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Developing an Expert and Reflexive Approach to Problem-Solving: The Place of Emotional Knowledge and Skills

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Abstract

Nowadays it is widely accepted that mathematics and, especially, problem-solving tasks, are particularly concerned by the issue of emotions. Yet, educational interventions designed to improve students' problem-solving competence and performance still mainly focus on cognitive and metacognitive knowledge and skills. The main purpose of this study was to design and assess the benefit of a training program that promotes the development of not only cognitive but also emotional knowledge and skills. This benefit was assessed using four variables, namely, problem-solving performance, problem-solving competence, academic emotions and emotion regulation strategies. 428 fifth and sixth graders took part in the study, split into four conditions: 1) a "cognition" condition which received an intervention on an eight-step problem-solving process; 2) an "emotion" condition in which emotional knowledge and skills were developed through various activities; 3) an "emotion and cognition" condition overlapping the two previous ones, and 4) a "control" condition. The findings showed that the "emotion and cognition" condition and the "cognition" condition had equivalent cognitive efficiency. However, only the former reduced negative emotions, aroused the emergence of positive ones, promoted the use of adaptive emotion regulation strategies and discouraged the use of maladaptive ones. The practical implications for educational practices and possible avenues for further research are discussed.

Keywords

Problem-Solving Competence, Elementary School Students, Emotion Knowledge and Skills, Heuristic Strategy, Intervention

1. Introduction

Students' poor grades in mathematical problem-solving have preoccupied researchers for several decades now (e.g., Blum, 2011; De Corte & Verschaffel, 2008; Demonty & Fagnant, 2014; Hanin & Van Nieuwenhoven, 2016a; Mevarech & Kramarski, 1997; Polya, 1957; Schoenfeld, 1992; Verschaffel, De Corte, Lasure, Van Vaerenbergh, Bogaerts, & Ratinckx, 1999). So far, the focus has been on the development of cognitive and metacognitive strategies to overcome the difficulties observed. The accumulating evidence, collected during the last decade, of the profound effects exerted by emotions on students' learning behaviors and academic emotions have changed the perspective of educational psychologists. Emotions influence students' attention, choice of learning strategies, source of regulation (external vs. internal), motivation to learn and their interactions with peers and teachers (Pekrun, 2014; Schutz & Pekrun, 2007). Studies have shown that mathematics and, especially, problem-solving tasks, are particularly concerned by the issue of emotions (Ahmed, van der Werf, Kuyper, & Minnaert, 2013; Kim & Hodges, 2012; Op't Eynde, De Corte, & Mercken, 2004). These findings have led to a reconceptualization of what expertise in mathematical problem solving requires. This is no longer limited to the mastery of cognitive and self-regulative knowledge and skills but also includes emotional components (De Corte & Verschaffel, 2008). However, just as the acquisition of cognitive and self-regulative skills does not occur spontaneously or automatically, the individual is not naturally endowed with emotional knowledge and skills. These latter need to be fostered and taught at school (Goetz, Frenzel, Pekrun, & Hall, 2005; Kim & Hodges, 2012). Since emotions are domain specific (e.g., Goetz, Frenzel, Pekrun, Hall, & Lüdtke, 2007; Goetz, Pekrun, Hall, & Haag, 2006), this apprenticeship has to be disciplinarily situated as well.

An overview of the available literature on this topic highlights the existence of two kinds of training-programs: on the one hand, programs that aim to develop an expertise in mathematical problem solving through the development of cognitive and self-regulative strategies (e.g., Blum, 2011; De Corte, Verschaffel, & Masui, 2004; Mevarech & Amrany 2008; Perels, Gürtler, & Schmitz, 2005) and, on the other hand, social and emotional learning (SEL) programs that aim to prevent risks and promote positive development among young people through the development of cognitive, affective, communication and behavioral competences (Payton, Wardlaw, Graczyk, Bloodworth, Tompsett, & Weissberg, 2000). Yet, to our knowledge, no attempt has been made so far to integrate both dimensions within the same training-program. Moreover, studies that have looked at the first kind of training-programs have measured problem-solving improvement through students' use of the problem-solving strategies taught, and through their performance. Yet, scholars (Elia, van den Heuvel-Panhuizen, & Kolovou, 2009; Kaizer & Shore, 1995) have shown that being competent in problem solving requires not only knowing and applying various strategies correctly, but also being flexible, namely, being able to select the strategies to use

depending on the problem given. The present contribution attempts to palliate these limitations. Our first goal consists in designing and assessing a training program that promotes the development of an expert and reflexive approach to mathematical problem-solving through the development of cognitive, emotional and regulative knowledge and skills among upper elementary students. Our second goal is to propose a more complete measure of problem-solving competence by taking into account not only the correctness of the use of strategies but also strategy flexibility.

2. Theoretical Background

2.1. Cognitive Dimension of Problem-Solving Competence

Nowadays, scholars acknowledge that the development of an expertise in mathematical problem solving requires the reconceptualization of mathematical problems as exercises in mathematical modeling; that is, the consideration of a problem's statement as the description of a situation in everyday life that can be modeled mathematically (Blum & Niss, 1991; Verschaffel, Greer, & De Corte, 2000). Mathematical modeling is viewed as a complex and cyclic process involving a number of phases (e.g., Blum & Niss, 1991; Fagnant, Demonty, & Lejonc, 2003; Verschaffel et al., 2000). In a previous study (Hanin & Van Nieuwenhoven, 2016a), we gave an account of the main processes used to solve non-routine word problems[1] as well as recommendations resulting from empirical studies (e.g., Fuchs, Fuchs, Prentice, Burch, Hamlett, et al., 2003; Fagnant & Demonty, 2005) within an integrative eight-step cycle (**Figure 1**, for a complete description see Hanin & Van Nieuwenhoven, 2016a). In addition, the majority of the steps constituting this solution process are heuristic strategies, namely, "search strategies for problem analysis and transformation which do not guarantee, but
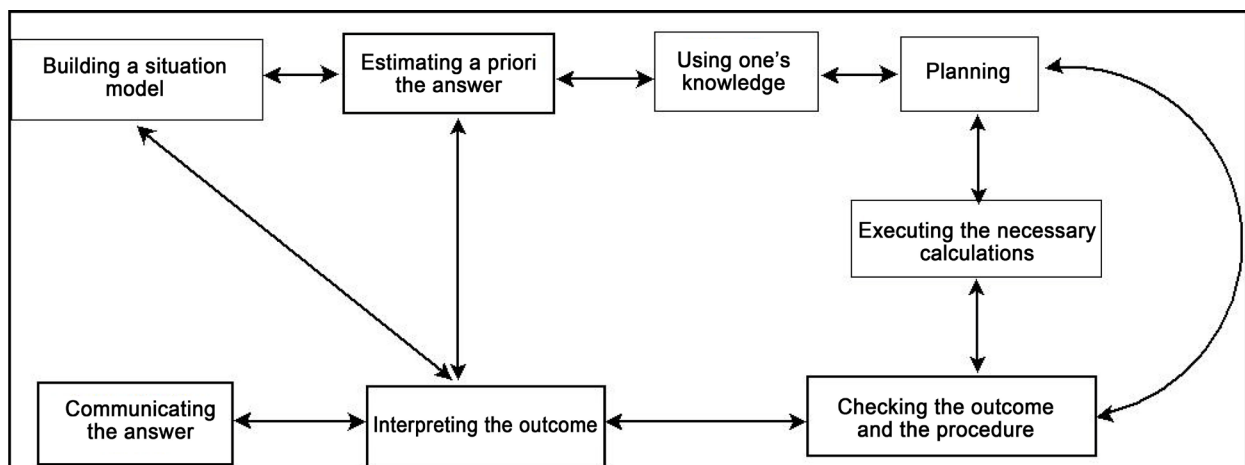
Figure 1. Schematic diagram of non-routine word problem-solving process.

[1]In this study, we use the terms "non-routine word problem" and "problem" interchangeably, in the sense of Elia et al. (2009), to refer to word problems whose solution does not appear immediately and whose resolution is not based on the application of the procedure that has just been seen in class.

significantly increase the probability of finding the correct solution of a problem because they induce a systematic approach to the task" (De Corte et al., 2004: p. 372). However, as noted above, being competent in problem solving is not limited to the use of heuristic strategies, but requires using them correctly, flexibly and thoughtfully, that is to say, self-regulating one's problem-solving approach (De Corte & Verschaffel, 2008; Focant & Grégoire, 2008). In the present study, we focused on what Elia et al. (2009) called inter-task strategy flexibility, defined as the student's ability to select and adjust his or her strategies to the demands of the problem-solving task. For instance, planning the steps of a problem that contains only two steps is an example of non-flexibility. Thoughtful use means the awareness, control and regulation of one's cognitive activities (Focant & Grégoire, 2008). So far, empirical studies have shown that cognitive and self-regulative interventions in mathematical problem-solving increase the use of heuristic strategies, support the development of a global self-regulative approach and improve performance (e.g., Blum & Leiss, 2007; De Corte, Verschaffel, & Masui, 2004; Mevarech & Amrany, 2008; Perels, Gürtler, & Schmitz, 2005).

