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New Procedure for Delineating the Mass of a Higgs Boson, While Interpolating Properties of the Scalar Singlet Dark Matter Model

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Abstract

We proceed to obtain a polynomial based iterative solution for early universe creation of the Higgs boson mass, using a derived polynomial of the form $A_1 m_h^4 + A_2 m_h^3 + A_3 m_h^2 + A_4 m_h^1 + A_5 = 0$ with the coefficients for A_i derived through a series of specific integral formulations with m_h^1 the mass of a Higgs boson, and the construction of the coefficients derived as of the A_i using a potential system, for the Higgs, largely similar to the usual Peskin and Schroeder quantum field theoretic treatment for a Higgs potential, which subsequently is modified, *i.e.* this is for the regime of space-time as up to the Electro-weak regime of cosmology, in terms of a spatial regime. The linkage to Dark matter is in terms of the Scalar Singlet Dark matter model proposed by Silvera and Zee, *i.e.* what we do is to use a procedure similar to the usual Standard model Higgs, but to find ways to iterate to isolate key inputs into electro weak symmetry breaking procedure for the creation of Dark Matter. Afterwards, we will use specific inputs into the Scalar Singlet Dark matter model which would isolate out input parameters which we think are amendable to experimental testing. We conclude with a discussion of entropy so generated, along the lines of a modification of the usual branching ratios used in Higgs physics, with spin offs we think are relevant to the Dark Matter problem. We also use it to critique some linkage between Dark Matter, and gravity, which may explain some of the findings of LIGO, which were reviewed in 2016 in one of our listed references.

Keywords

DM, DE, Entanglement, Gravitational Waves, Higgs Boson, Information Theory

1. Introduction

We begin first with an accounting of the traditional Higgs mass calculation, much of which is going to be taken from Peskin and Schroeder (1995) [1] and which is similar to Halzen and Martin [2] as well as Kleinert [3], whereas we will use material from Maggiore [4] as to identify a protocol to set up $\tilde{A}_1 m_h^4 + \tilde{A}_2 m_h^3 + \tilde{A}_3 m_h^2 + \tilde{A}_4 m_h^1 + \tilde{A}_5 = 0$, while assuming a Higgs style potential as given in [1], which is also discussed in [2] and[3] in a more general fashion, plus other factors, to come up with a different procedure as to analyze m_h mass, which in turn is relevant to a model of Dark Matter, as written up by [5], *i.e.* Majumdar, in 2015. After having made this analysis, we will compare what we are obtaining in terms of classical equivalent models as well as the Fluctuation and Dissipation theorem brought up by Thorne and Blandford [6] in pp 325 up to 327, as to mean energy values implied. The mean energy values implied, then will be a bridge to an analysis of entropy generation due to the production of Higgs particles, and its linkage to information theory [7] [8] as a close to our document. We will frame our conclusions as to entropy and information theory as to Branching ratios [9] [10]. This also will have a tie into an essay by Penrose [11] as to General relativistic energy flux, and optics, which raises interesting points we think are a fitting conclusion as to certain inquiry directions are document will be presented. This will tie in strongly to the thoughts the Author raised as to the intersection of General Relativity, and Quantum mechanics raised in [12] as well as informed speculation as to how this ties into how a particles affects a physical field, as given in page 163 of [13] and also classical versus quantum interfaces, as brought up in [14] [15] [16], and [17], as well as the embedding of classical versus quantum structures raised in [18]. As a Klein Gordon result, this leads directly to the idea of quantum mechanics, as embedded within a larger theory, *i.e.* this methodology as brought up by Kieffer, in page 177 of [18] in its own way is fully in sync with some of the investigations of the embedding of quantum mechanics within a larger structure, as has been mentioned in a far more abstract manner by t'Hooft, in [19], and which is why we used a semi classical argument from [4] as to mass, and then superimposed it with the Higgs potential as given in [1] [2] [3]. This will also lead to a question we will raise, as we link our formulation to Dark Matter, as sourced from [5], which is to what degree can we ascertain Dark matter as a classical or quantum geometry artifact [20]. We do not agree to the [20] interpretation, but the issues it brings forward can be addressed via our methods as to isolating out the mass of a Higgs particle m_h as a solution to the polynomial equation

$$\tilde{A}_1 m_h^4 + \tilde{A}_2 m_h^3 + \tilde{A}_3 m_h^2 + \tilde{A}_4 m_h^1 + \tilde{A}_5 = 0 . \quad (1)$$

Note that what we are doing is to eventually walk this to Equation (31) of our text, in terms of what we think the probable outcomes are, with the outcomes summarized in Equation (33). The entire plan of the first part of the paper is to ascertain criteria as to possibly make implementation of Equation (33) of probable outcomes obtainable.

In particular, the quote from [20]:

Quote:

In summary, although debate may continue over whether or not quantum wave functions themselves represent anything real, there is clear evidence that quantum effects such as coherence and quantum collapse can at least occur over many kilometers and at superluminal speeds.

Links into our supposition given in [21] as to information theory, and past to present universe, whereas we hope our approach will ultimately lead to how the formation of mass m_h is tied directly into information exchange, and transfer, possibly through entanglement [21] as well as issues about the interface between classical and quantum systems brought up in [22] [23] [24] [25]. Since there would be profound implications as to a formation of gravity theory, if we can in any way understand how Dark matter is formed [26]. Finally, our study is also important as to if the Standard model for Higgs, is the only way to treat this problem of the formation of mass and the Higgs scenarios given [27] [28] [29].

2. Brief Review of the Formation of the Potential System Used for the Higgs Based upon [1] [2] [3]

Essentially what we will do here is to discuss the usual procedures done to isolate out the Higgs mass. In doing so, we do not do it to refute the existence of the Higgs, itself, but more to the point to help in the elucidation of background which may give more evidence as to [5], and also to ascertain the necessity of certain fixes as brought up in [30] as well as what was brought up in [31], *i.e.*

Quote:

As in the SM, the electroweak vacuum is metastable, it is important to explore if an extended scalar has an answer in its reserve. As the scalar weakly interacting massive scalar particles protected by Z2 symmetry can serve as viable dark matter candidates, it is interesting to explore if they help prolong the lifetime of the Universe. The effective Higgs potential gets modified in the presence of these new extra scalars, improving the stability of electroweak vacuum.

End of quote.

As given in [1] explicitly. The scalar field, as used in the Higgs, which is involved in symmetry breaking, is part of a physical transformation which uses a Unitary Gauge, and in [1] [2] [3] which leads to a scalar field we can represent as

$$\phi = \frac{U(x)}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}. \tag{2}$$

Peskin and Schroeder [1] use unitary gauge transformation arguments to the Lagrangian allowing isolating a Higgs potential energy term which takes the form of, if we start off with a Lagrangian

$$\Im = \left|D_u \phi\right|^2 + \mu^2 \phi^\dagger \phi - \lambda\left(\phi^\dagger \phi\right)^2. \tag{3}$$

This after a unitary transformation will lead to the following representation of an explicit Higgs Boson potential

$$\Im_V = -\mu^2 h^2(x) - \lambda\mu h^3(x) - \frac{h^4(x)}{4}. \tag{4}$$

This can be written as:

$$\Im_V = -\frac{1}{2} m_h^2 h^2(x) - \sqrt{\frac{\lambda}{2}} m_h h^3(x) - \frac{\lambda h^4(x)}{4}. \tag{5}$$

The method in all of this is to isolate out the quadratic term and to read off, in doing so a mass value as from going from Equation (3) and Equation (4) and then go to Equation (5). We will use a different approach, *i.e.* one which relies upon an expression for mass times acceleration equal to spatial integration of derivatives of Equation (5) above, with an approximation of found in [4] relying upon spatial integration, whereas the difference in the upper and lower limits of integration will be $r(\text{spatial})$ to $r(\text{spatial}) + \Delta r$. The Δr will be to put in the Heisenberg uncertainty principle, and from there we will attempt to make connection with the physics of what Equation (5) entails. $h(x) \sim h(r) \sim \exp(-\tilde{\alpha} r)$ will be used as well. It will allow us, to make connection with several issues which will be outlined in the following text.

