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Ground State Properties of Closed Shell ^{4}He Nucleus under Compression

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Abstract

This paper addresses the effects of nuclear properties at different potentials. Reid Soft Core (RSC) and Nijmegen potentials were used to study the nuclear system of ^{4}He nucleus. It has been examined with and without large compression. Moreover, the constrained spherical Hartree-Fock (CSHF) approximations are used as a major tool of analysis. The dependence of the ground state properties was investigated to the degree of compression. It was noticed that it is possible to compress the nucleus to a smaller volume and the nucleus becomes more bounded using RSC than Nijmegen potential. It was also shown that the spectrum of single particle levels increases more rapidly for Nijmegen than RSC potential under compression. Finally, the radial density distribution remains constant, except in the interior region, while it is larger with RSC than Nijmegen potential. At large compression, the radial density distribution becomes larger than that in the interior region when RSC potential is used.

Keywords

Nuclear Structure, Compressed Finite Nuclei, Binding Energy, Single Particle Energy, Radial Density

1. Introduction

Traditional nuclear model assumes that nuclei are composed of neutrons and protons. Nuclear properties can be understood in terms of the interactions between nucleons.

The structure of compressed nuclei is a current challenge of both experimental and theoretical physicists. At present, the best available experimental and theoretical data on the structure of compressed nuclei come from the analysis of the breathing mode [1] [2] [3].

By using the nucleon-nucleon (N-N) interaction, a deep understanding of the structure of finite nuclei is a major issue that is needed to be resolved [4]. In non-relativistic nuclear model, the nucleus is considered as a nucleon system. It contains protons and neutrons without internal resonances. Compressed nuclei are expected to occur during heavy-ion experiments [5]. The problem of compression for nuclei is very useful in understanding astrophysics. The nuclei structure with their finite number of particles has to be calculated from simulating an effective N-N interaction and transition potentials which are not sufficiently well known.

In this paper, the nuclear properties of ^{4}He nucleus have been studied at equilibrium and large compression. Nijmegen and RSC potentials are used [6] [7]. Nijm II potential describes pp data. It is local potential. RSC potential is depend on partially erroneous phase shift in the single, triplet state and spin orbit coupling [8]. The sensitivity of the ground state properties to the potential is specifically examined.

The objective of this study is to investigate the effect of the potential used on softening the nuclear equation of state. This study also will shed some light on the behavior of nuclear matter under extreme conditions. It has importance in astrophysics [9]. Also it gives us a better understanding of its behavior in N-N collisions as in heavy ion collisions in high-energy supercollider's [10].

This work is written as: Sec. 2 shows a short description of a statement of problem. Sec. 3 specifies the results and discussions, while conclusion is given in Sec. 4.

2. Statement of Problem

A nuclear system of A-nucleon (N neutrons and Z protons) is considered with its spin s and isospin τ which is 1/2 for each. The Hamiltonian of this system consists of the single particle energy and the two-body interaction as:

$$\hat{H} = \sum_{i=1}^{A} \hat{T}_i + \sum_{i<j}^{A} V_{ij} \tag{2.1}$$

where $\hat{T}_i$ denotes the single particle kinetic energy operator, which in terms of single particle momentum $\boldsymbol{p}$ is:

$$\hat{T}_i = \boldsymbol{p}_i^2 / 2m \tag{2.2}$$

here m is the nucleon mass. V_{ij} is the two-body interaction term. It is two body V_{NN} and Coulomb V_C interactions.

The exact solution of the Schrödinger equation in the infinite Hilbert space was solved for mass number smaller than 20 only [11] [12] [13] [14]. For mass number greater than 20, the truncated model space with an effective Hamiltonian, H_{eff}, is used. So, Equation (2.1) can be written as:

$$\hat{H}'_{eff} = \sum_{i=1}^{A} \hat{T}_i + \sum_{i<j}^{A} \left(V_{eff}\right)_{ij} \tag{2.3}$$

A two-body effective Hamiltonian $\hat{H}'_{eff}$ is introduced by using the relative kinetic energy operator $\left(T_{rel}\right)_{ij}$ instead of the single particle energy operator.