Supporting the development of an accurate, flexible and thoughtful use of heuristic strategies appears positively and strongly related to problem-solving performance (e.g., Baroody, 2003; Elia et al., 2009; Schoenfeld, 1992; Torbeyns, De Smedt, Stassens, Ghesquière, & Verschaffel, 2009). People who develop such strategic behavior can modify their behavior according to changing situations and conditions, develop more refined strategies and procedures that are better adjusted to the special features of the situation and generate more creative and appropriate solutions to problems.

2.2. Emotional Dimension of Problem-Solving Competence

Academic emotions are emotions that pertain to learning, instruction and achievement in academic settings (Pekrun, 2006; 2014). On the whole, positive emotions have a beneficial effect on the student's attention, motivation to learn, choice of learning strategies, self-regulation of learning, academic performance and also on his/her psychological and physical health. Conversely, and globally, negative emotions have a deleterious effect on these dimensions (Ahmed, van der Werf, Kuyper, & Minnaert, 2013; Peixoto, Sanches, Mata, & Monteiro, 2017; Pekrun, 2006; 2014). Yet, empirical studies reported that students experienced mostly negative emotions during problem-solving tasks (Hanin & Van Nieuwenhoven, submitted; Op't Eynde, De Corte, & Mercken, 2004). So, just as it is of major importance for students to be aware of and control their cognitive activities, it is crucial that they do the same with their emotions. On this point, scholars agreed that students need to develop knowledge and skills related to emotions (Goetz et al., 2005; Mikolajczak, Quoidbach, Kotsou, & Nelis, 2009; Pekrun, 2014). They agreed on five core competencies, namely, emotion identification, emotion understanding, emotion expression, emotion regulation, and emotion utilization (Mikolajczak et al., 2009). The training-program designed in

the context of this study aims to develop the first four competencies. Emotion identification consists in awareness and identification of one's own emotions (Mikolajczak et al., 2009). This presupposes openness to both positive and negative emotions and a sufficient emotional vocabulary. Emotion expression implies taking into account what is to be expressed, the interlocutor to whom emotional information is addressed, and the way information is expressed in order to do it in a socially acceptable way (Rimé, 2005). Emotion understanding pertains to the understanding of the causes of one's emotions. It is a question of identifying the unsatisfied or threatened needs underlying the emotions felt. Finally, emotion regulation consists in influencing which emotions we feel, when we feel them, and how we experience and express them (Gross, 1998). Both positive and negative emotions may be down- or up-regulated. In the current study, we focused on the down-regulation of negative emotions through six strategies used by upper-elementary students to regulate their negative emotions during problem-solving tasks: task utility self-persuasion, help-seeking, brief attentional relaxation, emotion expression, negative self-talk and dysfunctional avoidance (Hanin, Grégoire, Mikolajczak, Fantini-Hauwel, & Van Nieuwenhoven, 2017).

3. The Present Study

In the current study, we sought to contribute to the growing knowledge base of the key cognitive and emotional ingredients of training-programs that aim to develop an expert and reflexive approach to problem solving. We designed and developed a training program that aims to develop cognitive, regulative and emotional knowledge and skills, and assessed its effects on upper elementary students' academic emotions, emotion regulation strategies, problem-solving competence and problem-solving performance. More precisely, four conditions were contrasted (Figure 2). First, a "cognitive" condition where the problem-solving process presented above, embedded in an overall self-regulated approach, is taught according to the methodology suggested by Veenman, Van Hout-Wolters and Afflerbach (2006). This methodology consists, for each strategy, in specifying the "what" (what it consists of), the "why" (its usefulness), the "when" (the most relevant moment in the problem-solving process to implement it), and the "how" (the way to implement it correctly). Next, an "emotion" condition where a wide range of activities (video-clips, analyses of mini-case studies, drawing, Chinese portrait, short lectures, facial expressions analyses, group tasks and discussions, role play) are presented to question the learner's emotional relationship to problem-solving tasks and to develop his/her emotion knowledge and competencies. Then an "emotion and cognition" condition which overlaps the two previous conditions. Finally, a control condition which received no specific training program but solved the same problems, at the same rate. In addition, both the "emotion" and the control conditions performed, on a weekly basis, additional mathematics activities in order to spend as much time in mathematics as the two other conditions. Thereby, we controlled for the "time of
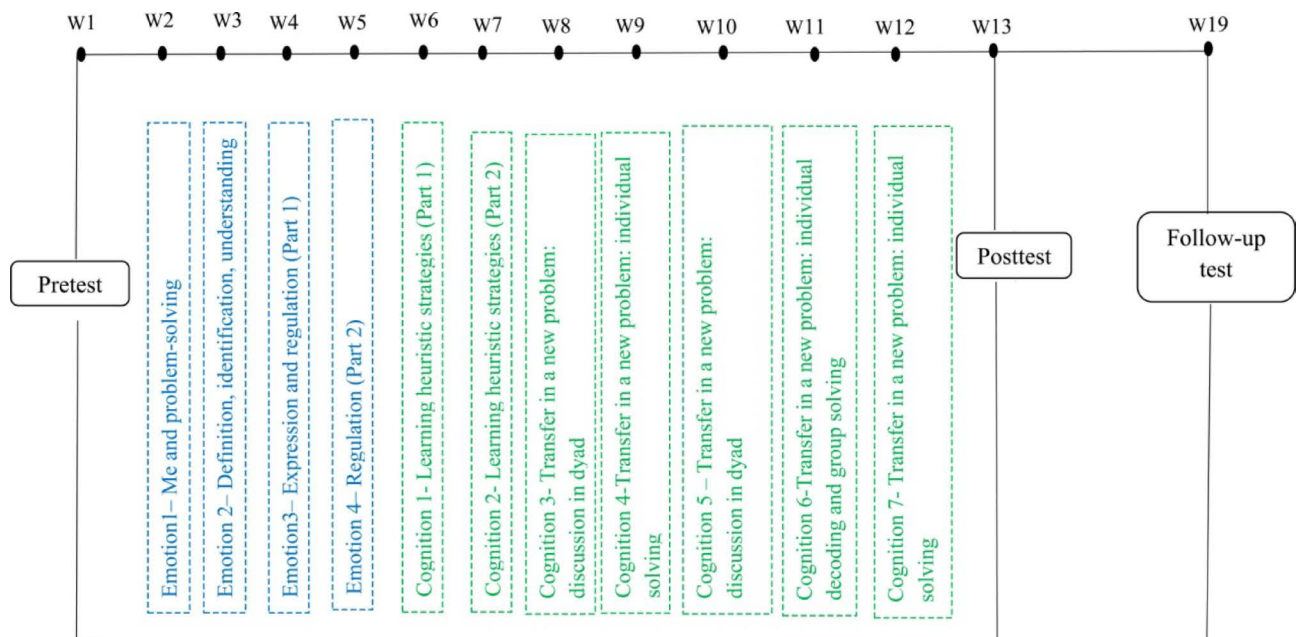
Note. w1 = week 1; w2 = week 2, etc.

Figure 2. Schematization of the experimental design.

task" variable (for a more detailed description of the conditions, see **Appendix 1**). For a question of ecological validity, the four conditions were carried out by regular teachers who had previously received two days' training from the first author.

On the basis of the literature presented above, we formulated the following hypothesis:

- The "emotion and cognition" condition will surpass the three other conditions regarding all the dependent variables (hypothesis 1);
- The "cognition" condition will surpass both the "emotion" and the "control" conditions on the cognitive dimensions (i.e., problem-solving competence and performance) (hypothesis 2);
- The "emotion" condition will surpass the "cognition" and the "control" conditions on the emotional dimensions (i.e., academic emotions and emotion regulation strategies) (hypothesis 3);
- The "emotion" condition will surpass the "control" condition in terms of problem-solving performance (hypothesis 4).

4. Method

4.1. Participants

428 upper elementary students[2] from nine French-speaking Belgian schools took part in the present study. We took care to make the four conditions as comparable as possible both regarding the schools' characteristics (i.e., geographical localization and socioeconomic level) and the teachers' characteristics (gender and

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[2]Those who were absent for one or more pretest, posttest or retention test were removed from the sample. They represent 15% of the sample.

Table 1. Characteristics of the sample constituting each condition.

