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Risk Correlation Based on Time-Varying Copula Function and Extreme Value Theory

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Abstract

The dependence structure of financial assets in financial risk measurement is very important, the tail relations in particular. Authors of extant studies in this field tended to focus on the linear analysis of the financial assets, rarely considering nonlinear, asymmetric and thick-tail characteristics. Here, we apply the copulas connection function with time-varying factors to discuss the risk dependency relationship between financial assets. Moreover, we develop an SV-EVT model to fit variables' marginal distribution combined with stochastic volatility and extreme value theory. Finally, we present an empirical comparative study of static and dynamic copula models applied to the sample comprising of the Chinese mainland A-shares and Hong Kong stock market. The results show that the CSJC copulas connection function describes the tail features of stock index better than the normal copulas connection function. Similarly, the time-varying model outperforms the static copulas model. Furthermore, we observe an asymmetry dependence change rule between Chinese mainland A-shares market and the Hong Kong stock market; the correlation of lower tail is significantly higher than that of the upper tail, and the bear market effect is remarkable. These findings indicate that time-varying Copulas-SV-EVT model can depict the correlation of financial asset tails exactly, and can thus be used to control investment risk and forecast abnormal fluctuations.

Keywords

Time-Varying Copula, SV-t-EVT Model, Risk, Correlation

1. Introduction

Owing to the rapid advances in the information technology, loose financial regulation and capital operation innovation, the financial resource allocation and capital flows have already surpassed the scope of national borders. Against the background of financial globalization, the economic development of all countries has become more closely interrelated, and fluctuated with each other in every capital markets. As a result, the risk contagion of the financial markets has become more serious, as is shown in a series of financial crises, starting with the American subprime mortgage crisis, followed by the European sovereign debt caused by the financial crisis in Greece, and culminating in the global dramatic decrease in the price of gold caused by the rise in the USD exchange rate and the alleviation of global inflation. In China's stock market, since our financial industry has started opening to the outside investors, the number and size of the QDII and QFII funds has experienced rapid growth, albeit with the increasingly enhanced correlation with global stock market volatility. This not only undermines the stability of the stock market itself, but also investor confidence. Risk diversification is the primary goal of diversified investment. However, achieving this aim has become difficult due to the global financial crisis. The changes in the nature of the stock market risk dependency have become a major concern for investors and regulators. Therefore, in this work, we examine risk contagion, focusing on the tail correlation. By measuring and comparing with the domestic capital market, we also provide some useful advice for financial regulators and all types of investors.

In this research field, the widely used measures of contagious risks are correlation coefficient method, the GARCH model, VAR co-integration as well as the Granger Causality Test. However, these methods have some notable measurement deficiencies [1] [2] [3], such as the insufficient depiction of non-linearity, asymmetry as well as the related features of the tail of financial variations, or the inadequate knowledge of the time-varying financial measurement. Copula, as a connection function, can create a multivariate distribution with flexibility and feasibility by connecting the marginal distribution of random variable and its joint distribution. This, in turn, can show the non-linearity and dynamic features better when analyzing the correlation between variables, thus effectively compensating for the insufficiency associated with measuring traditional risk correlation. Moreover, the copula function and its serial correlation indices can correctly and easily present the tail correlation when analyzing samples of variables, which is an obvious advantage relative to other widely used analytical computation methods. This is particularly relevant when studying the financial market risk features, such as the correlation among extreme risks and fluctuation overflow [4] [5]. Owing to these advantageous characteristics, copula function is widely used in its many forms, such as the Standard Copula, t-Copula, Gumbel Copula, Clayton Copula, Frank Copula, etc. [6] [7]. However, copulas are primarily used in constant correlation, while the correlation among financial variables is affected by market fluctuations. Thus, using fixed correlation is incorrect in this context, especially when the market is subject to some extreme events. The time-varying copula can be used to describe the non-linear dependence of financial capital and that of the tail and describe the dynamic structure of the financial capital better, due to its consideration of the time-variation of parameters and their structures [8] [9]. Hence, in this paper, we present a model of risk correlations based on the time-varying copula.