$$\hat{H}'_{eff} = T_{rel} + V_{eff} = T_{rel} + V_{eff}^{NN} + V_C \tag{2.4}$$

where $(T_{rel})_{ij}$ represents the pure two body natures. This is evident form the relative kinetic energy operator between pairs of nucleons

$$(T_{rel})_{ij} = (\boldsymbol{p}_i - \boldsymbol{p}_j)^2 / 2mA \tag{2.5}$$

The V_{eff}, however, is the sum of Brueckner G-matrix and the lower order folded diagram (2nd order in G) acting between pair of nucleons in the no-core model space The V_{eff}, however, is Brueckner G-matrix and the lower order folded diagram (2nd order in G) that acts between pair of nucleons in the no-core model space [15] [16].

The matrix element of the two-body part of the effective Hamiltonian is constructed by using two-particle harmonic oscillator basis. They have good total angular momentum J and isospin τ.

There are two problems: dimensional full Hilbert space and short range repulsion of the core potential. The first problem can be solved by truncating the full Hilbert space using Block-Horowitz theory. The second problem is removed by solving the Brueckner-Bethe Goldstone equation; the potential V matrix elements are replaced by the Brueckner G-matrix elements in the series expansion of V_{NN}.

$$G(\omega) = V + VQ / (\omega - H_0) G(\omega) \tag{2.6}$$

where the variable ω is the starting energy and Q is the Pauli operator. It prohibits particles from scattering into occupied states. H_0 is the unperturbed single particle Hamiltonian [17].

In order to evaluate matrix element of $\hat{H}'_{eff}$, the harmonic oscillator basis are chosen with $\hbar\omega$ = 14.0 MeV. By using harmonic oscillator wave function, the relative and center of mass coordinates can be separated. Therefore, a great simplification results in the calculation of the two-body matrix elements.

By using the effective Hamiltonian within the chosen model space, the Hartree-Fock equation for nucleon orbital can be derived, by applying the variation principle. By applying a static load, Compressed system is achieved. The radial constraint represents an external force to compress or expand the nucleus. For details see Refs. [18]-[27].

No-core model space with six major oscillator shells was used for calculations. In this space, orbits were: $0s_{1/2}$, $0p_{3/2}$, $0p_{1/2}$, $0d_{5/2}$, $1s_{1/2}$, $0d_{3/2}$, $0f_{7/2}$, $1p_{3/2}$, $0f_{5/2}$, $1p_{1/2}$, $0g_{9/2}$, $1g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, $0h_{11/2}$, $0h_{9/2}$, $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$ and $2p_{1/2}$. The finite truncated model space which is used in this study was described.

A no-core model space was used to avoid calculating core polarization effects with realistic effective interaction. Also, all nucleons were considered active, so there were no terms in the expansion that involves particle-hole excitations.

Techniques used in this study as same as Refs. [18]-[27] without nuclear resonances.

3. Results and Discussion

The results of the ground state properties of ^{4}He nucleus namely the binding energy, the single particle energy (SPE) and the radial density distribution are presented. These results are obtained by using model space of six shells within the CSHF approximation based on RSC and Nijmegen (Nijm. II) potentials.

The adjusting parameters are listed in **Table 1**. With these parameters, an equilibrium root mean square radius r_{rms} and E_{HF} are found using RSC and Nijm. II potentials. The value of λ_1 is less than one. This because of the operator for kinetic energy is a positive and is normalized by itself this will reduce its magnitude. The value of λ_2 is larger than one to compensate for the lack of sufficient binding when the full Hilbert space is truncated to a finite model space.

The E_{HF} energies versus r_{rms} using RSC and Nijmegen potentials are displayed in **Figure 1**. It can be seen from the figure that there is a reduction in

Table 1. Values of adjusting parameters λ_1, λ_2 and $\hbar\omega'$ of the effective Hamiltonian for ^{4}He in six oscillator shells to get an agreement between the HF results and experimental data [28]. The binding energy (nuclear radius r_{rms}) was −28.296 MeV (1.46 fm) for ^{4}He.