		Geographical localization	Socio-economic index	Teacher's gender	Teacher's years teaching (years)	Condition
	Class 1	Brabant Wallon	19	F	8 - 15	Cognition
School 1	Class 2	Brabant Wallon	19	F	16 - 25	Cognition
	Class 3	Brabant Wallon	19	F	16 - 25	Cognition
School 2	Class 1	Brabant wallon	20	F	8 - 15	Emotion and cognition
	Class 1	Namur	9	F	16 - 25	Cognition
School 3	Class 2	Namur	9	F	4 - 7	Cognition
	Class 3	Namur	9	M	16 - 25	Control
	Class 1	Namur	19	F	16 - 25	Control
School 4	Class 2	Namur	19	F	8 - 15	Control
	Class 3	Namur	19	F	16 - 25	Control
School 5	Class 1	Brussels	9	F	16 - 25	Emotion and cognition
	Class 2	Brussels	9	F	0 - 1	control
	Class 1	Brussels	8	F	4 - 7	Emotion and cognition
School 6	Class 2	Brussels	8	F	4 - 7	Emotion and cognition
	Class 3	Brussels	8	M	16 - 25	Emotion and cognition
	Class 4	Brussels	8	F	2 - 3	Control
School 7	Class 1	Brussels	8	F	16 - 25	Emotions
	Class 2	Brussels	8	F	8 - 15	Emotions
School 8	Class 1	Brussels	3	F	8 - 15	Cognition
School 9	Class 1	Brussels	1	F	8 - 15	Control

years teaching) (Table 1). The "control" condition was made up of 148 students (M age = 10.7; SD = 0.63), 54.7% were girls; the "cognition" condition consisted of 131 students (M age = 10.1; SD = 0.36), of which 51.9% were girls. The "emotion" condition was composed of 41 students (M age = 11; SD = 0.53), of which 58.5% were girls. Finally, the "emotion and cognition" condition comprised 108 students (M age = 10.8; SD = 0.59), of which 49.1% were girls. Participants provided data on three occasions: prior (Time 1), immediately after (Time 2) and six weeks after the training-program (Time 3).

4.2. Measures

Problem-solving performance was assessed by means of a performance test made up of four non-routine word problems (the pretest problems are available in **Appendix 2**). This test was designed on the basis of the expertise of the first

author and on Fagnant and Demonty's (2005) textbook. In selecting these problems, our main concern was to respect the three characteristics of openness, realism, and complexity, recommended by Verschaffel and his colleagues (2000) to develop, among students, an expert and reflexive approach to problem solving. Students' performance was appraised by a global score obtained by summing up the correct answers on a binary scale (0 = *wrong answer*, 1 = *right answer*). Interscorer agreement, computed on 40% of protocols, by two independent blind scorers, was 0.95. The test took on average 45 minutes to complete. It is noteworthy that for each measurement time, it was the same four mathematical structures that were used to design the problems' statements, and only the "dressed-up" of the problems was modified.

Problem-solving competence was assessed through three indicators, namely, "strategy use", "correct strategy use" and "strategy flexibility". The heuristic strategies were measured on the basis of students' written productions. Regarding "strategy use", we scrutinized students' pretest, posttest, and retention test for traces of the application of five[3] out of the seven heuristic strategies taught, namely; building a situation model, using one's knowledge (i.e., searching for connections with familiar problem structures in order to identify the structure of the problem, and identifying the mathematical tool to be used to solve this type of problem), verifying the outcome and the procedure, interpreting the outcome and communicating the solution. Then, we computed a presence score for each heuristic strategy according to a binary scale (0 = *missing*; 1 = *present*). As for "correct strategy use", a ratio between the number of correct implementations and the total number of implementations was calculated for each heuristic. As regards "strategy flexibility", a ratio between the number of students who used the heuristic and the total number of students was computed. As a flexible use of heuristics requires to be familiar with heuristic, only the posttest and the retention test problems were analyzed, and that only for the "cognition" and the "emotion and cognition" conditions. The scoring of the use of heuristics and of their correct use had a high interrater reliability $\left(k = 0.89 \right)$.

Academic emotions experienced by students while solving non-routine mathematical problems were evaluated through a questionnaire presenting facial expressions. These included positive emotions (enjoyment, pride, relief), and negative emotions (boredom, fear, anger, hopelessness, shame, worry, frustration, and nervousness) most frequently experienced by elementary students when dealing with problem-solving tasks (Hanin & Van Nieuwenhoven, submitted; D'Mello & Graesser, 2011; Frenzel, Pekrun, & Goetz, 2007; Prawat & Anderson, 1994). Students were asked to indicate to what extent they felt each emotion when solving a math problem using a 5-point Likert scale (1 = *never* to 5 = *always*).

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[3]We chose not to report the results pertaining to the estimation and planning strategies. First, students' estimations were very broad and vague making the data unexploitable. Second, the planning strategy is not relevant given that the problems of the problem-solving tests require a maximum of two steps to be solved. "Executing the necessary calculations" is more a step than a heuristic strategy.

Emotion regulation strategies were appraised using the Children's Emotion Regulation Scale in Mathematics (CERS-M) (Hanin et al., 2017). This questionnaire consists of 14 items, rated on a 4 point Likert scale (ranging from 1 = *(almost) never* to 4 = *(almost) always*) and targets six strategies used by 5th and 6th graders to regulate their emotions when solving mathematical problems (see Table 2).

Classroom social climate, one of the control variables, was assessed using a French version of the questionnaire designed by Patrick, Kaplan and Ryan (2007) for use with upper elementary students. Students responded to 27 items assessing their perception of teacher emotional and academic support, their peers' emotional and academic support, the promotion of mutual respect and the promotion of interaction (see Table 3). Responses ranged from 1 (*false*) to 4 (*true*). Due to poor internal consistency, the student academic support scale and the promotion of mutual respect scale were removed from the subsequent analysis.

4.3. Fidelity of Implementation

As outlined by Durlak and Dupre (2008), one cannot interpret the results of the implementation of a training-program without first ensuring that it has been delivered as planned. To monitor the program's fidelity of implementation, several tools were used. First, a check-list containing the prescribed instructions (activities,

Table 2. Description of the emotion regulation strategies' scale.

Scale name	Item number	Cronbach α	Sample item
Help-seeking	2	0.74	"I ask the teacher to help me to solve the problem"
Brief attentional relaxation	3	0.60	"I put down my pen a few seconds and stretch my arms"
Task-utility self-persuasion	2	0.70	"Even if I do not like solving math problems, I tell myself that it is important to do so in order to be able to understand them and thereby to succeed"
Emotion expression	2	0.65	"I tell my neighbor that the problem makes me angry, sad, hopeless, or bored"
Negative self-talk	3	0.77	"I tell myself that it is terrible not being able to solve the problem and that I am sure that it only happens to me"
Dysfunctional avoidance	2	0.66	"In order not to experience an unpleasant moment, I tell myself that I will solve the problem later"

Table 3. Description of the classroom social climate scale.

Scale name	Item number	Cronbach α	Sample item
Teacher emotional support	5	0.72	"My teacher really understand how I feel about things"
Teacher academic support	5	0.63	"My teacher cares about how much I learn"
Student emotional support	6	0.77	"In my class, other students are nice to me"
Student academic support	2	0.47	"In my class, other students want me to do well in school"
Promotion of mutual respect	5	0.50	"My teacher does not allow students to make fun of other students' ideas in class"
Promotion of interaction	4	0.66	"My teacher often allows us to discuss our work with classmates"

methodology and duration) was filled-in by each teacher at the end of each lesson. Examination of the checklists showed that teachers completed 95% of the steps as prescribed. Second, teachers were asked to keep a diary with their feelings, students' responsiveness, potential amendments, and any other item of information considered relevant. Third, at the end of their two-day training, teachers received a manual containing a full and detailed description of the training-program attributed to them, that is, a number of theoretical aspects, a scenario of each lesson (goals, working methods, teacher's and student's activities, equipment and duration) and a detailed answer to each problem based on the heuristic strategies taught and highlighting the diversity of possible paths. Fourth, the researcher and the teachers met weekly to exchange views about positive experiences and difficulties encountered as well as to take stock of the past lessons and to plan the following ones.

4.4. Analysis

As significant baseline variations were found between the four conditions for several variables under investigation, the data were analyzed using normalized gain scores. Normalized gain scores are recommended when pretest scores differ widely between groups (Marx & Cummings, 2007; Gilley & Clarkston, 2014). By normalizing the gain score, that is, by dividing the absolute gain by the maximum possible gain, the variance in pretest scores is accounted for. Normalized change scores were calculated, using the formula developed by Marx and Cummings (2007):

$$
\begin{aligned}
&\frac{post-pre}{Max.-pre} && \text{if } post > pre \\
&drop && \text{if } post = pre = \text{maximum or minimum} \\
&0 && \text{if } post = pre \\
&\frac{post-pre}{pre} && \text{if } post < pre
\end{aligned}
$$

More precisely, when the pretest is lower than the posttest (or the retention test), we talk about normalized gain score which represents the amount students learned (post-pre) divided by the amount they could have learned (max-pre) (Marx & Cummings, 2007). However, when students score higher on the pretest than the posttest (or the retention test), it is a normalized loss score (post-pre divided by pre). Normalized scores were expressed as a percentage and so may reach a maximum of 100%. According to Marx and Cummings's formula, participants with the maximum or the minimum score on both the pretest and the posttest (or retention test) were dropped from the analysis, which explains why the sample size varies for each dependent variable.

To compare the mean normalized scores of the four conditions on each of the variable under study, univariate analyzes of variance were computed. These analyses were conducted controlling for gender, schools' socioeconomic level, students' perception of the classroom social environment and their previous

problem-solving performance. Then, in order to refine our results, Bonferroni post-hoc procedures were applied.

5. Results

Table 4 summarizes the significant differences observed between the four conditions. It is noteworthy that we examined short-term changes (gains scores between Time 1 and Time 2) as well as long-term changes (gains scores between Time 1 and Time 3).

