_{j=1}^{m} b_{n+j} M([S_{n+j}]) \right) \tag{2}$$

To be conservative, the term associated with the negative terms is set to zero. That is, if $M(S_0)$ is large enough to be greater than max ($\sum_{i=1}^{n} a_i M([S_i])$), then Equation (2) necessarily holds.

However it may not hold that

$$M(S_0) > a_1 (M_0(S_1) - 1) + a_2 (M_0(S_2) - 1)$$
$$+ a_n (M_0(S_n) - 1) = \sum_{i=1}^{n} a_i (M_0(S_i) - 1)$$
$$= \sigma_{aM_0} - \sum_{i=1}^{n} a_i$$

That is S_0 may not be controlled when each S_i is limit controlled. After lowering $M([S_i])$ to $M_0(S_i) - \xi_{S_i}$, $\xi_{S_i} \geq 1$ where ξ_{S_i} is the control depth variable mentioned in Li and Zhou [13], for each S_i, it may hold that

$$M(S_0) > a_1 (M_0(S_1) - \xi_{S_1}) + a_2 (M_0(S_2) - \xi_{S_2})$$
$$+ a_n (M_0(S_n) - \xi_{S_n}) = \sum_{i=1}^{n} a_i (M_0(S_i) - \xi_{S_i})$$
$$= \sigma_{aM_0} - \sigma_\xi.$$

This is exactly the *MLI* (marking linear inequality mentioned in [13]). In the sequel, we do not set the term associated with the negative terms to zero; therefore achieving a better controllability.

S_1, S_2, ⋯, and S_5 (resp. S_6, S_7, and S_8) are basic (resp. compound) siphons; they are limit-controlled by setting $M_0(V_{S_1}) = a + b + c - 1$, $M_0(V_{S_2}) = b + c + d + e - 1$, $M_0(V_{S_3}) = d + e + f - 1$, $M_0(V_{S_4}) = b + c - 1$, and $M_0(V_{S_5}) = d + e - 1$. $M_0(S_6) = a + b + c + d + e$, $M_0(S_7) = b + c + d + e + f$, and $M_0(S_8) = a + b + c + d + e + f$. **Table 4** shows the disturbanceless control model of the net in **Figure 2**.

$M_{\max}([S_i]) = M_0(V_{S_i})$, *i* = 1, 2, ⋯, 5, and

$M_{\max}([S_7]) = M_{\max}([S_1]) + M_{\max}([S_2]) - M_{\min}([S_4])$

(by Theorem 4 and part 3 of Lemma 1)

$$= a + b + c - 1 + b + c + d + e - 1 - c$$
$$= a + b + c + d + e + b - 2.$$

Thus, S_7 is limit-controlled (no need for control elements) *iff* $M_0(S_7) > M_{\max}([S_7])$, which implies that $b < 2$ or $b = M_0(p_{13}) = 1$.

On the other hand, if $b > 1$, we need to add control elements for S_7 to be limit-controlled. For instance, $M_0(p_{13}) = b = 3$, of which, one token goes to p_4 and two tokens to p_{17}, also a tokens in p_{16} to p_5 and e tokens in p_{12} to p_{21}. This makes S_7 emptied and yet both S_1 and S_2 are marked (consistent with the fact that both S_1 and S_2 are controlled) since $p_{17} \in S_1$ and $p_4 \in S_2$ and both p_4 and p_{17} are marked.

Similarly, one can argue that S_8 is limit-controlled (no need for control elements) *iff* $M_0(p_{13}) = 1$. Now consider the controllability of S_6.

$$M_{\max}([S_6]) = M_{\max}([S_1]) + M_{\max}([S_2]) + M_{\max}([S_3]) - M_{\min}([S_4]) - M_{\min}([S_5])$$

(by Part 3 of Theorem 4)

$$\begin{aligned} &= (a+b+c-1)+(b+c+d+e-1) \\ &\quad +(d+e+f-1)-c-e \\ &= (a+b+c+d+e+f)+(b+d-3) \end{aligned}$$

(by Part 3 of Lemma 1).

where $M_{\min}([S_4]) = c$ and $M_{\min}([S_5]) = d$. Thus, S_6 is limit-controlled (no need for control elements) *iff* $M_0(S_6) > M_{\max}([S_6])$, which implies that $b+d-3<0$ or $b = M_0(p_{13}) = d = M_0(p_{15}) = 1$. If $b = d = 1$, then $(b+d-3) = -1$ and $M_0(S_6) = M_{\max}([S_6]) + 1$, otherwise, $b+d \geq 3$ and $M_0(S_6) \leq M_{\max}([S_6])$; S_6 is emptied.

6. Conclusions

We have derived the controllability for both strongly (SDS) and weakly dependent siphons (WDS). It is surprised that they have the same controllability. Thus, this paper improves the conservative control policy for WDS in [8].

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