Nucleus	Potential	λ_1	λ_2	$\hbar\omega'$ (MeV)
^{4}He	RSC	0.980	1.186	17.872
^{4}He	Nijmegen	0.990	1.041	15.700

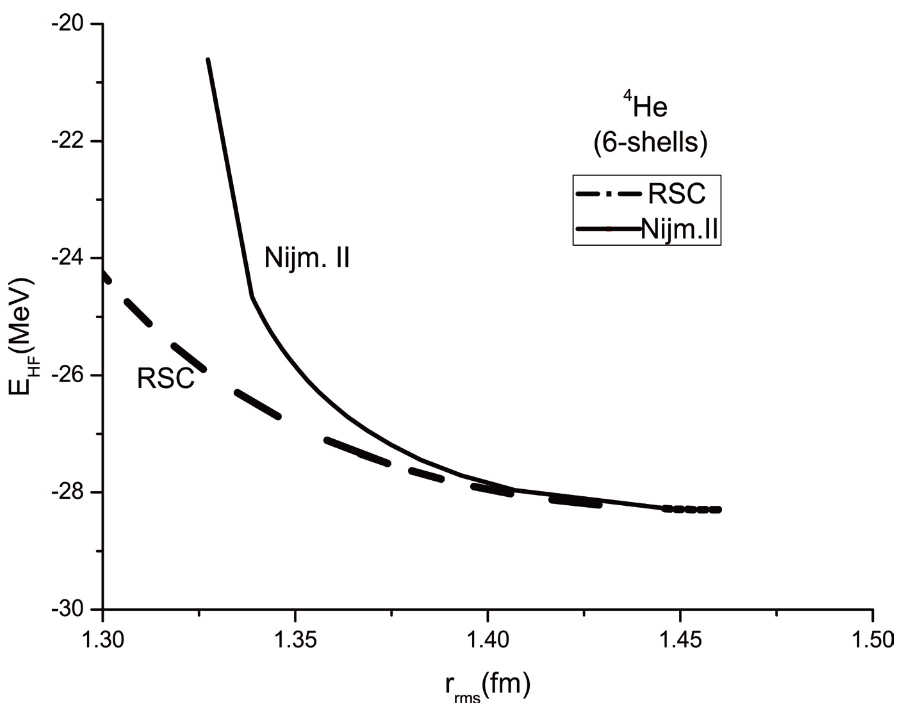

Figure 1. Constrained spherical Hartree-Fock energy for ^{4}He in six-oscillator shells versus r_{rms}. The dashed and solid curves are for RSC and Nijmegen potentials, respectively.

the volume of the nucleus by about 15% for RSC potential. Thus, the binding energy is reduced by 61% for this reduction in volume. It appears from these results that 11.05 MeV of the excitation energy, is enough to reduce the volume and the binding energy by 15%, 61% respectively more for RSC potential. This means that the lager dense inner part of the nucleus initially responds to the external load more radially than the outer part.

By using Nijmegen potential, the reduction of volume is about 25% compared to its volume at equilibrium case. In this case, the reduction in the binding energy is 27%. *i.e.* the binding energy of $^4He^4He$ nucleus at equilibrium case E_{HF} is −28.296 MeV and at large compression is −20.611 MeV. This means that nuclear equation of state becomes stiffer for the compressed nucleus. It can be noted that at large compression the nuclear binging energy for Nijmegen potential is larger than the binding energy for RSC potential.

In Figure 2, the lowest neutron single particle energy levels as a function of compression are shown. The order of the orbitals is exact with standard shell model. The orbitals curved up as the load on the nucleus increases. For Nijmegen potential, the levels cure up more rapidly than the RSC potential when the nucleus is compressed. This is because the positive kinetic energy of the nucleons becomes more effective than the attractive mean field core of the nucleons.