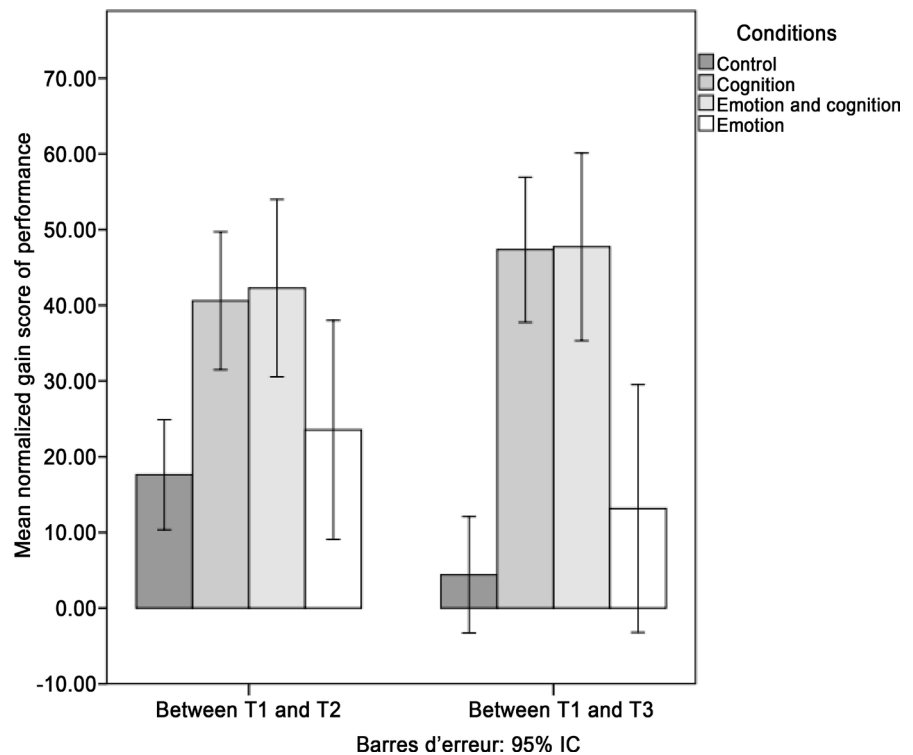The results of the Bonferroni post-hoc tests are reported in the following text.

5.1. Problem-Solving Performance

Between Time 1 and Time 2 a significant difference of mean normalized gain score of problem-solving performance was observed between the "control" condition and both the "cognition" condition $(MD = -31.95, p < 0.001)$ and the "emotion and cognition" condition $(MD = -34.85, p < 0.001)$ (Figure 3). Further, a difference on the edge of significance was observed between the "emotion and cognition" condition and the "emotion" group $(MD = 23.49, p = 0.066)$. Between Time 1 and Time 3, only the differences between the "control" condition and both the "cognition" $(MD = -31.58, p < 0.001)$ and the "emotion and cognition" conditions $(MD = -35.92, p < 0.001)$ were maintained. In terms

Table 4. Univariate analyses of variance of mean normalized gain scores.

	Between time 1 and time 2			Between time 1 and time 3		
	$F(ddl)$	p	eta²	$F(ddl)$	p	eta²
Problem-solving performance	13.45 (3372)	<0.001	0.10	12.25 (3340)	<0.001	0.10
Situation model	4.77 (3393)	0.003	0.04	15.35 (3364)	<0.001	0.11
Verification	1.24 (3125)	0.285		2.5 (3112)	0.063[b]	0.07
Interpretation	7.02 (3268)	<0.001	0.07	13.10 (3262)	<0.001	0.13
Communication	15.94 (3265)	<0.001	0.16	23.00	<0.001	0.22
Pride	2.47 (3310)	0.062[a]		6.44 (3295)	<0.001	0.06
Relief	2.65 (3338)	0.050[a]		1.40 (3323)	0.242	
Boredom	4.47 (3320)	0.004	0.04	1.83 (3314)	0.140	
Frustration	4.46 (3225)	0.005	0.06	3.37 (3207)	0.019	0.05
Hopelessness	1.99 (3243)	0.116		2.66 (3240)	0.050[a]	
Task utility self-persuasion	1.80 (3360)	0.146		4.10 (3312)	0.007	0.04
Help-seeking	2.74 (3390)	0.043[a]		1.80 (3338)	0.147	
Brief attentional relaxation	6.23 (3400)	<0.001	0.05	5.14 (3343)	0.002	0.04
Negative self-talk	0.71 (3332)	0.549		2.61 (3280)	0.052[b]	0.03
Dysfunctional avoidance	1.91 (3321)	0.127		3.76 (3271)	0.011	0.04

[a]The Bonferroni post-hoc procedure indicates that there is no significant difference between the four conditions. [b]The Bonferroni post-hoc procedure shows significant difference between the four conditions.
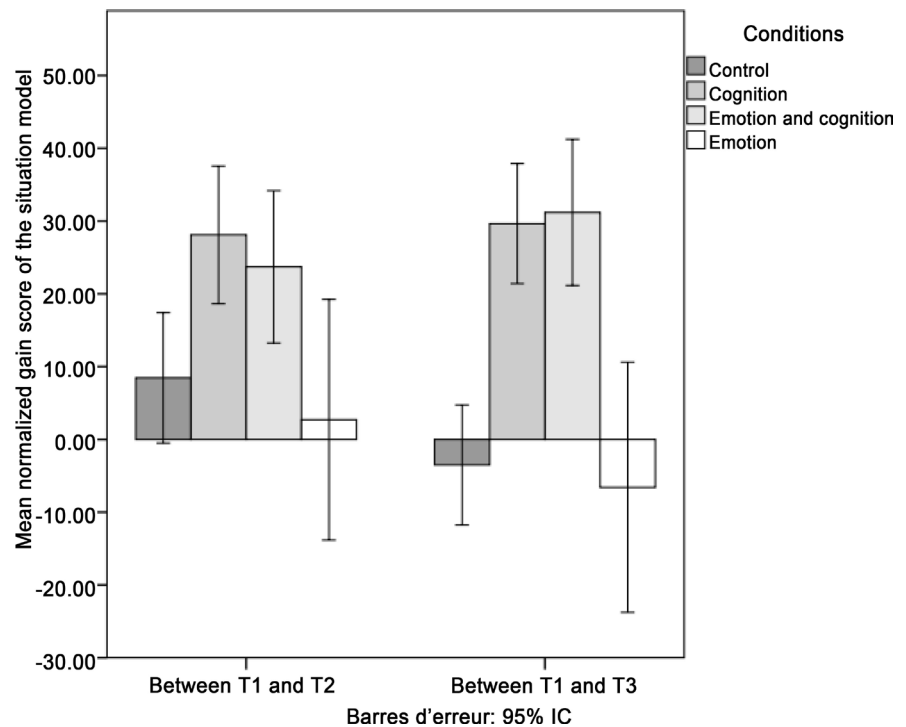
Figure 3. Mean normalized gain score and standard errors for the problem-solving performance for each condition per measurement time.

of problem-solving performance, hypothesis 1 and 2 are thus only partly confirmed. As no significant difference between the "emotion" condition and the "control" condition was observed, hypothesis 4 was not corroborated.

5.2. Cognitive Dimension of Problem-Solving Competence

We start with the findings related to the "strategy use" indicator. First, regarding the "situation model" strategy, between Time 1 and Time 2, a significant difference of mean normalized gain score was found between the "cognition" condition and the "control" one $\left(MD = 20.49, p = 0.013 \right)$ as well as a difference at the limits of significance between the "emotion and cognition" condition and the "control" condition $\left(MD = 18.38, p = 0.070 \right)$ (Figure 4). Between Time 1 and Time 3, if the difference of mean normalized gain score between the "control" group and both the "cognition" group $\left(MD = -33.93, p < 0.001 \right)$ and the "emotion and cognition" group $\left(MD = -35.45, p < 0.001 \right)$ is maintained; the "cognition" group $\left(MD = 35.82, p = 0.003 \right)$ as well as the "emotion and cognition" group $\left(MD = 37.34, p = 0.003 \right)$ also differ from the "emotion" group.