3. Using Reference [4], We Can Write a Mass, as Given by the Following Semi Classical Approximation

Use the following from [4]

$$m_h = \frac{4\pi}{\left(\frac{\mathrm{d}^2 z}{\mathrm{d}t^2}\right)_{\text{acceleration}}} \cdot \int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot r^2 \cdot \{\ \}$$

$$\{\ \} = \left(\frac{1}{r^2} \cdot \frac{\partial}{\partial r} \cdot r^2 \cdot \frac{\partial}{\partial r} \cdot \Im_V(r)\right) \cdot \left(\frac{1}{r^2} \cdot \frac{\partial}{\partial r} \cdot r^2 \cdot \Im_V(r)\right)$$

$$\Im_V(r) = -\frac{1}{2} m_h^2 h^2(r) - \sqrt{\frac{\lambda}{2}} m_h h^3(r) - \frac{\lambda h^4(r)}{4}$$

$$h(r) \sim \exp(-\tilde{\alpha} r)$$

$$\begin{aligned} m_h = \frac{4\pi}{\left(\frac{\mathrm{d}^2 z}{\mathrm{d}t^2}\right)_{\text{acceleration}}} \cdot \Bigg[& \left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot r \cdot \left[A_1 \cdot h^4(r)\right]\right) \cdot \lambda^0 \cdot m_h^4 \\ & + \left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot r \cdot \left[A_2 \cdot h^5(r)\right]\right) \cdot \lambda^{1/2} \cdot m_h^3 \\ & + \left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot r \cdot \left[A_3 \cdot h^6(r)\right]\right) \cdot \lambda^1 \cdot m_h^2 \\ & + \left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot r \cdot \left[A_4 \cdot h^7(r)\right]\right) \cdot \lambda^{3/2} \cdot m_h^1 \\ & + \left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot r \cdot \left[A_5 \cdot h^8(r)\right]\right) \cdot \lambda^2 \Bigg] \end{aligned} \tag{6}$$

We will fill out the A_i terms in the next block of equations as follows:

$$
\begin{aligned}
A_1(r) &= \frac{5\tilde{\alpha}^2}{r} - \left(3\tilde{\alpha}^3 + \frac{2\tilde{\alpha}}{r^2}\right) \\
A_2(r) &= \frac{\left(\left(15/\sqrt{2}\right) + 15\sqrt{2}\right)\tilde{\alpha}^2}{r} - \left(\left(12/\sqrt{2}\right)\tilde{\alpha}^3 + \frac{\left(\left(1/\sqrt{2}\right) + 6\sqrt{2}\right)\tilde{\alpha}}{r^2}\right) \\
A_3(r) &= \frac{(81/2)\tilde{\alpha}^2}{r} - \left(33\tilde{\alpha}^3 + \frac{(47/2)\tilde{\alpha}}{r^2}\right) \\
A_4(r) &= \frac{\left(\left(24/\sqrt{2}\right) + 20\sqrt{2}\right)\tilde{\alpha}^2}{r} - \left(\left(21/\sqrt{2}\right)\tilde{\alpha}^3 + \frac{\left(20/\sqrt{2}\right)\tilde{\alpha}}{r^2}\right) \\
A_5(r) &= \frac{6\tilde{\alpha}^2}{r} - \left(4\tilde{\alpha}^3 + \frac{2\tilde{\alpha}}{r^2}\right).
\end{aligned}
\tag{7}
$$

This leads to a decomposition of the mass, as looking like what is given in Equation (1) with the following coefficients filled in for Equation (1)

$$
\begin{aligned}
\tilde{A}_1 &= \left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot r \cdot \left[A_1(r) \cdot \mathrm{e}^{-4\tilde{\alpha}r}\right]\right) \cdot \lambda^0 \\
\tilde{A}_2 &= \left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot r \cdot \left[A_2(r) \cdot \mathrm{e}^{-5\tilde{\alpha}r}\right]\right) \cdot \lambda^{1/2} \\
\tilde{A}_3 &= \left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot r \cdot \left[A_3(r) \cdot \mathrm{e}^{-6\tilde{\alpha}r}\right]\right) \cdot \lambda^1 \\
\tilde{A}_4 &= \left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot r \cdot \left[A_4(r) \cdot \mathrm{e}^{-7\tilde{\alpha}r}\right]\right) \cdot \lambda^{3/2} - \left(\frac{\mathrm{d}^2 z}{\mathrm{d}t^2}\right)_{\text{acceleration}} \Big/ 4\pi \\
\tilde{A}_5 &= \left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot r \cdot \left[A_5(r) \cdot \mathrm{e}^{-8\tilde{\alpha}r}\right]\right) \cdot \lambda^2
\end{aligned}
\tag{8}
$$

The sheer amount of algebra is, in this situation, unbelievable, and this will be sourced to the following three integrals in from [32].

The three integrals, from [32] are

$$
\begin{aligned}
&\int \mathrm{d}r \cdot r \cdot \exp\left(-\tilde{B}_1 \cdot r\right) = \frac{\exp\left(-\tilde{B}_1 \cdot r\right)}{\tilde{B}_1^{\,2}} \cdot \left(-\tilde{B}_1 \cdot r - 1\right) \\
&\int \mathrm{d}r \cdot \exp\left(-\tilde{B}_1 \cdot r\right) = \frac{\exp\left(-\tilde{B}_1 \cdot r\right)}{-\tilde{B}_1} \\
&\int \mathrm{d}r \cdot r^{-1} \cdot \exp\left(-\tilde{B}_1 \cdot r\right) = \left[\log(r)\right] - \frac{\tilde{B}_1 \cdot r}{1!} + \frac{\left(\tilde{B}_1 \cdot r\right)^2}{2 \cdot 2!} - \cdots
\end{aligned}
\quad . \tag{9}
$$

Having said, this we should in the next section say a bit as to the physics behind the formula

$$
\begin{aligned}
&m_h \left(\frac{\mathrm{d}^2 z}{\mathrm{d}t^2}\right)_{\text{acceleration}} = -4\pi \int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot r^2 \cdot \{\ \} \\
&\{\ \} = \left(\frac{1}{r^2} \cdot \frac{\partial}{\partial r} \cdot r^2 \cdot \frac{\partial}{\partial r} \cdot \Im_V(r)\right) \cdot \left(\frac{1}{r^2} \cdot \frac{\partial}{\partial r} \cdot r^2 \cdot \Im_V(r)\right)
\end{aligned}
\quad . \tag{10}
$$

We are, here, implying use of symmetry, in our ideas, and making the creation of the mass of the Higgs, as seen above, dependent upon a regime of space-time roughly contingent upon the regime of the electro weak regime. In doing so we will be making the following assumptions [33].

Quote from [33]:

In physical cosmology, the ***electroweak epoch*** *was the period in the evolution of the early universe when the temperature of the universe was high enough to merge electromagnetism and the weak interaction into a single electroweak interaction* (>100 GeV). *The electroweak epoch began when the strong force separated from the electroweak interaction. Some cosmologists place this event at the start of the inflationary epoch, approximately* 10^{-36} *seconds after the Big Bang. Others place it at approximately* 10^{-32} *seconds after the Big Bang.*

End of quote.

We will, as a start, begin with the assumption that $r\left(\text{electroweak}\right) \sim \text{planck length } l_P$ and that the expression Δr is connected to the Heisenberg uncertainty principle, *i.e.* we will be approximately using the following for making sense of the HUP with the following arguments, Afterward, we will then begin to ascertain what is to be done with the integrals, as for the input for Equation (8) and Equation (9). But before that we will say something of the HUP to be used, in this input into Equation (9), so to begin our process, we will give a review of HUP Basics. The HUP basics are extremely important in terms of giving an upper bound to the variance of Δr which is of extreme importance, especially in our modeling of if we have Tachyon's in the Pre Planckian Space time era, and then regular Higgs physics from after the Electroweak era to today. So a lot of basic physics is covered in this supposition.