The energy spectrum also displays the gaps between the shells. For the compressed nucleus, the ordering of the energy spectrum levels and the gaps among them are conserved. The splitting of the energy spectrum levels in each subshell is an indicator that the orbital-spin (L-S) coupling is strong enough in

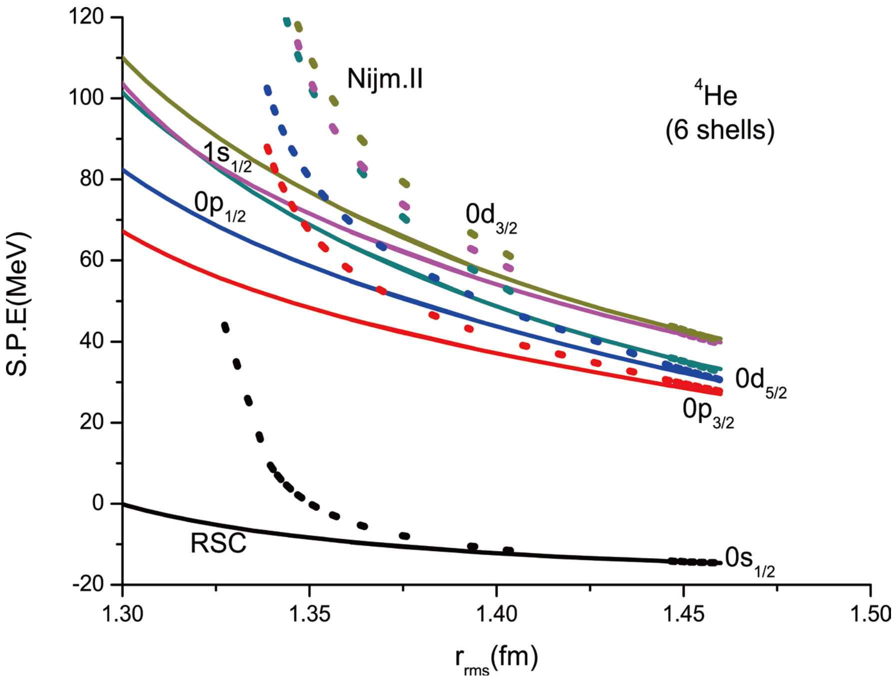

Figure 2. Spectrum energy of lowest six neutron orbitals for ^4He in 6 shells as a function of r_{rms}. The solid and dashed levels are for RSC and Nijmegen potentials, respectively.

both RSC and Nijmegen potentials. If the static load increases then the L-S coupling becomes stronger. The behaviour of the spectrum energies (except the deepest bound orbital which actually drops with compression) shifts to higher energies for the compressed nucleus so that the binding energies become lower. The curvature goes up more and more for the surface orbits under compression. This means that the surface is more responsive to compression than the interior of the nucleus.

In addition, energy spectrum is formed entirely from the underlying microscopic Hamiltonian. This is a good point since the calculated energy spectrum agrees with the expected ordering of the theoretical shell model in the dominantly nucleon orbitals, and the energy levels exhibit clear gaps among the shells. It is also worth noting that the closest orbitals to binding energy are more sensitive in the compressed nucleus for both potentials.

Figure 3 displays the total density distribution for ρ_T versus the displacement from the center of the nucleus at equilibrium. It is noted that the ρ_T is larger for RSC potential than the ρ_T for Nijmegen potential in the interior region of the nucleus, but this difference is very small. In the exterior region, the ρ_T is approximately same for both potentials.

Figure 4 shows the total density at large compression and equilibrium cases using Nijmegen potential. This figure displays that when the nucleus volume is reduced by 23% of the equilibrium case, the radial density increases by about 1.20 of its value at the equilibrium case. In the compressed nucleus, the nuclear radial density becomes denser in the interior than the exterior regions. This