Second, a difference on the edge of significance, of mean normalized gain score regarding the "verification" strategy was observed between Time 1 and Time 3 to the benefit of the "emotion and cognition" group as compared to the "control" group $\left(MD = 72.96, p = 0.054 \right)$. Third, as regards the "interpretation" strategy, both the "cognition" $\left(MD = 38.69, p = 0.001 \right)$ and the "emotion and cognition" $\left(MD = 41.17, p = 0.002 \right)$ conditions stood out from the "control"
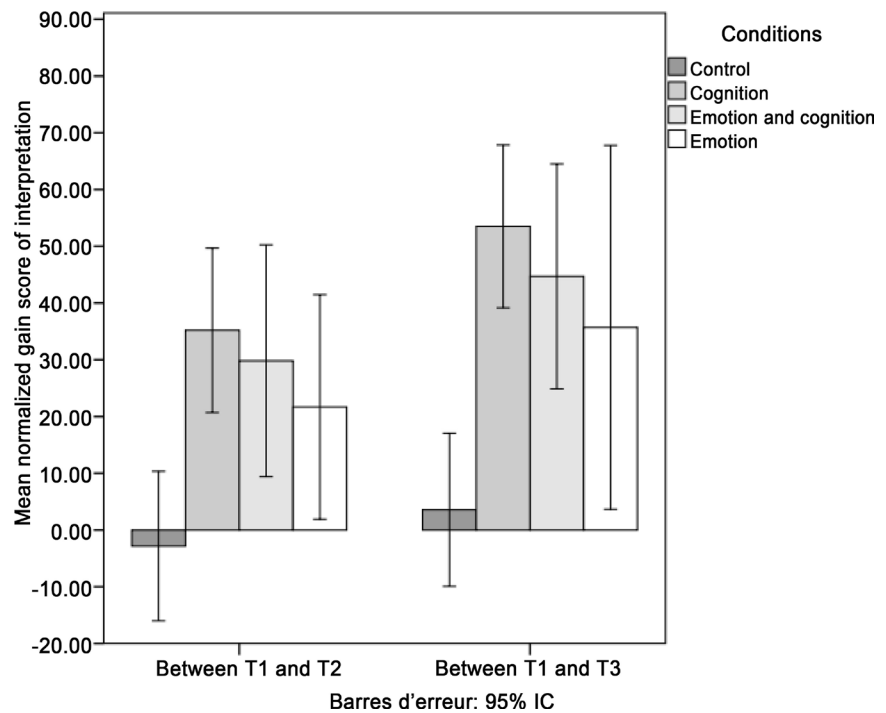
Figure 4. Mean normalized gain score and standard errors for the "situation model" strategy for each condition per measurement time.

condition, between Time 1 and Time 2. These differences are maintained between Time 1 and Time 3 (Between the "control" condition and the "cognition" condition: $MD = 42.68, p = 0.041$; between the "control" condition and the "emotion and cognition" condition: $MD = 51.49, p < 0.001$) (**Figure 5**). Between Time 1 and Time 3, a significant difference was also found between the "emotion" condition and the "control" condition $(MD = 42.68, p = 0.041)$.

Fourth, with respect to the "communication" strategy, the "cognition" and the "emotion and cognition" groups stood out from, on the one hand, the "control" group ($MD = 39.14, p < 0.001$; $MD = 31.66, p = 0.014$; respectively), and, on the other hand, from the "emotion" group ($MD = 68.58, p < 0.001$; $MD = 76.07, p < 0.001$ respectively) (**Figure 6**). These four differences were still observable at Time 3 (Between "cognition" and "control": $D = 48.92, p < 0.001$; between "emotion and cognition" and "control": $MD = 63.78, p < 0.001$ between "cognition" and "emotion": $MD = 71.39, p < 0.001$; between "emotion and cognition" and "emotion": $MD = 86.25, p < .001$).

We then examined a second indicator of problem-solving competence, namely, correct strategy use. In this regard, findings highlighted a significant improvement regarding the "situation model" and the "interpretation" strategies between Time 1 and Time 2, within the four conditions (**Table 5**). However, this improvement persists, at Time 3, mainly among the "emotion and cognition" condition as regards the "situation model" strategy and among the three experimental conditions (i.e. "cognition", "emotion" and "emotion and cognition") as regards the "interpretation" strategy. As for the strategies of "using one's

knowledge" and "verification", findings showed that when used, they are used correctly. Finally, with respect to the communication strategy, it seemed more relevant to assess the completeness than the correctness of its implementation.

Figure 5. Mean normalized gain score and standard errors for the "interpretation" strategy for each condition per measurement time.
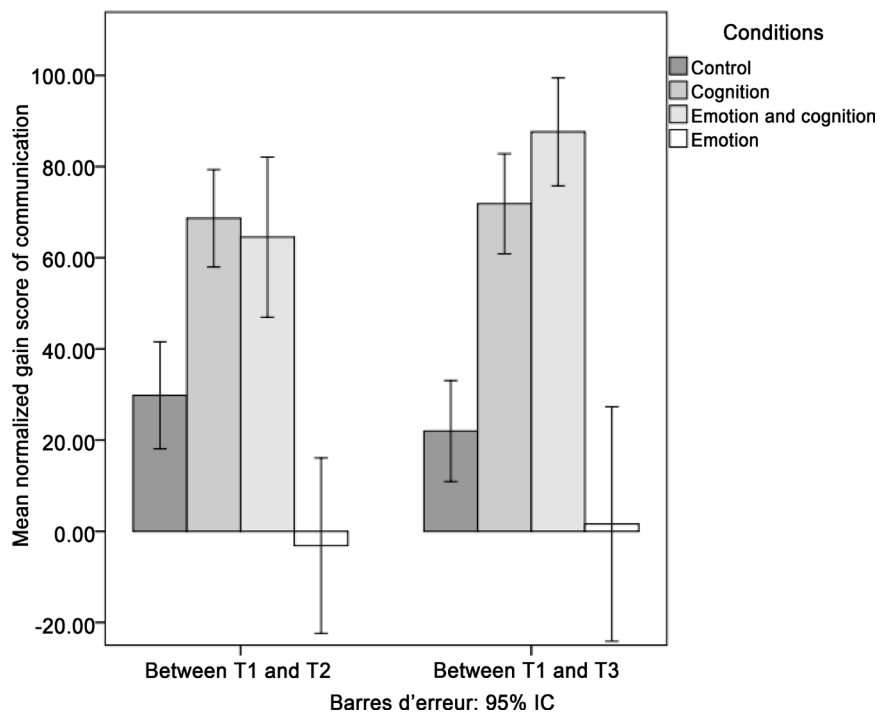
Figure 6. Mean normalized gain score and standard errors for the "communication" strategy for each condition per measurement time.

Table 5. Ratio between the number of correct implementations and the total number of implementations per group and per heuristic.

Condition	Control			Cognition			Emotion and cognition			Emotion		
	T1 (%)	T2 (%)	T3 (%)	T1 (%)	T2 (%)	T3 (%)	T1 (%)	T2 (%)	T3 (%)	T1 (%)	T2 (%)	T3 (%)
Situation model	50	65	55	62	74	66	65	78	80	42	55	50
Using one's knowledge	/	/	/	/	98	99	/	93	90	/	/	/
Answer's verification	/	/	/	/	94	100	/	95	100	/	/	/
Interpretation	80	99	76	74	98	91	81	100	96	68	100	88
Communication	85	81	82	86	94	96	91	96	100	88	84	85

Note. T1 = pretest; T2 = posttest; T3 = retention test.

Table 6. Ratio between the number of students who used the heuristic and the total number of students per group and per heuristic.

		Posttest		Retention test	
		Cognition	Emotion and cognition	Cognition	Emotion and cognition
Situation model	P1	55	48	38	49
	P2	91	94	94	100
	P3	32	36	33	32
	P4	58	76	71	80
Answer's verification	P1	39	44	31	41
	P2	23	24	28	27
	P3	33	24	34	33
	P4	16	13	16	25
Interpretation	P1	76	79	78	82
	P2	/[4]	/	/	/
	P3	70	71	85	85
	P4	/	/	/	/
Communication	P1	88	95	94	96
	P2	96	96	95	99
	P3	93	95	92	100
	P4	99	94	95	99

Note. P1 = problem 1; P2 = problem 2; P3 = problem 3; P4 = problem 4.

Findings present no change between the three measurement times for both the "control" and the "emotion" conditions as well as a continuous increase between Time 1 and Time 3 for both the "cognition" and the "emotion and cognition" conditions.

Finally, we are interested in "strategy flexibility". On this point, it turned out that the "situation model" strategy is, by both the "cognition" and the "emotion and cognition" conditions, notably more used in problems 2 and 4, whether it be the posttest or the retention test (Table 6). This is not surprising since they both contain a lot of information, which is not the case of problems 1 and 3. As regards the "verification of the calculations", this strategy seems, on the whole,

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[4]Problems 2 and 4 do not make it possible to collect tangible traces of interpretation.

more used in problems 1 and 3, both at the posttest and retention test. These latter problems are the only two that contain numerical data and thus that require calculations. As regards the "interpretation" and "communication" strategies, they seem to be used at a similar frequency for all the problems, both by the "cognition" and the "emotion and cognition" conditions. Nonetheless, as the analysis does not take into account students' inner thinking, the results must be interpreted with caution.

In sum, findings showed that the "emotion and cognition" condition surpassed both the "control" and the "emotion" conditions in terms of "situation model" and "communication" and only the "control" group in terms of "verification" and "interpretation", thus partly corroborating hypothesis 1. Further, the "cognition" condition also surpassed both the "control" and the "emotion" conditions in terms of "situation model" and "communication" but only the "control" condition regarding "interpretation", partly supporting hypothesis 2. Regarding correct strategy use, both the "cognition" and the "emotion and cognition" conditions seem to stand out. Further, while both conditions displayed a flexible use of the situation model strategy, they did not seem to use the verification strategy in a relevant way.