4. Heisenberg Uncertainty Principle Basics Used into the Input of Values into Equation (9) Coefficients Which Are in Turn Part of Equation (1) for a Fourth Order Polynomial for Mass of the Higgs

Starting with background given in [34] [35] [36] [37] [38] we start with

$$\begin{aligned} &\left(\Delta l\right)_{ij} = \frac{\delta g_{ij}}{g_{ij}} \cdot \frac{l}{2} \\ &\left(\Delta p\right)_{ij} = \Delta T_{ij} \cdot \delta t \cdot \Delta A \\ &\phi \sim \xi^{+} \ll M_{\text{Planck}}. \end{aligned} \tag{11}$$

Then

If we use the following, from the Roberson-Walker metric [34] [35] [36] [37] [38]

$$\begin{aligned} &g_{tt} = 1 \\ &g_{rr} = \frac{-a^2\left(t\right)}{1 - k \cdot r^2} \\ &g_{\theta\theta} = -a^2\left(t\right) \cdot r^2 \\ &g_{\phi\phi} = -a^2\left(t\right) \cdot \sin^2\theta \cdot d\phi^2. \end{aligned} \tag{12}$$

Following Unruh [37] [38] write then, an uncertainty of metric tensor as, with the following inputs

$$\begin{aligned} &a^2(t)_{\min} \sim 10^{-110}, \\ &r \sim l_P \sim 10^{-35} \text{ meters.} \end{aligned} \tag{13}$$

Then, if $\Delta T_{tt} \sim \Delta\rho$ [34] [35] [36]

$$\begin{aligned} &V^{(4)} = \delta t \cdot \Delta A \cdot r \\ &\delta g_{tt} \cdot \Delta T_{tt} \cdot \delta t \cdot \Delta A \cdot \frac{r}{2} \geq \frac{\hbar}{2} \\ &\Leftrightarrow \delta g_{tt} \cdot \Delta T_{tt} \geq \frac{\hbar}{V^{(4)}}. \end{aligned} \tag{14}$$

This Equation (14) is such that we can extract, up to a point the HUP principle for uncertainty in time and energy, with one very large caveat added, namely if we use the fluid approximation of space-time [39]

$$T_{ii} = diag(\rho, -p, -p, -p). \tag{15}$$

Then by [34] [39]

$$\Delta T_{tt} \sim \Delta\rho \sim \frac{\Delta E}{V^{(3)}}. \tag{16}$$

Then, by [35]

$$\begin{aligned} &\delta t \Delta E \geq \frac{\hbar}{\delta g_{tt}} \neq \frac{\hbar}{2} \\ &\text{Unless } \delta g_{tt} \sim O(1). \end{aligned} \tag{17}$$

We will then if we make the following approximations have a connection to the Δr value used in Equation (8)

$$\Delta r \approx (\Delta l)_{ij}\, \delta_{ij} = \frac{\delta g_{ii}}{g_{ii}} \cdot \frac{l_{\text{planck}}}{2} \xrightarrow{i \to t = \text{time in Planck regime}} \frac{\delta g_{tt}}{g_{tt}} \cdot \frac{l_{\text{planck}}}{2}. \tag{18}$$

Use, then

$$\delta t \Delta E \approx \frac{\hbar}{\delta g_{tt}} \Rightarrow \Delta r \approx \left|\frac{\delta g_{tt}}{g_{tt}}\right| \cdot \frac{l_{\text{planck}}}{2} \approx \left|\frac{\hbar}{\delta t \Delta E}\right| \cdot \frac{l_{\text{planck}}}{2}. \tag{19}$$

It means that the spatial fluctuation, if δt is of the order of Planck time, and ΔE an emergent energy value, is then crucially dependent upon what we chose for ΔE, then we will have a way to input values into Equation (8).

5. Numerical Procedure Used to Give First Order Integration Values in Equation (8)

Due to the smallness of the value $\Delta r \approx \left|\frac{\hbar}{\delta t \Delta E}\right| \cdot \frac{l_{\text{planck}}}{2}$ which will be evaluated, the following is a default choice, *i.e.* Gauss quadrature [40] [41] [42]

$$\int_{r_0}^{r_0+\Delta r} f(r)\mathrm{d}r \approx \frac{\Delta r}{2}\cdot\left[f\left(r_0+\left(\frac{1}{2}-\frac{\sqrt{3}}{6}\right)\Delta r\right)+f\left(r_0+\left(\frac{1}{2}+\frac{\sqrt{3}}{6}\right)\Delta r\right)\right]+\mathrm{O}\left((\Delta r)^4\right) \tag{20}$$

In this case, the values of function f will be dependent upon the details of what is chosen for the different integrands in the five integrals given in Equation (8), but due to the smallness of $\Delta r \approx \left|\frac{\hbar}{\delta t \Delta E}\right|\cdot\frac{l_{\text{planck}}}{2}$, we claim that this will put a premium upon the evaluation of a suitable ΔE value, and we will next, in order to address this, detail procedures as to isolating optimal ΔE values in order to ascertain, optimal strategies for implementing Equation (20) strategies on the integrals given in Equation (8) above.

6. Optimal ΔE Strategies, Possibly from Astrophysical Data, for Looking at Input Parameters from Equation (8) into Equation (1)

We will look at what is given in [6] as to an elementary Fluctuation-Dissipation theorem given in Pages 325-327, *i.e.* first of all we define a spectral density type of derivation for an average energy, given by, for a Bose system in a heat bath, of quantum oscillators, which becomes

$$E_{\text{mean}} = \frac{\hbar\omega_0}{\mathrm{e}^{\hbar\omega_0/k_B T(\text{temperature})}-1}+\frac{\hbar\omega_0}{2}. \tag{21}$$

Assume that T, temperature is of the order of over 100 GeV, and that we will also have frequency ω_0, for the background as to the creation of particles in the electroweak era, *i.e.* our estimate is, then that we will be able to assert, then that with temperature of a very large magnitude, and with frequency ~1/(time for initiation of electro weak regime) that if we have say

$$\begin{aligned}
&E_{\text{mean}} = \frac{\hbar\omega_0}{\mathrm{e}^{\hbar\omega_0/k_B T(\text{temperature})}-1}+\frac{\hbar\omega_0}{2} \\
&\hbar\omega_0 \ll k_B T\left(\text{temperature Electro weak}\right) \\
&\Delta E \approx E_{\text{mean}} - \frac{\hbar\omega_0}{2} \doteq \frac{\hbar\omega_0}{\mathrm{e}^{\hbar\omega_0/k_B T(\text{temperature})}-1} \sim \text{very big.}
\end{aligned} \tag{22}$$

This would push down the magnitude of Δr

$$\begin{aligned}
\Delta r &\approx \left|\frac{\hbar}{\delta t \Delta E}\right|\cdot\frac{l_{\text{planck}}}{2} \approx \left|\frac{\mathrm{e}^{\hbar\omega_0/k_B T(\text{temperature})}-1}{\delta t\cdot\omega_0}\right|\cdot\frac{l_{\text{planck}}}{2} \\
&\approx \left|\frac{\mathrm{e}^{\hbar/(\text{time for initiation of electro weak regime})\cdot k_B T(\text{temperature})}-1}{\delta t/\left(\text{time for initiation of electro weak regime}\right)}\right|\cdot\frac{l_{\text{planck}}}{2}.
\end{aligned} \tag{23}$$

7. Implications as to Different Integral Terms in Equation (1)

What we are doing is to group the terms, in terms of powers of m_h, and to do this we have a block of five inputs into Equation (1) above which we write up as follows in the following equation.

$$\tilde{A}_1 = \left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot \left[\left[(5)\cdot\tilde{\alpha}^2 \right] - (2)\cdot\tilde{\alpha}^3 r - \left[(3)\cdot\tilde{\alpha}/r \right] \right] \mathrm{e}^{-4\tilde{\alpha}r} \right)$$

$$\tilde{A}_2 \propto \left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot \left[\left[\left(\left[15/\sqrt{2} \right] + 15\sqrt{2} \right) \cdot \tilde{\alpha}^2 \right] - \left(12/\sqrt{2} \right) \cdot \tilde{\alpha}^3 r \right. \right.$$
$$\left. \left. - \left[\left(\left[1/\sqrt{2} \right] + 6\sqrt{2} \right) \cdot \tilde{\alpha}/r \right] \right] \mathrm{e}^{-5\tilde{\alpha}r} \right) \cdot \lambda^{1/2} \tag{24}$$

$$\tilde{A}_3 = \left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot \left[\left[(81/2)\cdot\tilde{\alpha}^2 \right] - (47/2)\cdot\tilde{\alpha}^3 r - \left[(33)\cdot\tilde{\alpha}/r \right] \right] \mathrm{e}^{-6\tilde{\alpha}r} \right) \cdot \lambda$$

$$\tilde{A}_5 \propto \left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot \left[\left(6\tilde{\alpha}^2 - 4\tilde{\alpha}^3 r - \left[2\tilde{\alpha}/r \right] \right) \cdot \mathrm{e}^{-8\tilde{\alpha}r} \right] \right) \cdot \lambda^2$$

Integrals of the form due to the smallness of Δr would likely be deal with by Gaussian Quadrature, as for using Equation (20) on Equation (25)

$$\int \mathrm{d}r \cdot r^{-1} \cdot \exp\left(-\tilde{B}_1 \cdot r\right) = \left[\log(r) \right] - \frac{\tilde{B}_1 \cdot r}{1!} + \frac{\left(\tilde{B}_1 \cdot r\right)^2}{2 \cdot 2!} - \cdots. \tag{25}$$

The other integrals, in Equation (9) would most likely be dealt with via their integral table values, but if we wish to group the integrals from Equation (24) via their integrands, according to r, 1 and 1/r we observe the following pattern, *i.e.* it is in essence a recreation of the driven harmonic oscillator model, which in terms of physics, especially after Gauge invariance is invoked one of the work horses of elementary particle physics, *i.e.* observe the following Equation (26). And in spirit it is not that dissimilar to [1] [2] [3] with one huge difference, *i.e.* what we refer to later, and comment upon, due to Equation (24) and Equation (26) is the possibility of real and complex values as to the Higgs mass.