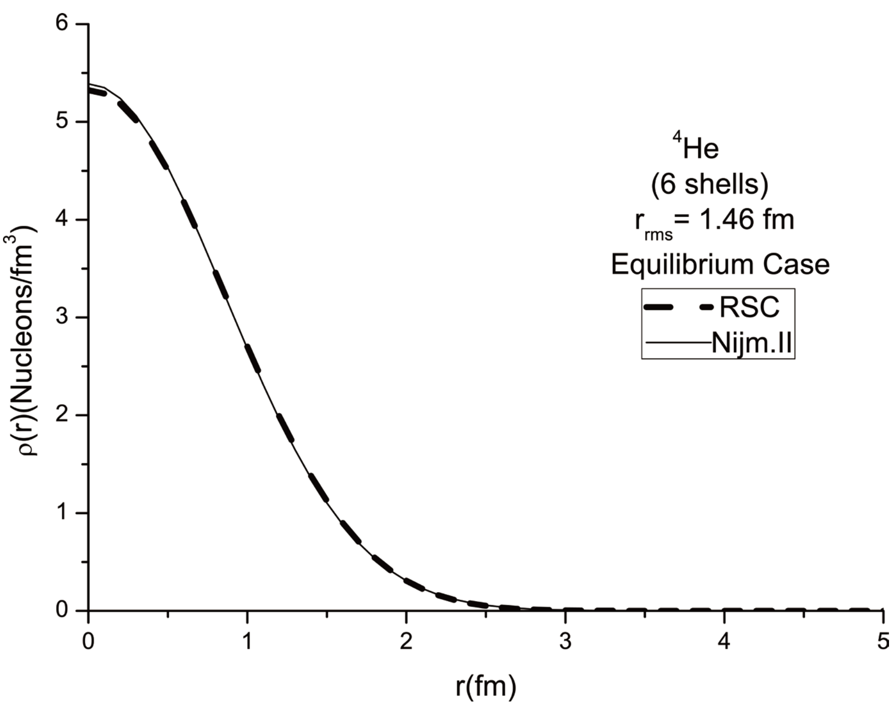

Figure 3. Total ρ_T (dashed curve for Nijm. II potential and solid line for RSC potential) density for ^{4}He at radius $r_{rms} = 1.46$ fm in a 6 shells(Equilbrium case).

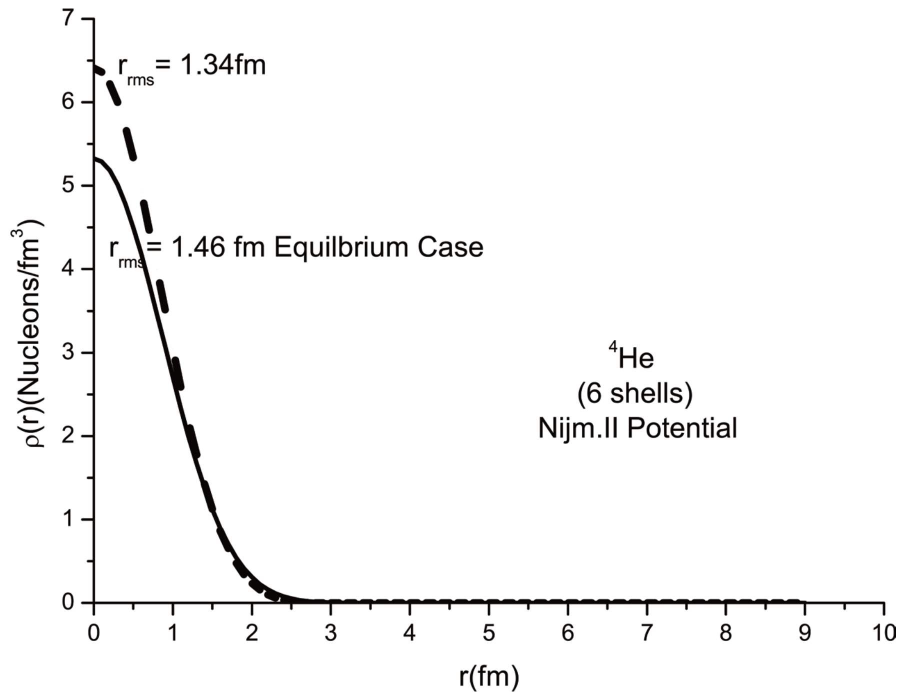

Figure 4. ρ_T (dashed curve) radial density distribution for ^{4}He at point mass $r_{rms} = 1.34$ fm and solid cure for equilibrium case in a 6 shells model space by using Nijm. II potential.