## 5.3. Emotional Dimension of Problem-Solving Competence

First, pride is the only positive emotion on which the four conditions stood out. More precisely, between Time 1 and Time 3, the "emotion and cognition" group as well as the "emotion" group distinguished themselves from both the "control" group ($MD = 19.9, p = 0.056$ [5]; $MD = 40.35, p = 0.003$; respectively) and the "cognition" group ($MD = 21.46, p = 0.032$; $MD = 41.90, p = 0.003$; respectively). However, these differences faded at Time 3 (Figure 7).

Second, boredom and frustration are the two negative emotions on which the four conditions differed. With respect to boredom, the "emotion" group displayed a mean normalized loss score significantly higher than that of the "cognition" group $(MD = -23.63, p = 0.042)$ and presented a difference on the edge of significance with the control group $(MD = -22.90, p = 0.058)$. The "emotion and cognition" differed significantly from the "cognition" group as well $(MD = -17.86, p = 0.041)$. However, these differences were only observed between Time 1 and Time 2 (Figure 8).

As regards frustration, the "emotion" group presented a mean normalized loss score significantly higher than the "control" group $(MD = -28.4, p = 0.025)$. Quite surprisingly, the "cognition" group displayed a mean normalized loss score of frustration significantly higher as compared to the "control" group $(MD = -24.87, p = 0.011)$. These differences did not persist at Time 3.

With respect to adaptive emotion regulation strategies, the "emotion and cognition" condition presented a mean normalized gain score between Time 1 and Time 2 of "brief attentional relaxation" significantly higher than that of the three
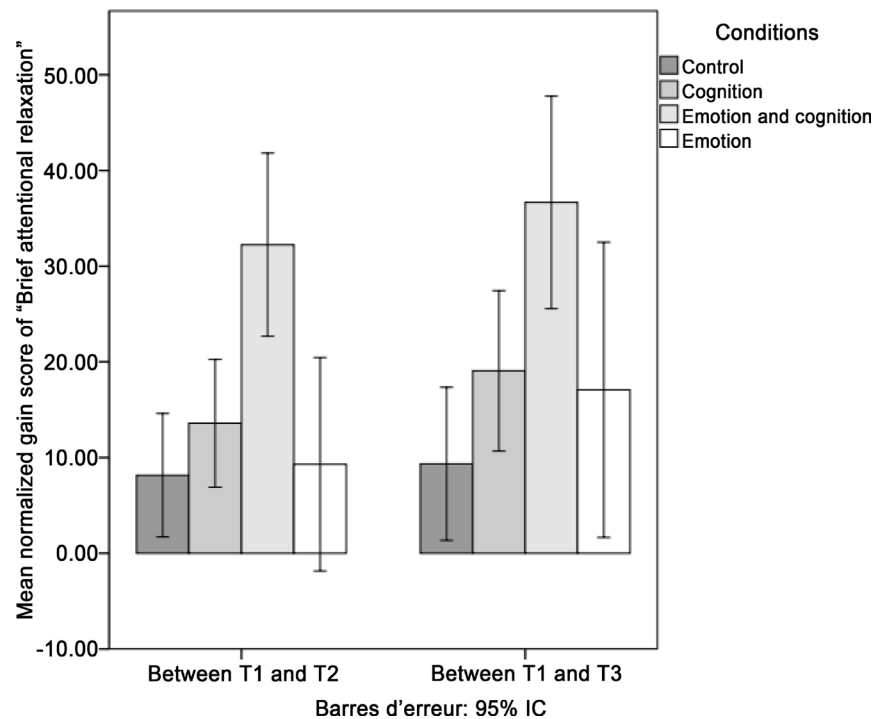
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[5]On the edge of significance.
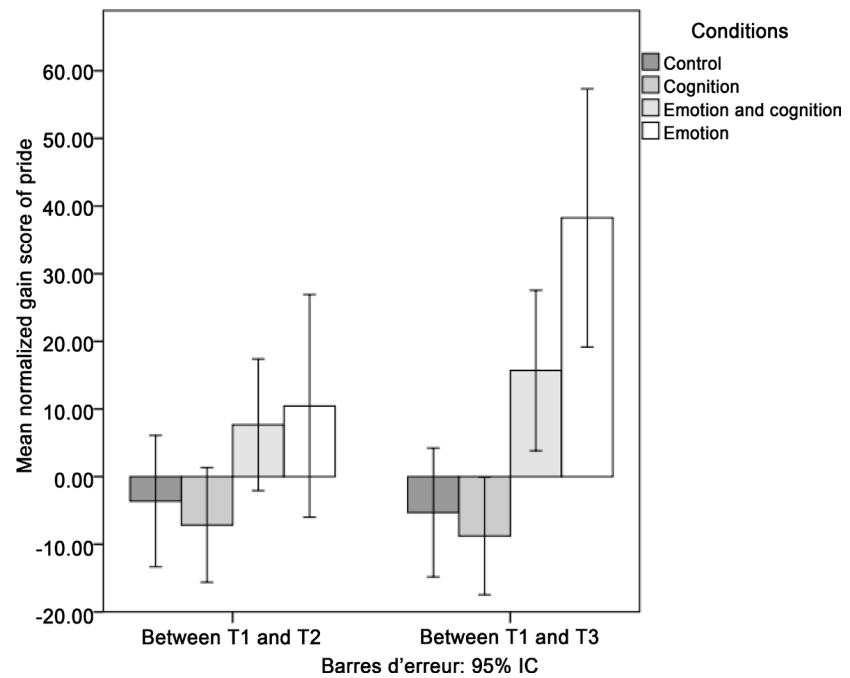
Figure 7. Mean normalized gain score and standard errors for pride for each condition per measurement time.
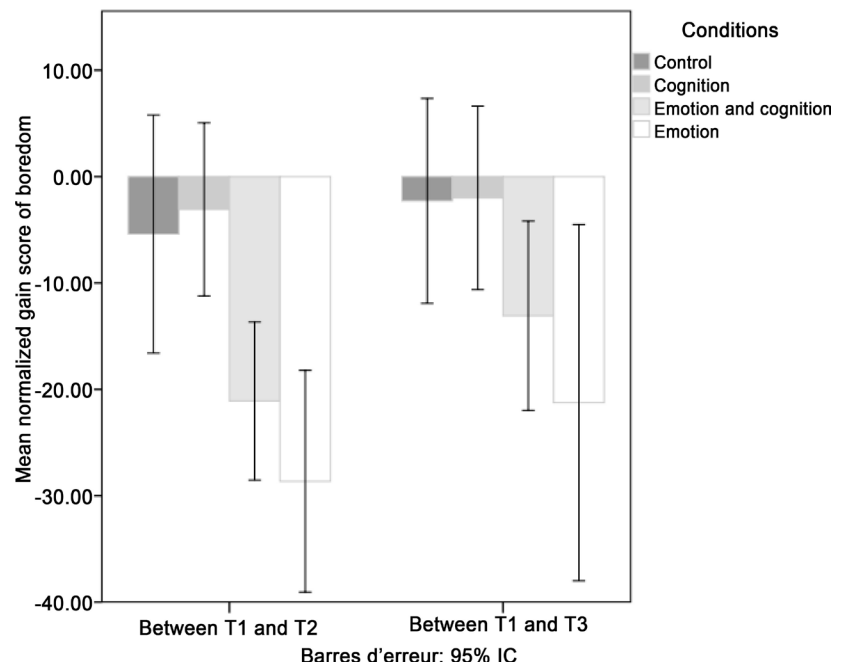
Figure 8. Mean normalized gain score and standard errors for boredom for each condition per measurement time.

other conditions (control: $MD = 21.70, p = 0.001$; "cognition": $MD = 16.05$, $p = 0.024$; "emotion" $MD = 25.59, p = 0.007$). Between Time 1 and Time 3, only the difference with the "control" condition $\left(MD = 27.92, p = 0.001 \right)$ and, on the edge of significance, with the "cognition" condition $\left(MD = 17.31, p = 0.062 \right)$ were maintained (**Figure 9**).

Figure 9. Mean normalized gain score and standard errors for the "brief attentional relaxation" strategy for each condition per measurement time.

Still at Time 3, the "emotion and cognition" condition recorded a mean normalized gain score regarding the "task utility self-persuasion" strategy significantly higher than both the "control" condition $\left(MD = 22.77, p = 0.038\right)$ and the "cognition" condition $\left(MD = 24.43, p = 0.012\right)$. With respect to dysfunctional strategies, differences are observed from Time 3 as well. More precisely, the "emotion and cognition" group stood out, to its advantage, from the "control" group in terms of "dysfunctional avoidance" $\left(MD = -15.26, p = 0.024\right)$. Further, a difference regarding the "negative self-talk" strategy approaching significance was observed between the "emotion" group and the "control" group to the benefit of the first $\left(MD = -26.21, p = 0.07\right)$.

In sum, the "emotion and cognition" condition differs to its advantage from the "control" and the "cognition" conditions, both in terms of academic emotions and emotion regulation strategies, thereby partly supporting hypothesis 1. The "emotion" condition also differs from the "control" and the "cognition" conditions but mainly regarding academic emotions, so partly confirming hypothesis 3.

6. Discussion

The present study sought to demonstrate the benefit for upper elementary students of a mathematical problem-solving intervention that combines the development of cognitive, emotional and regulative knowledge and skills. Students' academic emotions, emotion regulation strategies, problem-solving competence and problem-solving performance constitute the four dependent variables used

to assess this benefit.