In doing so, we are also aware that this is so far akin to much of the phenomenology of the standard model, but with the outstanding difference which we have put in the prior paragraph.

$$C_0 \propto \left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot \tilde{\alpha}^2 \cdot \left[(6)\cdot \mathrm{e}^{-8\tilde{\alpha}r}\lambda^2 + \left(\left[24\sqrt{2} \right] + \left[20/\sqrt{2} \right] \right) \cdot \mathrm{e}^{-7\tilde{\alpha}r}\lambda^{3/2} m_h \right. \right.$$
$$\left. \left. + (81/2)\cdot \mathrm{e}^{-6\tilde{\alpha}r}\lambda^1 m_h^2 + \left(\left[15/\sqrt{2} \right] + \left[15\sqrt{2} \right] \right) \cdot \mathrm{e}^{-5\tilde{\alpha}r}\lambda^{1/2} m_h^3 + (5)\cdot \mathrm{e}^{-4\tilde{\alpha}r}\lambda^0 m_h^4 \right] \right)$$

$$C_1 \propto -\left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot r \cdot \tilde{\alpha}^3 \cdot \left[(4)\cdot \mathrm{e}^{-8\tilde{\alpha}r}\lambda^2 + \left(21/\sqrt{2} \right) \cdot \mathrm{e}^{-7\tilde{\alpha}r}\lambda^{3/2} m_h \right. \right.$$
$$\left. \left. + (81/2)\cdot \mathrm{e}^{-6\tilde{\alpha}r}\lambda^1 m_h^2 + (47/2)\cdot \mathrm{e}^{-5\tilde{\alpha}r}\lambda^{1/2} m_h^3 + (2)\cdot \mathrm{e}^{-4\tilde{\alpha}r}\lambda^0 m_h^4 \right] \right)$$

$$C_{-1} \propto -\left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot r^{-1} \cdot \tilde{\alpha}^1 \cdot \left[(2)\cdot \mathrm{e}^{-8\tilde{\alpha}r}\lambda^2 + \left(20/\sqrt{2} \right) \cdot \mathrm{e}^{-7\tilde{\alpha}r}\lambda^{3/2} m_h \right. \right.$$
$$\left. \left. + (33)\cdot \mathrm{e}^{-6\tilde{\alpha}r}\lambda^1 m_h^3 + \left(\left[1/\sqrt{2} \right] + 6\sqrt{2} \right) \cdot \mathrm{e}^{-5\tilde{\alpha}r}\lambda^{1/2} m_h^3 + (3)\cdot \mathrm{e}^{-4\tilde{\alpha}r}\lambda^0 m_h^4 \right] \right)$$

(26)

This is further grouped as

$$m_h \cdot \left(\mathrm{d}^2 z / \mathrm{d}t^2 \right) / 4\pi = C_1 + C_0 + C_{-1}. \tag{27}$$

In order to solve this in terms of powers of m_h, Equation (1) and Equation (24) are preferred. This grouping in Equation (26) and Equation (27) is done for ease in terms of the rules of integration, alone.

8. Examining Issues as to Root Finders and Equation (1)

Doing this means we will be considering an optimal set of procedures so as to find a numerical protocol as to solve for a fourth order polynomial for iteration of mass of m_h.

Simple on line root finders abound. Here is one of them [43]. So what do we want in terms of finding ROOTS of the polynomial given in Equation (1)?

If we wish to solve for a fourth order equation and we are examining to obtain real values for m_h one of the very few papers giving the Cardano solution is sourced here, *i.e.* [44], and then one also look at [45] [46] [47] [48], for the idea of the Descartes rule of signs.

In other words, if Equation (1) and its inputs are defined properly and sourced mathematically, one has the probability one is looking at a real and complex part of a solution to m_h.

9. Ground Zero, What If We Obtain m_h with Real and Complex Coefficients as Far as a Solution to Equation (1) above?

Fortunately, we are not completely without guidance, *i.e.* in Wikipedia, there is a solution [49] [50].

Quote [49]:

*In complex analysis, a branch of mathematics, the **Gauss-Lucas theorem** gives a geometrical relation between the roots of a polynomial P and the roots of its derivative P'. The set of roots of a real or complex polynomial is a set of points in the complex plane. The theorem states that the roots of P' all lie within the convex hull of the roots of P that is the smallest convex polygon containing the roots of P. When P has a single root then this convex hull is a single point and when the roots lie on a line then the convex hull is a segment of this line. The Gauss-Lucas theorem, named after Carl Friedrich Gauss and Félix Lucas, is similar in spirit to Rolle's theorem.*

I.e. we can look at the derivative, [49] [50] in terms of the Equation (1) for m_h we have then,

$$\begin{aligned} & \tilde{A}_1 m_h^4 + \tilde{A}_2 m_h^3 + \tilde{A}_3 m_h^2 + \tilde{A}_4 m_h^1 + \tilde{A}_5 = 0 \\ & \Rightarrow 4\tilde{A}_1 m_h^3 + 3\tilde{A}_2 m_h^2 + 2\tilde{A}_3 m_h^1 + \tilde{A}_4 = 0 \\ & \Rightarrow \tilde{A}_1 m_h^3 + \frac{3}{4}\tilde{A}_2 m_h^2 + \frac{1}{2}\tilde{A}_3 m_h^1 + \frac{1}{4}\tilde{A}_4 = 0. \end{aligned} \tag{28}$$

Beyer, [32] has standard root finding procedures for Equation (28) on page 9. But what is at times over looked, is that [32] also has on page 12 a way to reduce a fourth power quartic equation to a resolvent cubic equation which we will put down below, *i.e.* this is how to get the fourth order solution in full generality, *i.e.* go to Equation (31). Note that in doing it, we STILL have to obtain the cubic solution to the equation, given below, namely as given by [32], page nine, we need to use the standard cubic root finders to solve for the rescaled cubit polynomial.

$$y^3 - by^2 + (ac - bd)y - a^2 d + 4bd - c^2 = 0 \tag{29}$$

where

$$a = \frac{\tilde{A}_2}{\tilde{A}_1}, b = \frac{\tilde{A}_3}{\tilde{A}_1}, c = \frac{\tilde{A}_4}{\tilde{A}_1}, d = \frac{\tilde{A}_5}{\tilde{A}_1}. \tag{30}$$

The terms in Equation (24) are huge numerical monsters in their own right. And of course we will be requiring that in Equation (31), below, is that to solve for Equation (1) in full generality, we will almost certainly be observing complex valued solutions to m_h. If we solve for m_h and obtain complex valued solutions to the Higgs mass, as obtained this way, we then have some fundamental questions to answer.

Before we do that, let us review below, in Equation (31) the full generality of our answer. It is to put it very specifically, a huge mess. Why would we do this or want to do it?

Note that Equation (31) is, superficially like a driven harmonic oscillator. The program for identification of the Higgs mass, has been to identify terms in the potential which proportionality are linkable to the mass of a Harmonic oscillator, *i.e.* go to [1] [2] and [3]. While in terms of phenomenology, this has been brilliantly successful, it also has made it impossible to ascertain if the mass, itself, and the formation of mass, may have complex roots, or, even quaternion number roots.

We have been ascertaining possible solutions to the Higgs, and seeking solutions to beyond the standard model of the Higgs, for decades, *i.e.* see [51].

The solutions as to what was brought up in [1] [2] [3] as extended beyond the standard model may involve the idea of a unique, or non-unique Higgs doublet, and match some of the experimental evidence so far, but references [52] [53] [54] [55], *i.e.* so far they have not been able to dethrone the standard model, as given in [1] [2] [3]. However, the Standard model also has problems. Hence we will examine what we are able to ascertain via Equation (31) below.

Due to how nonstandard all this is, we will commence in our section past Equation (31) below to comment upon the implications of real and complex parts to mass terms, especially in something as foundational as the Higgs mass.