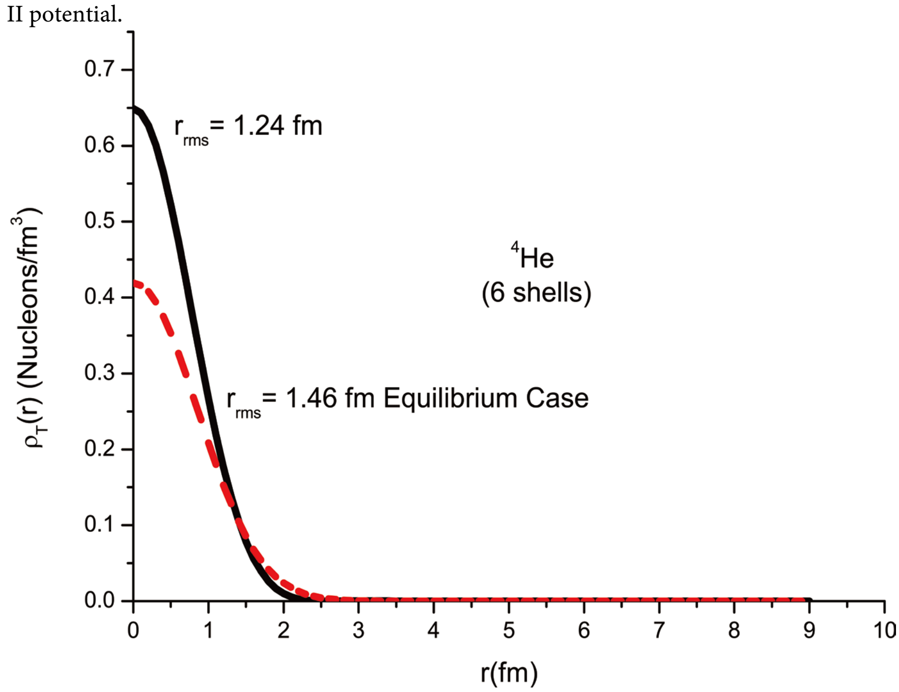

Figure 5. ρ_T (dashed curve) radial density distribution for ^{4}He at point mass $r_{rms} = 1.24$ fm and solid cure for equilibrium case in a 6 shells space by using RSC potential.

result shows that the surface of the nucleus becomes more and more responsive as the load increases more and more.

In **Figure 5**, the total density distribution at equilibrium and large static compression ($r_{rms} = 1.24$ fm) is shown for RSC potential. This figure sees that when the nucleus volume is reduced by 39% of the equilibrium volume, the radial density increases by about 1.19 of its value at the equilibrium case.

It is clear that the density distribution following: the nuclear density becomes denser in the interior than the exterior regions for both potentials under compression. Also, the nuclear density becomes denser in the interior regions for Nijmegen than RSC potential. It is less dense in the exterior regions for Nijmegen than RSC potential. This means when the static load increases more and more on the nucleus, the surface of the nucleus becomes more and more responsive. Finally, it is possible to compress the nucleus by using RSC potential more than Nijmegen potential.

4. Conclusions

In the CSHF approximation, the ground state properties of the double magic spherical ^{4}He nucleus have been investigated by using RSC and Nijmegen potentials. A realistic effective N-N Hamiltonian is used in a six shells model space. The nucleus can be compressed to a smaller volume using RSC than Nijmegen potential.

If the compression increases, E_{HF} will increase very sharply towards zero energy (unbound state). The behavior of the energy spectrum levels is found to be in a good agreement with those of the traditional phenomenological shell model. At higher compression levels, the overlapping of energies of single particles become more pronounced. Therefore, the nucleus becomes free (*i.e.* unbounded nucleus). The single particle energy levels curve up under compression more rapidly for Nijmegen than RSC potential.

Finally, if the compression increases then the total radial density will increase. The radial density distribution is the same except in the interior region; it is larger with RSC than Nijmegen potential. At large compression, the situation is reversed especially in the the interior region, the radial density distribution becomes larger than the radial density distribution when RSC potential is used.

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