Our first research question was about the effects of such an intervention. We at first expected that the "emotion and cognition" condition would surpass the three other conditions regarding all the dependent variables (*Hypothesis* 1). This is partly supported by our data. First, the "emotion and cognition" condition stood out, to its advantage, from the "control" condition regarding problem-solving performance, the frequency of use of four heuristic strategies (i.e., situation model, verification, interpretation, communication), pride, and three emotion regulation strategies (i.e., brief attentional relaxation, task utility self-persuasion, dysfunctional avoidance). Only the difference observed for pride was no longer observed after the end of the intervention. Second, the "emotion and cognition" condition distinguished itself from the "emotion" condition as well. Not surprisingly, the two groups differed mainly in cognitive terms, namely, regarding problem-solving performance and the frequency of use of two heuristic strategies (situation model, communication). However, the difference in performance did not last after the end of the intervention. Furthermore, and unexpectedly, the "emotion and cognition" condition differed advantageously from the "emotion" condition regarding the "brief attentional relaxation" strategy as well, but only during the intervention. This suggest that students may need to be cognitively and metacognitively well equipped to engage in brief attentional relaxation moments. This hypothesis is partly supported by a study conducted among upper elementary students that stressed that the brief attentional relaxation strategy is more used by expert than by novice problem solvers (Hanin & Van Nieuwenhoven, submitted). Third, quite logically, the "emotion and cognition" group distinguished himself from the "cognition" group on emotional variables, namely, pride, boredom and on two functional regulation strategies, that is, "brief attentional relaxation" and "task utility self-persuasion". If the differences regarding the two strategies are maintained after the end of the intervention, this is not the case of the emotions.

Second, we anticipated that the "cognition" condition would surpass the "control" and the "emotion" conditions in terms of problem-solving competence and performance (*Hypothesis* 2). This, too, has been partially corroborated. First, the "cognition" condition distinguished itself significantly and advantageously from the "control" condition regarding both problem-solving performance and the use of three heuristic strategies (i.e., situation model, interpretation, communication). These differences endured after the end of the intervention. Surprisingly, the "cognition" condition also displayed a significant decrease of frustration in comparison to the "control" condition. However, the latter decrease of frustration faded after the end of the intervention. Although surprising at first, the examination of the sources of frustration points out that they are partly cognitive, such as the use of inefficient strategies, a lack of resources available and the difficulty in grasping the mathematical reasoning (Misra & Castillo, 2004; Sierpinska, Bobos, & Knipping, 2008). On that basis, the prob-

lem-solving process taught may have contributed to the reduction of the frustration of students in the "cognition" condition. A question remains, however, as to why no significant change of frustration was observed among the "emotion and cognition" condition. In this respect, several scholars have pointed out that duration, intensity and frequency of the intervention's activities impact the findings (e.g., Durlak, Dymnicki, Taylor, Weissbergand, & Schellinger, 2011; Greenberg, Domitrovich, Graczyk, & Zins, 2005). In the present case, the "emotion and cognition" condition was rather intense in that the "cognition" intervention followed immediately after the "emotion" intervention, leaving no time for the students to assimilate the information provided by the first intervention. In addition, four lessons on emotions knowledge and skills may not have been enough to allow them to fully appropriate this whole new academic content (Verschaffel et al., 2000). However, as this is only speculation, future research is warranted to clear up these inconsistent findings.

Hypothesis 3 stated that the "emotion" condition would surpass the "cognition" and the "control" conditions with respect to academic emotions and emotion regulation strategies. Several findings lend support to this hypothesis. First, the "emotion" condition differentiated itself, profitably, from the "cognition" condition in terms of pride and boredom. However, these differences did not last after the end of the intervention. Surprisingly, no significant difference was found regarding the emotion regulation strategies between these two groups. Second, the "emotion" condition outperformed the "control" condition regarding pride, boredom, frustration and the "negative self-talk" strategy. Again, the differences regarding the emotions did not last after the end of the intervention. In addition, and unexpectedly, at Time 3, the "emotion" condition outperformed the "control" group regarding the "interpretation" strategy. It should be recall that students in the "emotion" condition did not receive any training in heuristic strategies. One possible explanation may have to do with the students' problem-solving-related belief system. On this point, scholars have shown that students hold inadequate beliefs about problem solving, such as assuming that the reality described in a problem is different from real life (e.g., McLeod, 1992; Schoenfeld, 1988, 1992; Verschaffel et al., 2000). But to interpret correctly the mathematical result of a problem requires modifying one's beliefs system and, more precisely, breaking down the barriers between mathematical reality and everyday life. On that basis, we hypothesize that the "emotion" intervention, in that it invites students to become aware, to understand, to express and to regulate their emotions in the context of problem solving, made mathematics more human and thereby could have contributed to bring mathematical reality closer to everyday life.

Finally, hypothesis 4, based on Pekrun (2006) control-value theory of achievement emotions, which postulates a direct link between emotions and achievement, and supported by several empirical studies (e.g., Frenzel et al., 2007; Peixoto et al., 2017), was not corroborated. As a reminder, we postulated that the "emotion" group would have a higher problem-solving performance than the "control"

group. The unfamiliarity of students with non-routine problems may be a possible explanation. Several scholars have shown that the strategies used by students to solve the problems traditionally presented in classes prove to be ineffective in solving non-routine problems (Depaepe, De Corte & Verschaffel, 2015; Fagnant, Demonty, & Lejonc, 2003). For novice students, solving non-routine problems required both emotional knowledge and skills and effective cognitive and meta-cognitive strategies (Hanin & Van Nieuwenhoven, submitted; De Corte & Verschaffel, 2008). And neither the "control" group nor the "emotion" group possessed the strategies required to solve non-routine problems. Another complementary explanation may be that the two conditions differ regarding two activity emotions (boredom, frustration) and one outcome emotion (pride). Yet, a study conducted by Hanin & Van Nieuwenhoven (2016b) showed that mathematics performance is only predicted by outcome emotions. Future research is warranted to clear up this finding.

Two related general observations deserve to be examined more closely. A first one is the few differences regarding the emotional variables between the groups who received an intervention on emotional knowledge and skills and those who did not. A second observation is the non-persistence after the end of the intervention of the differences in emotions observed at the end of the intervention. A first possible reason for these two observations could be the weight of motivational beliefs, direct antecedents of academic emotions (Ahmed, Minnaert, van der Werf, & Kuyper, 2010; Pekrun, 2006, 2014; Tzohar-Rozen & Kramarski, 2014). As these motivational beliefs were not taken into account in the intervention, they may have remained fragile for some students and thereby may have countered the positive effects induced by the intervention on emotions. This observation is in line with the conclusion made by other researchers, that it is necessary to act on both fronts simultaneously to reach more optimally the learner's emotions (Dweck, 1992; Kim & Hodges, 2012; Pekrun, 2006). A second explanation may be the too short duration of the intervention on emotion, as outlined previously.

Our second research question looked at a more complete measure of the problem-solving competence. Therefore, building on the work of Elia et al. (2009), we took into account not only the use of heuristic strategies but also two additional indicators, namely, correct strategy use and strategy flexibility. Findings provide additional confirmation of the cognitive effectiveness of the "emotion and cognition" condition, which made a more correct use of the "situation model" and the "interpretation" strategies both at the end of the intervention and after it. Moreover, this group showed a different frequency of use of two heuristic strategies between the four problems both at the posttest and at the retention test. As for the situation model, it was used more in the problem containing a large amount of information, whose format makes the construction of such a model necessary (problem 2), than in the problem that contains less information and presents it in such a way as to make possible a mental treatment

or, at least, does not make necessary the construction of a situation model (problem 4) (Elia et al., 2009; Fagnant & Vlassis, 2013). This reflects a flexible use of the strategy. On the other hand, verifying one's answer is a relevant strategy regardless of the problem. Yet, the "emotion and cognition" condition did not resort to it in all the problems. Two hypotheses may be put forward to explain this. First, as problems 2 and 4 do not imply calculations, the verification may have been carried out mentally by the learner. Second, as most problems usually dealt with in classes contain numerical data (Houdement, 2014), students may have associated the relevance of the verification strategy with the verification of calculations rather than with that of the whole resolution process. In this second case, our result would illustrate the influence of the didactic contract on the students' approach to problem-solving tasks (Fagnant et al., 2003; Greer, 1997; Verschaffel et al., 2000).

In sum, our results showed that to act significantly on emotions and on emotion regulation strategies, a specific intervention on emotion knowledge and skills is needed just as acting on the problem-solving competence required a cognitive and self-regulative intervention.