We wish to also, afterwards, go to the 2nd part of this paper, which will be to make linkage to the dark matter model, which may involve the Higgs, in modified form, as given in [5]. And then to also close the paper with observations on classical versus quantum foundations of our present cosmos as has been referenced already via [12] [18] and [19].

Part of the problem especially even with extensions as to the Higgs models, is still the fix upon derivations of matter and reality as based squarely on modifications of the Harmonic oscillator. While it has served us very well in the last 100 years, extensions beyond it may require a re think of basic assumptions. We will be revisiting some of these assumptions in the second half of our paper.

To make the case for possible complex parts to the Higgs mass, we reference Equation (31) below and also then will use it as to going to the foundational question in the next part which is what happens if we have not solely real valued mass terms. What are we in for?

$$
\begin{aligned}
&\tilde{A}_1 m_h^4 + \tilde{A}_2 m_h^3 + \tilde{A}_3 m_h^2 + \tilde{A}_4 m_h^1 + \tilde{A}_5 = 0 \\
&\Rightarrow m_h^4 + \frac{\tilde{A}_2}{\tilde{A}_1} m_h^3 + \frac{\tilde{A}_3}{\tilde{A}_1} m_h^2 + \frac{\tilde{A}_4}{\tilde{A}_1} m_h^1 + \frac{\tilde{A}_5}{\tilde{A}_1} = 0 \\
&\text{is linkable to} \\
&x^4 + ax^3 + bx^2 + cx + d = 0 \\
&x = m_h^1, a = \frac{\tilde{A}_2}{\tilde{A}_1}, b = \frac{\tilde{A}_3}{\tilde{A}_1}, c = \frac{\tilde{A}_4}{\tilde{A}_1}, d = \frac{\tilde{A}_5}{\tilde{A}_1} \\
&\text{Then, to} \\
&y^3 - by^2 + (ac - bd)y - a^2 d + 4bd - c^2 = 0 \\
&x = -(a/4) + R/2 \pm (D/2) \\
&\text{or } \; x = -(a/4) - R/2 \pm (E/2) \\
&\text{where } \; R = \sqrt{\frac{a^2}{4} - b + y} \\
&\text{If } \; R \neq 0 \\
&D = \sqrt{\frac{3a^2}{4} - R^2 - 2b + \left(\frac{4ab - 8c - a^3}{4R}\right)} \\
&E = \sqrt{\frac{3a^2}{4} - R^2 - 2b - \left(\frac{4ab - 8c - a^3}{4R}\right)} \\
&\text{If } \; R = 0 \\
&D = \sqrt{\frac{3a^2}{4} - 2b + 2\sqrt{y^2 - 4d^2}} \\
&E = \sqrt{\frac{3a^2}{4} - 2b - 2\sqrt{y^2 - 4d^2}}
\end{aligned}
\tag{31}
$$

10. Implications If a Mass for the Higgs Has Real and Complex Components, *i.e.* Imaginary Mass Has Tachyon Mass Properties (Faster Than Light). As Cited from Baez, Reference [56]

What we can expect if the Higgs, say has

$$m_h = \mathrm{Re}(m_h) + i \cdot \mathrm{Im}(m_h). \tag{32}$$

Baez says that if the mass of a particle is purely imaginary that, in [56], this means faster than light travel. We suppose from what we know of real valued Higgs physics, as to what is given in the middle to latter part of the Electroweak era [57], that [1] to [3] actually hold up very well.

According to Baez, [56], we then have if the mass is purely imaginary, a situation where one has particles which travel faster than the speed of light, *i.e.* the Tachyon.

What we are supposing, taking a line from [56] is a situation in which then at least part of the mass will contribute to faster than light "travel". This is also discussed on page 16 and 17 of [57], albeit as a supposed fault of prior to present day String theory models.

Note that in [12] that we discussed the case of entanglement, especially with regards to Gravitons, and came to the conclusion that it is probable that information, would be traveling say up to 100,000 times faster than light, whereas physical exchange of space time, exclusive of information would be traveling at the speed of light.

Our supposition is as follows, *i.e.* to a large degree what we are predicting is that solving Equation (31) would lead to [1] [12] and [56]

$$\begin{aligned}
& m_h = \mathrm{Re}(m_h) + i \cdot \mathrm{Im}(m_h) \xrightarrow[\text{Prior To Electroweak era}]{} i \cdot \mathrm{Im}(m_h) \\
& \&\ \text{'Quantum-entanglement}(\text{information-transfer}) \\
& m_h = \mathrm{Re}(m_h) + i \cdot \mathrm{Im}(m_h) \xrightarrow[\text{Electroweak era to today}]{} \mathrm{Re}(m_h) \\
& \&\ \text{Standard Model Higgs mass formation properties}
\end{aligned} \tag{33}$$

Getting a description of this modeling would require extensive numerical and analytical treatment of Equation (1), Equation (31), and of also reviewing what we think we know of how information could be transferred through the ideas of finding EPR style entangled wave function states from a prior to the present universe. The readers are encouraged to review [12] for the ideas of entanglement, and to also consider [56] as to what Baez is implying.

Having made our predictions, as far as a goal to be investigated, we remind the readers to see what entanglement may be doing in the Pre Planckian to Planckian era, as in [21]. After having said this, we will next go to the idea of Dark matter physics, which is alluded to in Majumdar [5], and allude to what we think the modification of the Higgs mass hypothesis may be telling us next, *i.e.* the idea will be to examine again what is called "inert Doublet Dark matter", *i.e.* by necessity this will mean that the gauge invariance arguments will have to be written out to an extraordinary degree, undoubtedly boring some particle physics researchers whom have done this sort of thing for years. What we will do though will be in lieu of predictions given in Equation (33) above, go to making testable modifications of the usual Higgs doublet hypothesis which may lead to some astrophysical traces in the CMBR. After we allude to the astrophysical traces, as a result of a review of the material given in [5], as modified in part by

Equation (33) above, we will be finally asking about the inter relationship between classical and quantum mechanics which will conclude our document.

We will make brief reference, to what Equation (1) has as a solution to We will before moving on, make brief reference as to what the value of Equation (1) is if we take the derivative of it, with respect to We will before moving on, make brief reference as to what the value of Equation (1) is if we take the derivative of it, with respect to m_h is, and then state that this is a brief foray into what would be needed even if we merely had the DERIVATIVE of Equation (1) with respect to m_h, set equal to zero. What would we have to have so that the mass, m_h would have a definite imaginary component? *i.e.* we will be looking at in the case of a DERVIATIVE of Equation (1) with respect to the presumed Higgs mass the following procedure which may happen in the Pre Planckian to Planckian regime of space-time about the Electroweak spatial regime the following procedure as seen in the next page.

Keep in mind the following that this is to illustrate the complexities involved, as far as how to isolate in the spatial regime, to the beginning of the electro weak era how complex values of the mass could arrive.

In doing so, it is important to note that this is for the DERIVATIVE of Equation (1) with respect to Higgs mass, and that this is in order to appeal to this well-known example, *i.e.*

This is from the Wiki discussion given in [49].

Quote: from [49].

Special Cases:

It is easy to see that if $P(x) = ax^2 + bx + c$ *is a second degree polynomial, the zero of* $P'(x) = 2ax + b$ *is the average of the roots of P. In that case, the convex hull is the line segment with the two roots as endpoints and it is clear that the average of the roots is the middle point of the segment.*

End of quote.

What we did below is to appeal to a very approximate method in order to isolate conditions in which we could have an imaginary component to the Higgs mass. In doing so, we do this with the expectation that in the regime of space-time so written out, that we have the dreaded by String theorists Tachyon solution, which is going faster than light.