The basic principle of copula is connecting the joint distribution function of multiple random variables by the one-dimensional marginal distribution function. Given that the measure of an individual financial asset by the marginal distribution function itself shall influence the accuracy of the correlation of the final variable, it is vital to obtain the marginal distribution function in order to correctly evaluate the risk of contagion. In practice, the patterning of capital fluctuation is mainly performed via the GARCH model and random fluctuation in the SV model. In particular, the former is easy to use and understand, and is thus usually combined with copula theory when studying correlation of risks [10] [11] [12]. However, the definite relationship between the financial capital profits and fluctuation contained by the GARCH model cannot be proved theoretically [13]. In order to overcome this deficiency, some scholars combine the SV model with copula theory when conducting analyses. In spite of the fact that Copula-SV is seldom used in practical research, available evidence indicates that the SV model is superior to GARCH in depicting financial data. Similarly, Copula-SV is preferable to Copula GARCH when the aim is to elucidate the risks pertaining to joint investment [14]. Additionally, the SV model also has some deficiencies, such as inability to identify extreme financial events, typically exhibited by the abnormal data of the tail, despite its accuracy in depicting the fluctuation of financial capital. Although the EVT can measure the risky losses in extremity for the tail distribution of profits by GPD, it does not study the overall distribution of profits. Moreover, in extreme conditions characterized by high risk, the empirical distribution by EVT is very close to reality, which is beyond the predictability of samples and can deal with the tail thick with effectiveness [15] [16] [17]. Given the aforementioned facts, along with the abnormal distribution and tick tail of the financial capital variables, in this study, we contribute to combine the SV and EVT to depict and construct the joint distribution of samples in measuring the risks of contagion of financial capital. Also, considering that Chinese market is one of the biggest emerging markets which is different from developed market, this study thus contributes to employ the data from both Chinese A-share market and Hong Kong stock market to examine the theory by empirical testing the performance in the emerging market.

In sum, in order to construct a joint model capable of reflecting the actual distribution of all the financial capital variables and their correlative fluctuation, which is necessary for measuring the contagious risks of related financial capital with accuracy, our aim is to elucidate the marginal distribution of capital variable fluctuations by combining SV and EVT. In addition, this paper constructs a financial chronological correlation by combining time-varying copula theory in order to develop a new model—a time-varying Copula-SV-EVT—for measuring the correlation between risks pertaining to financial capital. The data utilized in model verification comprise of Hushen 300 Index and the Hang Seng Index of Hong Kong. More specifically, after establishing the correlation of risks in the two markets, suggestions for investment institutions are provided. The remainder of this paper is organized as follows: In Section 2, we apply the acquisition of the research samples to elaborate the time-varying copula theory and its marginal distribution. Section 3 introduces the development of the time-varying Copula-SV-EVT model and its parameter estimates. In Section 4, the analysis of the data features is presented, along with the empirical test of the model. Section 5 concludes this paper.

2. Time-Varying Copula Theory

2.1. Basic Theory of Copula Function

The theoretical work on copulas dates back to Sklar's research in 1959, who holds that a joint distribution can be divided into K marginal distributions and a copula function, which depicts the correlation among the variables. Copula function can be considered a multi-dimensionally distributed function $C:[0,1]^n \to [0,1]$, with its marginal distribution $F_1, F_2, \cdots, F_n$ evenly allocated in the [0,1] range. More specifically, let A denote a joint distribution function in N dimensions, $F(x_1), F(x_2), \cdots, F(x_n)$. Then, there exists a copula function C in accordance with the equation: $F(x_1, x_2, \cdots, x_\text{n}) = C(F(x_1), F(x_2), \cdots, F(x_n))$. In addition, if $F(x_1), F(x_2), \cdots, F(x_n)$ are constantly distributed, the copula function is definite. Otherwise, if function C is n-dimensional copula, and $F_1, F_2, \cdots, F_n$ is a distributed function, function F is n-dimensional random variable joint distribution function.