7. Conclusion

To develop an expert and reflexive approach to mathematical problem solving supposes acting on each dimension of learning. While many researchers hold this view (e.g., Boekaerts, 2007; De Corte & Verschaffel, 2008; Kim & Hodges, 2012; Op't Eynde, De Corte, & Verschaffel, 2006), this study is the first to test it empirically. Findings show that a training program designed to develop in students heuristic strategies embedded in an overall self-regulative approach as well as emotional knowledge and competencies improves students' problem-solving performance and competence, reduces their negative emotions, arouses the emergence of positive emotions, promotes the use of adaptive emotion regulation strategies and discourages the use of maladaptive ones. In this way, our results contribute to the progress of research in mathematics education. Furthermore, these findings are also important for the practice of teaching and learning how to solve non-routine problems. First, our conclusions draw the attention of educators to the benefit of developing the emotional knowledge and skills of their students. More precisely, it appears important to consider emotion as an academic topic in its own right with which most students are unfamiliar. This implies spending time developing their emotional knowledge and skills and giving them opportunities to take ownership of this new content. As shown by this study, such a classroom environment may lead to a change in attitude and beliefs regarding problem-solving tasks. This would be no small feat given the tenacity of students' beliefs regarding problem-solving tasks. Such beliefs include, assuming that the reality described in a problem is different from real life, that there is no place for emotions and feelings in mathematics or that solving a problem involves performing calculations with the numbers given and that the

reasoning carried out is only secondary (Schoenfeld, 1992; Verschaffel et al., 2000). Second, our findings show that an accurate, flexible and thoughtful use of heuristic strategies may be enhanced through instruction which intergratively fosters the development of strategic knowledge addressing the procedural (how?) and the conceptual (why? when?) dimensions. In this way, the findings align with previous studies (Baroody, 2003; Elia et al., 2009; Mevarech & Kramarski, 1997). Third, the close intertwining between emotions and motivational beliefs, extensively documented in the literature (e.g., Ahmed et al., 2010; Hanin & Van Nieuwenhoven, 2016b; Peixoto et al., 2017; Pekrun, 2006, 2014; Tzohar-Rozen & Kramarski, 2014), suggests that such an educational intervention could be even more fruitful by fostering students' emotional knowledge and skills simultaneously with their motivational beliefs (e.g., perceived competence, problem-solving tasks value).

However, the present study suffers from several limitations that may motivate future investigations. First, one important dimension of problem-solving learning has not been considered in our intervention, namely, students' motivational beliefs. In line with educational models of emotions that posit a direct and close relation between academic emotions and motivational beliefs (e.g., Meyer & Turner, 2006; Pekrun, 2006; Schutz, Hong, Cross, & Osbon, 2006), our results suggest that providing a motivation enhancement intervention along with an emotional intervention will reinforce the effects induced by the latter. To our knowledge, such a training-program does not exist yet, and represents a useful direction for future research. Second, a number of studies have showed that students' reports of habitual emotions (trait-like) are biased by subjective beliefs, but that these subjective beliefs are much less likely to impact real-time reports of academic emotions (state-like) (Robinson & Clore, 2002). In order to access and to draw conclusions about the emotions experienced by students in the real-time, scholars suggest crossing the two measures. The present study could therefore be supplemented by a state-like measure of academic emotions. Third, data were self-reported, which raises the question of a possible social desirability bias. To address this social desirability issue and deepen our understanding of academic emotions, future studies should take a multi-method approach to include, among other factors, discussion, observation, survey and verbal report (Greene, Robertson, & Croker Costa, 2011; Kim & Hodges, 2012). Fourth, despite our efforts, the four conditions were not totally equivalent in terms of school characteristics (i.e. geographical localization and socioeconomic level) as well as teacher characteristics (i.e. gender and years teaching), especially for the "emotion" condition. Given that both the "teacher effect" and the socioeconomic level impact academic learning and achievement (Nye, Konstantopoulos, & Hedges, 2004; Wayne & Youngs, 2003), it would be interesting to confirm our results with perfectly equivalent conditions. Fifth, in line with previous studies that showed interventions benefited differently according to the learner's profile (novice vs. expert), it would be interesting to address this issue regarding an

emotional and cognitive intervention (Hanin & Van Nieuwenhoven 2016a; Montague, 2007; Muir, Beswick, & Williamson, 2008). Sixth, the problems constituting the pretest, posttest and retention test do not necessitate the use of the planning strategy. However, with our objective of offering a more complete and accurate measure of problem-solving competence—by taking into account correct strategy use and strategy flexibility—it is necessary to vary even more the problems of these tests in terms of "unmissable" heuristic strategies to solve them. Last, if the intervention in its current state already leads to promising results, we believe that implementing it over the year would be even more fruitful (Durlak et al., 2011).

Acknowledgements

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Appendix 1. Description of the Training Programs

The **training program on emotions** took place over four weeks. Each week, between one and a half and two hours were devoted to a specific emotional content in line with the adapted version of Mayer and Salovey's (1997) model, i.e. the five-branch emotional competencies model. Concretely, the first lesson questioned the learner about his emotional relationship to math problem solving. The goal here was to get students to think about and become aware of how they experience problem solving. The second lesson aimed to familiarize learners with the concept of emotion by working on its components and on the basic emotions. The ability to identify and understand one's own emotions was also part of this session. The third lesson emphasized the importance of expressing rather than repressing one's emotions, addressed the social norms associated with emotional expression and introduced the concept of emotion regulation. The fourth and last lesson proposed to discover and experienced six strategies used by upper elementary students to regulate their negative emotions when solving math problems: task utility self-persuasion, brief attentional relaxation, help-seeking, emotion expression, dysfunctional avoidance and negative self-talk. While the first three are effective, the last three are ineffective (Hanin et al., 2017). A wide range of activities were provided including video-clips, analyses of mini-case studies, drawings, Chinese portrait, short lectures, facial expressions analyzes, group tasks and discussions and role play. At the beginning of each lesson, a summary of the previous lesson was made by the teacher in interaction with the students. To facilitate the transfer, at the end of each lesson an activity of linking of what had just been learned to problem-solving tasks was offered. At the end of the fourth lesson, students received an illustrative plate of twenty basic emotions as well as a booklet containing a succinct and pictorial description of the six emotion regulation strategies. Students in the "emotion and cognition" condition were invited to use the booklet during the subsequent intervention on heuristic strategies and self-regulated behaviors.

The training program aiming to develop **heuristic strategies embedded in an overall self-regulative approach to problem solving** comprised seven lessons. These lessons took place weekly and lasted from one to four hours. The first two lessons were devoted to the construction of an expert and reflexive approach to problem solving, at the cognitive level. More specifically, after solving the problem individually, students were invited to share their procedure in groups of 3 to 4. At the end of their discussions, they had to agree on a procedure and a solution that they displayed on a poster. The posters were then put up on the board. Comparison of the different posters made it possible to update the characteristics of a good situation model, estimation, planning, verification, interpretation and communication. The heuristic strategies that did not appear on the students' posters were introduced by the teacher. Then, on the basis of these exchanges, the students, in interaction with the teacher, constructed, for each heuristic, a summary sheet based on the WWW & H rule (what, why, when and how)

(Veenman et al., 2006). This summary sheet was then used to solve the subsequent problems. Each of the following lessons consisted in solving a non-routine problem according to a specific methodology (individual resolution, individual resolution followed by a discussion in dyad or individual representation followed by group solving). A collective discussion promoting a reflective and correct use of heuristic strategies closed each lesson. Several activities aimed at developing a self-regulated approach among students, such as the completion of a self-assessment grid, the comparison of one's self-assessment with the assessment made by another student (co-assessment), the emphasis put on the utility of each heuristic strategy (the why) and a mid-term formative evaluation on heuristic strategies. Note that before starting to solve each problem, students in both the "emotion" and the "emotion and cognition" conditions were invited, systematically, to choose an emotion regulation strategy. These strategies and their effectiveness were then discussed during the collective discussion.

Concerning the choice of problems, in order to remain in line with the training-program's objective, i.e. to develop an expert and reflexive approach to problem solving, only application problems were offered, and use of the calculator was permitted. In addition, the problems chosen for the training-program presented a structure distinct from those of the problems addressed in the problem-solving tests so as to avoid a potential learning effect.

Appendix 2. Statements of the pretest problems

Problem 1 | To celebrate my sister's birth, my parents organized a party with the whole family. They ordered 170 balloons to inflate to decorate the garden. After the party, they distributed the balloons between the 8 cousins. How many balloons did each one receive?

Problem 2 | Christine, Julie and Camille, three friends in the 6th year of primary school, would like to organize an outing to "Adventure Park" during the holiday week. They are all very busy: Julie is free on Monday and Saturday, as well as on Tuesday, Thursday and Friday. Christine is not available on Monday, Thursday and Sunday. Camille can come on Monday, Wednesday, Friday and Saturday.
The "Adventure Park" is open every day from 10am to 6pm except Mondays.
When will they be able to organize the outing? Show all possible answers.

Problem 3 | The 5th and 6th elementary school students of the "Stream" school go to the snow class. A bus can contain 31 people. Knowing that there are 182 people (children and adults) who are going to the snow class, how many buses will it take?

Problem 4 | To decorate the Christmas tree, Madam Nathaly proposes various DIY projects: painting stars, pasting glitter on balls, drawing angels, cutting out lanterns.
Sylvie does not like to draw or cut.
Tom loves stars.
Steve says that glitter is for girls.
Clara hates cutting with scissors.
Every DIY project can only be done by one child. Show which DIY project each child will do. Explain.