In the case of the solution given in Equation (34) below, we have an imaginary component to the Higgs mass as specified in the spatial regime for which the Electro weak processes occur in cosmology, *i.e.* sometime after a distance of at least Planck length from the boundary of creation of a new universe. The results of this are given below. We will then, say something as to the restriction as to having what we call C2 as nearly zero, in order to draw out this approximate imaginary behavior, for Higgs mass, which is what we will be appealing to

$$m_h^4 + \frac{\tilde{A}_2}{\tilde{A}_1} m_h^3 + \frac{\tilde{A}_3}{\tilde{A}_1} m_h^2 + \frac{\tilde{A}_4}{\tilde{A}_1} m_h^1 + \frac{\tilde{A}_5}{\tilde{A}_1} = 0$$
$$\text{Take} - \text{derivative} - w.r.t. - m_h^1$$

$$
\begin{aligned}
&\Rightarrow m_h^3 + \frac{3\tilde{A}_2}{4\tilde{A}_1} m_h^2 + \frac{\tilde{A}_3}{2\tilde{A}_1} m_h^1 + \frac{\tilde{A}_4}{4\tilde{A}_1} = 0 \\
&a_1 = \frac{3\tilde{A}_2}{4\tilde{A}_1},\ a_2 = \frac{\tilde{A}_3}{2\tilde{A}_1},\ a_3 = \frac{\tilde{A}_4}{4\tilde{A}_1} \\
&\Rightarrow m_h^3 + a_1 m_h^2 + a_2 m_h^1 + a_3 = 0 \\
&\text{Then rewrite} \\
&y^3 + c1 \cdot y + c2 = 0 \\
&m_h^1 = y - (a_1/4) \\
&c1 = \frac{1}{3} \cdot \left(\frac{3}{2} a_2 - \left[\frac{3}{4} a_1 \right]^2 \right) \\
&c2 = \frac{1}{27} \cdot \left(2 \left[\frac{3}{4} a_1 \right]^2 - \frac{27}{8} \cdot a_1 \cdot a_2 + \frac{27}{4} \cdot a_3 \right) \\
&A = \left(-\frac{(c2)}{2} + \sqrt{\frac{(c2)^2}{4} + \frac{(c1)}{27}} \right)^{1/3},\ B = -\left(-\frac{(c2)}{2} + \sqrt{\frac{(c2)^2}{4} + \frac{(c1)}{27}} \right)^{1/3} \\
&y = A + B, -\left(\frac{A+B}{2} \right) + \left(\frac{A-B}{2} \right) \cdot \sqrt{-3}, -\left(\frac{A+B}{2} \right) - \left(\frac{A-B}{2} \right) \cdot \sqrt{-3} \\
&\&\ \ m_h^1 = A + B - (a_1/4), -\left(\frac{A+B}{2} \right) + \left(\frac{A-B}{2} \right) \cdot \sqrt{-3} - (a_1/4), \\
&\text{and } -\left(\frac{A+B}{2} \right) - \left(\frac{A-B}{2} \right) \cdot \sqrt{-3} - (a_1/4) \\
&\ \text{then} \\
&c2 \approx \varepsilon^+ \Rightarrow m_h^1 \approx \left(\frac{A-B}{2} \right) \cdot \sqrt{-3} - (a_1/4) \approx 2 \left(+ \sqrt{\frac{(c1)}{27}} \right)^{1/3} \cdot \sqrt{-3} - (a_1/4); \\
&m_h^1 \approx 2 \left(+ \sqrt{\frac{\frac{1}{3} \cdot \left(\frac{3}{2} a_2 - \left[\frac{3}{4} a_1 \right]^2 \right)}{27}} \right)^{1/3} \cdot \sqrt{-3} - (a_1/4)
\end{aligned}
\tag{34}
$$

Here, C2 nearly zero is if we can make the following identification, *i.e.* this is what would be necessary, *i.e.*

If

$$
\begin{aligned}
&\tilde{A}_1 = \left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot \left[\left[(5) \cdot \tilde{\alpha}^2 \right] - (2) \cdot \tilde{\alpha}^3 r - \left[(3) \cdot \tilde{\alpha}/r \right] \right] \mathrm{e}^{-4\tilde{\alpha}r} \right) \\
&\tilde{A}_2 \propto \left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot \left[\left[\left(\left[15/\sqrt{2} \right] + 15\sqrt{2} \right) \cdot \tilde{\alpha}^2 \right] - \left(12/\sqrt{2} \right) \cdot \tilde{\alpha}^3 r \right. \right. \\
&\qquad \left. \left. - \left[\left(\left[1/\sqrt{2} \right] + 6\sqrt{2} \right) \cdot \tilde{\alpha}/r \right] \right] \mathrm{e}^{-5\tilde{\alpha}r} \right) \cdot \lambda^{1/2} \\
&\tilde{A}_3 = \left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot \left[\left[(81/2) \cdot \tilde{\alpha}^2 \right] - (47/2) \cdot \tilde{\alpha}^3 r - \left[(33) \cdot \tilde{\alpha}/r \right] \right] \mathrm{e}^{-6\tilde{\alpha}r} \right) \cdot \lambda
\end{aligned}
$$

$$\tilde{A}_4 \propto \left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot \left[\left[\left(24\sqrt{2} + 20/\sqrt{2} \right) \tilde{\alpha}^2 \right] - \left(21/\sqrt{2} \right) \tilde{\alpha}^3 r \right. \right.$$
$$\left. \left. - \left[\left(20/\sqrt{2} \right) \tilde{\alpha}/r \right] \right] \mathrm{e}^{-7\tilde{\alpha}r} \right) \cdot \lambda^{3/2} - \left(\mathrm{d}^2 z/\mathrm{d}t^2 \right) / 4\pi$$
$$\tilde{A}_5 \propto \left(\int_{r(\text{electroweak})}^{r(\text{electroweak})+\Delta r} \mathrm{d}r \cdot \left[\left(6\tilde{\alpha}^2 - 4\tilde{\alpha}^3 r - [2\tilde{\alpha}/r] \right) \cdot \mathrm{e}^{-8\tilde{\alpha}r} \right] \right) \cdot \lambda^2$$
Then
$$c2 = \frac{1}{27} \cdot \left(2\left[\frac{3}{4}a_1\right]^2 - \frac{27}{8} \cdot a_1 \cdot a_2 + \frac{27}{4} \cdot a_3 \right) \approx \varepsilon^+$$
$$\Leftrightarrow \frac{27}{8} \cdot a_1 \cdot a_2 \approx 2 \cdot \left[\frac{3}{4}a_1\right]^2 + \frac{27}{4} \cdot a_3$$
$$\& \;\; a_1 = \frac{3\tilde{A}_2}{4\tilde{A}_1},\ a_2 = \frac{\tilde{A}_3}{2\tilde{A}_1},\ a_3 = \frac{\tilde{A}_4}{4\tilde{A}_1} \tag{35}$$

What we could do would be to try to ascertain if this is possible in the regime of space-time going up to the electroweak regime in cosmology. Our supposition as to the transfer between the regime of imaginary mass, which may involve Faster than light transference of "information" to the Higgs we believe may be similar to the physics, given in page 70 of Ohanian, and Ruffini [58] as given by

$$\partial_v T^{uv} = f^u \tag{36}$$

i.e. the relativistic correction to Newton's second law. The force, *f*, in Newtonian physics becomes force density, or four force per unit volume arising from forces external to the system.

This is the trick we are using, *i.e.* assuming that there is an external to the initial universe recycling of matter-energy, with the idea of mass, initially re injected into the system. To get an idea of what the author is suggesting, we recommend that readers peruse [59]. Our suggestion is that this is also linked to information transfer, as discussed in [12], by the Author, as what occurs in [60] as to a recycling cosmology, which the author generalized in [59], as a candidate as to why Equation (36) would even be considered in this situation as to early universe cosmology.

Having said this, and specifically enjoining readers that this discussion has to be confirmed and vetted with numerical studies, we then will go to how we think this set of ideas may impact a model of Dark matter, called "inert doublet Dark matter" which is the next section of our manuscript. We will first, in [5] allude to Scalar Singlet Dark Matter, and then afterwards go to the inert Doublet Dark Matter model given in [5].

11. Modified Higgs Theory, as Alluded to in Equation (33) and Linkage to Reference [5] Higgs Fields & 2 Dark Matter Models

In [5] there is a statement as of page 133 to 134 about Scalar Singlet dark matter, as to adding to the Standard Model Lagrangian the following addition, namely

$$\begin{aligned} V(H,S) &= \frac{m^2}{2}\cdot H^\dagger H + \frac{\lambda}{4}\cdot\left(H^\dagger H\right)^2 + \frac{\delta_1}{2}\cdot H^\dagger HS + \frac{\delta_2}{2}\cdot H^\dagger HS^2 \\ &+ \frac{\delta_1}{2}\cdot\frac{m^2}{\lambda}S + \frac{\kappa_2}{2}S^2 + \frac{\kappa_3}{3}S^3 + \frac{\kappa_4}{4}S^4 \end{aligned} . \tag{37}$$

This leads to a total Lagrangian for the Higgs singlet model

$$\begin{aligned} \Im &= \Im_{SM} + \frac{1}{2}\partial_\mu S\partial^\mu S - \left(\frac{m^2}{2}\cdot H^\dagger H + \frac{\lambda}{4}\cdot\left(H^\dagger H\right)^2 + \frac{\delta_1}{2}\cdot H^\dagger HS\right. \\ &\left. + \frac{\delta_2}{2}\cdot H^\dagger HS^2 + \frac{\delta_1}{2}\cdot\frac{m^2}{\lambda}S + \frac{\kappa_2}{2}S^2 + \frac{\kappa_3}{3}S^3 + \frac{\kappa_4}{4}S^4\right) \end{aligned} . \tag{38}$$

Here, we have that h is the physical Higgs field, $v = 246\ \text{GeV}$, and the vacuum expectation value (vev) of scalar H is defined by m and λ as $\sqrt{-2m^2/\lambda}$, with H defined as given below

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}. \tag{39}$$

Furthermore, [5] on page 134 designates that after electroweak symmetry breaking one has $H^\dagger HS^2$ reset as

$$\frac{\delta_2}{2}\cdot H^\dagger HS^2 = \frac{\delta_2}{2}\cdot\left(\frac{v^2S^2}{2} + vhS^2 + \frac{h^2S^2}{2}\right) \tag{40}$$

where we also have a scalar mass term defined by, (if $\lambda = \frac{\delta_2 v^2}{2}$ is a coupling between two scalars and the Higgs field, and $\frac{\delta_2}{2}\cdot H^\dagger HS^2$ the interaction term between the two scalars and the Higgs)

$$V_{\text{mass}} = \frac{1}{2}\cdot\left[\left(m_h^2 = -m^2 = \frac{\lambda v^2}{2}\right) + \left(m_s^2 = \kappa^2 + \frac{\lambda v^2}{2}\right)\right]. \tag{41}$$

Our supposition, is that if we have an initially imaginary mass, that this will lead to

$$m^2\Big|_{\text{initial}} = \frac{\lambda v^2}{2} \Rightarrow \lambda = \frac{2m^2\Big|_{\text{initial}}}{v^2} = \frac{\delta_2 v^2}{2}. \tag{42}$$

The consequence would be in having

$$\delta_2 = \frac{4m^2\Big|_{\text{initial}}}{v^4}. \tag{43}$$

Leading to a recasting of the coupling of the two scalars and the Higgs field, to read as follows

$$\lambda = \frac{2m^2\Big|_{\text{initial}}}{v^2}. \tag{44}$$

What we are saying, is that when we have this, that we will be referring to $m^2\Big|_{\text{initial}}$, which is due to the imaginary mass contribution being dominant, as a direct result of information transfer, a.k.a. the same way we have quantum entanglement. So, to highlight the importance of this idea, we will next revisit what we can say from [21] as far as entanglement and information transfer, as it applies, as we think to the problem of up to the initial start of the electroweak era.

12. Information Transfer, Possibly from a Prior to the Present Universe and How It May Affect What We Can Say about Equation (44) above

From [21] we isolated the way to have an equivalent mass, based upon a non-standard way to illustrate a basic cosmology as given by.

Quote, pages 38-39 of [21]:

We begin with a non-standard representation of mass from Plebasnki and Krasiniski [61] which affirms the likelihood of the synthesis of mass, if and when the radii of a universe, due to a metric is non zero, *i.e.* from [61], page 295, and page 296.

If one assumes a metric given by [61], page 295

$$\mathrm{d}S^2 = \left[\exp\left(C\left[t,r\right]\right)\right]\cdot \mathrm{d}t^2 - \left[\exp\left(A\left[t,r\right]\right)\right]\cdot \mathrm{d}r^2 - R^2\left(t,r\right)\cdot\left[\mathrm{d}\vartheta^2 + \left(\sin^2\vartheta\right)\cdot \mathrm{d}\phi^2\right] \tag{45}$$

Pick, in this case, R is equal to r, usual spatial distance, due to the following argument given below

$$\begin{aligned} &S\left(\text{surface}\right) = 4\pi\cdot R^2\left(x,t\right) \\ &\Leftrightarrow R^2\left(x,t\right) = \text{square, of areal radius} \\ &\Leftrightarrow R^2\left(x,t\right) = r^2\left(x,t\right). \end{aligned} \tag{46}$$

Leading to an effective mass which we can define via page 296 of [61] as given by

$$\begin{aligned} m\left(\text{effective}\right) = \frac{C^2\left(r,t\right)}{2G}\cdot\Bigg(& R + \left(\exp\left(-C(r,t)\right)\right)\cdot R\cdot\left(\frac{\partial R}{\partial t}\right)^2 \\ & -\left(\exp\left(-A(r,t)\right)\right)\cdot R\cdot\left(\frac{\partial R}{\partial r}\right)^2 + \frac{\Lambda\cdot R^3}{3}\Bigg). \end{aligned} \tag{47}$$

This expression for an effective mass would be zero if the R goes to zero, but we are presuming due to Beckwith, [21] that we have a finite, nonzero beginning to the radius of expansion of the universe.

Furthermore, to get this in terms of $R = r$, that the above is, then at (Friedman equation Hubble parameter H, with $H = 0$ re written as, if $R = r = $ Planck length),

$$\begin{aligned} & m\left(\text{effective}\right)\Big|_{H(\text{Hubble})=0} \\ & = \frac{C^2\left(r,t\right)}{2G}\cdot\left(r^2 - \left(\exp\left(-A\left(r,t\right)\right)\right)\cdot r + \frac{\Lambda\cdot r^3}{3}\right)\Bigg|_{r\propto l(\text{Planck})} \\ & \approx \frac{C^2\left(r = l_{\text{Planck}}, t_{\text{Planck}}\right)}{2G}\cdot\left(l_{\text{Planck}}^2 - \left(\exp\left(-A(l_{\text{Planck}}, t_{\text{Planck}})\right)\right)\cdot l_{\text{Planck}} + \frac{\Lambda\cdot l_{\text{Planck}}^3}{3}\right)\Bigg|_{r\propto l(\text{Planck})} \end{aligned} \tag{48}$$

End of quote.

We say that in the very early universe, perhaps before the electroweak era, that the above expression for mass, would have to be equivalent in "information" with respect to Equation (42), *i.e.* note that in [21] what we were doing with this above was to come up with a way to identify due to the action of a line metric in space-time the existence of space-time mass for radii of the universe not equal to zero. We will state that if the two are equivalent, *i.e.* Equation (42) and Equation (48), that then what we may be looking at is entanglement information transfer, and this in terms of imaginary mass, as delineated in Equation (42) having its counterpart, as to what was written in [21] which we quote below as:

Quote, [21] page 39:

We claim that this effective mass should be put into the following wave function at the boundaries of the $H = 0$ causal boundary, as outlined by Beckwith, in [62]. Then we make the following approximation at the $H = 0$ causal boundary, for the entangled wave function for Faster than light transmission of "quantum information", *i.e.*

$$\begin{aligned}
&\psi_{\text{Entangled}} \\
&\propto \frac{1}{\sqrt{8\pi}} \cdot \left\{1 - \frac{1}{3!}\frac{n_A^2 \cdot \pi^2 \cdot r_{\text{Prior}}^2 \cdot \hbar^2}{2mE_0}\right\} \otimes \left\{1 - \frac{1}{3!}\frac{n_B^2 \cdot \pi^2 \cdot r_{\text{Present}}^2 \cdot \hbar^2}{2mE_0}\right\} \\
&\quad + \frac{1}{\sqrt{8\pi}} \cdot \left\{1 - \frac{1}{3!}\frac{n_A^2 \cdot \pi^2 \cdot r_{\text{Present}}^2 \cdot \hbar^2}{2mE_0}\right\} \otimes \left\{1 - \frac{1}{3!}\frac{n_B^2 \cdot \pi^2 \cdot r_{\text{Prior}}^2 \cdot \hbar^2}{2mE_0}\right\}.
\end{aligned} \tag{49}$$

End of quote.

What we are suggesting is that we may have something very similar as to the formation of mass problem, for a Higgs, with the positions r(prior) referring to prior universe positions, and r(present) referring to positions as of about the formation(start) of the electro weak era. The indices (A) and (B) refer to information states in the prior to present universe which would be relatively instantaneously transferred from the prior to the present universe, *i.e.* the position r(present) would be about at the start of the electroweak era in the present universe, *i.e.* an "imaginary" mass corresponding to "information" packet which is almost instantaneously transferred.

The entanglement protocol is also described in [12] in a generalized description of entanglement and also is sourced in [14] [15] [16].

Our supposition is that imaginary mass, not being measurable in our space-time, would be in its transfer from a present to a prior universe, similar in the entanglement information exchange as referenced in [12] [14] [15] [16] [21].

This is a detail which has to be worked out. The numbers $n(A)$ and $n(B)$ in this case would refer to specifically numbered states of matter-energy packets whose information would be exchanged almost instantaneously from a prior to a present universe. In this case, our supposition is that any would be imaginary mass, going in between the prior to present universe, would probably be unmeasurable, until they went to being a real valued traditional mass value.

This is our supposition. And what we would be working with. Our next goal will be to see if there is any linkage of this topic with that as to gravity waves and gravitation.

13. Does Our Formulation of Higgs Bosons with Imaginary Mass, Initially (Information Transfer?) Help to Explain Initial Gravitational Wave Generation? If So, How and What Does This Say as to Dark Energy?

So far, we have discussed the formation of a Higgs mass, with, prior to and then up to the start of the Electro weak era having at least some imaginary mass component.

To rephrase our question, what would happen if the mass went from an unmeasurable by physics instruments imaginary value to real value, say as in the middle of the electroweak transformation? We assert that such an emergence, from imaginary to real values, as we assert could occur through evaluation of Equation (1) would in itself create turbulence in space-time which may lead to dark energy, *i.e.* to see what we are referring to, we go to [63] where massive gravitons are conflated with Dark energy, and also we have that [21] has explicit linkage of early universe graviton states, as generated by information transfer from prior to present universe conditions as seen in the counterpart to (49) in [21], which we have already cited.

Our supposition is that the emergence of Higgs mass due to the following transformation created local space-time turbulence, hence thereby generation of GW, and what we suppose are gravitons.

So we will make the following linkage between a change from imaginary (Higgs boson mass), to real (Higgs boson mass), and then the generation of GW, from what would be a phase transition induced in the Electro-weak era, as we go directly to real valued Higgs boson mass, from an earlier imaginary valued Higgs boson mass. This would induce turbulence, in local space time which would also then induce GW, and by default, massive gravitons.

Chiara Caprinia, Ruth Durrer and G'eraldine Servant [64] state specifically that.

Quote, page 2 of [64]:

Although the first GW detections will come from astrophysical processes, such as merging of black holes, another mission of GW astronomy will be to search for a stochastic background of GWs of primordial origin. An important mechanism for generating such a stochastic GW background is a relativistic first-order phase transition [65] [66]. *In a first-order phase transition, bubbles are nucleated, rapidly expand and collide. The free energy contained in the original vacuum is released and converted into thermal energy and kinetic energy of the bubble walls and the surrounding fluid. Most of the gravitational radiation comes from the final phase of the transition, from many-bubble collisions and the subsequent MHD turbulent cascades. The associated GW spectrum encodes infor-*

mation on the temperature of the universe T at which the waves were emitted as well as on the strength of the transition. The characteristic frequency of the waves corresponds to the physics that produces them.*

End of quote.

Note that in [64], we have the following approximations given in page 36 of that document, in [64] to the effect that

$$\tilde{\tilde{\beta}} \cdot \tilde{\tilde{h}} \sim 8\pi G \tilde{\tilde{T}} \tag{50}$$

$$\begin{aligned}
&\tilde{\tilde{h}} = \text{Amplitude of GW producing Perturbation} \\
&1\big/\tilde{\tilde{\beta}} = \text{Characteristic time which GW producing Perturbation occurs} \\
&\quad \propto 1\big/\left(\omega_{\text{GW}} = \text{Gravitational wave frequency}\right) \\
&\tilde{\tilde{T}} = \text{Energy momentum tensor of source for creation of GW} \\
&\rho_{\text{GW}} = \left(\text{GW energy density}\right) = \left(1\big/\tilde{\tilde{\beta}}\right) \cdot 8\pi \cdot G \cdot \tilde{\tilde{T}}
\end{aligned}$$

In [21] we estimated a relic GW frequency of the order of 1/Planck length, *i.e.* ~10^35 GHz, or higher, *i.e.* very high, at the start of the Electroweak regime, or just before at the beginning of Planckian physics.

What we will be doing as a future project will be to make a convincing argument as to what $\tilde{\tilde{T}}$ should be following an emergence of imaginary (initially) Higgs bosons, transferred to real Higgs bosons, in the middle of the Electroweak era.

We submit that the emergence from imaginary to real space time of such Higgs bosons, would precipitate the creation of the bubbles in space time which would subsequently collide, thereby generating gravitational waves and turbulence, with the energy of such collisions, and other factors of Gravitational generation leading to massive graviton production, *i.e.* this is our prediction,

Prediction. Should the author in careful analysis, create due to the emergence of space-time bubbles, co currently with Higgs bosons emerging from imaginary to real space time, in the electroweak era be able to write out $\tilde{\tilde{T}}$ as a suitable mass-energy tensor, then this analysis will be co currently solving not only where the information (similar to the [21] discussion of entanglement, as well as what was discussed in [12]) transfer does in terms of creation of a template for Higgs bosons, but there will also be a mechanism in the Electro weak era, giving motivation to Equation (50) above.

14. Conclusions. Future Research Directions and 7 Unanswered Modeling Questions as Well as Potentially Rich GW/Graviton Production Physics, from the Electro-Weak Era

First of all, this paper supposes that if there was implementation of Equation (1) of this document, there would be largely, due to the smallness of the spatial dimensions, a regime in space-time where solving the Higgs boson mass could lead to imaginary Higgs masses, *i.e.* an argument sourced from Baez [56] identifies imaginary mass with Tachyons, *i.e.* particles which travel faster than the speed of light. What we did was to source this argument with the methodology of entanglement as given in [12] [21], and also some of the geometry brought up in [62].

1) 1st question

Can rigorous numerical simulations verify in fact, what we are supposing as to how mass could become at least in part imaginary, as solutions to the Equation (1) nonlinear Higgs mass solver? Qualitative solutions to this, in terms of what is known about the numerical procedures for solution to cubic and quartic equations were referenced. We also state that many of the integrals in the coefficients in Equation (1) will be needed to be solved via Gaussian quadrature, as given in Equation (20) which puts a premium upon Δr.

2) 2nd question

To vet and to analyze is how far we can go to solve Δr. This means confirmation of $\Delta r \approx \left|\frac{\hbar}{\delta t \Delta E}\right| \cdot \frac{l_{\text{planck}}}{2}$. Note that this is akin to, with caveats to picking, in the case of Equation (50), *i.e.* we think that the energy momentum tensor, as talked about in Equation (50).

3) 3rd question

If we can solve for the shift in energy, and we discuss that about Equation (21) and Equation (22) we will be taking steps to obtain the energy momentum tensor of Equation (50). Can this be done?

I.e. in addition, what we did, is to invert the procedure as to solve the Higgs mass problem and to argue that its mass emergence from imaginary, to real values would be the template as to create in local space time the bubbles, which can collide in the electroweak era to create gravitational physics, which could create relic GW and hopefully massive gravitons.

4) 4th question

Can we state that emergence of imaginary Higgs mass to real Higgs mass create bubbles of space-time in the electro weak era?

5) 5th question

We have argued in terms of entanglement, in terms of transferal of information from a past to a present universe. Is entanglement linkable to the following supposition?

Our supposition is that entanglement instantaneous transfer of information, would be akin to having Faster than Light Tachyons go from the prior to the present universe, with a dump of that information into real mass valued Higgs bosons, in the electro weak era.

Can a rigorous mathematical model of an answer to this question above be ascertained?

Being able to confirm this would be largely updating Equation (49), which is from [21].

6) 6th question

Is of DM, *i.e.* the singlet DM hypothesis, *i.e.* the key question centers about what we call Equation (44), *i.e.* is Equation (44) actually allowable?

We claim that if Equation (44) is true, that we will have CMBR signatures to pick up and that this will have a lot to do with.

7) 7th question

How reliable is [63] *i.e.* massive gravitons for Dark Energy?

In addition, how could we link our inquiry into issues brought up in [67]-[72]? The Abbot papers [67] [70] [71] have gravitational wave astronomy basics which cannot be contravened, *i.e.* we have [64] which we are claiming as a motivation for creation for GW, and by extension Gravitons, and can it be reconciled with the Abbot papers?

Corda's document, [69] and [72] should be compared with the ideas given in [64] which we claim is our motivation for imaginary Higgs masses being transferred to real Higgs masses, creating space time bubbles. Are these sets of documents congruent with each other?

Finally is the question of extra dimensions, *i.e.* [68]. Are our suppositions in support of, or dismissive of extra dimensions in space time?

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