Furthermore, the Copula theory provides a simple method for constructing a model of complex and multiple variables and is conducive to the analysis and understanding of many financial problems. There are three main applications of Copula in Finance: the multi-derivative asset pricing, financial risk management and credit risk management. Cherubini, Luciano and Vecchiato have extended the application of Copulas to all fields of finance [18]. In their study, they emphatically researched the frontier issues about the market synergies, credit derivatives pricing, hedging and risk management. From the perspective of probability, they applied the Copula function into these areas and discussed the applications in the fields of the credit derivative assets (credit-default swaps, CDOs) and multi-asset options pricing (binary digital option, rainbow option, fragile and barrier options) in detail. In addition, Bouyé *et al.* and Durrleman *et al.* also made great contributions in introducing the Copulas into the finance area [19] [20].

2.2. Time-Varying Copula Function

The traditional correlation index of multi-dimensional variables is often depicted by liner correlation. However, this correlation obviously has some limitations [21], because fluctuations vary according to conditions, whereby the correlation between variables inevitably changes. This is particularly the case for financial markets subject to sudden and extensive changes. Such abnormal fluctuations in the market cannot be described via liner correlation. Consequently, the time-varying correlation model should be employed when describing the tail. The tail correlation description allows establishing whether another significant market fluctuation shall be evoked by an earlier occurrence of a similar fluctuation. Copula function is particularly appropriate for establishing the correlation of the tail. If copula $C(u,v)$ does exist, the lower tail correlation and the upper tail correlation of the financial capital variables can be expressed as follows:

$$\tau^L = \lim_{u\to 0}\frac{C(u,u)}{u},\ \tau^U = \lim_{u\to 1}\frac{1-2u+C(u,u)}{1-u} \tag{1}$$

However, there are many copulas, each with distinct characteristics. For example, the Standard Copula and the t-Copula cannot depict the asymmetry of the financial capital; Gumbel Copula fails to grasp the lower tail correlation; Clayton Copula fails to grasp the upper tail correlation; Frank Copula cannot establish both the upper and the lower tail correlation; while compared with the common Copula, the JC Copula is superior in describing both the upper and the lower tail correlation. Nevertheless, it is affected by the asymmetry in describing the joint distribution of the same correlation in both the upper and the lower tail. To overcome this deficiency, Patton [8] proposed SJC Copula. This function has been widely used in the correlation calculations pertaining to the financial market and financial capital. In this work, we also adopt the time-varying SJC-Copula as the connecting function, described as follows:

$$C_{\mathrm{SJC}}\left(u,v\middle|\tau^U,\tau^L\right) = 0.5\left[C_{\mathrm{JC}}\left(u,v\middle|\tau^U,\tau^L\right) + C_{\mathrm{JC}}\left(1-u,1-v\middle|\tau^U,\tau^L\right) + u + v - 1\right] \tag{2}$$

In practice, when studying the tail correlation of the financial chronological sequences by using copula function, the relevant parameters of the tail τ^U and τ^L are assumed to be constant for simplicity. However, assuming that fluctuations in the financial market are constant, the correlation of the sequences in fluctuation is obtained in relation to time. In order to highlight the feature of constant variation, a process similar to ARMA can be used to describe the correlation of the upper and the lower tail of SJC-Copula [22]. The function equation is given below:

$$\begin{cases} \tau_t^L = \wedge\left(\omega_L + \beta_L\tau_{t-1}^L + \alpha_L\dfrac{1}{10}\sum_{j=1}^{10}\left|u_{t-j} - v_{t-j}\right|\right) \\ \tau_t^U = \wedge\left(\omega_U + \beta_U\tau_{t-1}^U + \alpha_U\dfrac{1}{10}\sum_{j=1}^{10}\left|u_{t-j} - v_{t-j}\right|\right) \end{cases} \tag{3}$$

where $\wedge(x) = (1-e^x)/(1+e^x)$ guarantees that τ^U and τ^L remain in the $[-1,1]$ range, τ^U and τ^L are the two parameters measuring the dynamic structure of the tail, and thus characterize the time-variation function. These parameters and the tail correlation parameters of the time-varying function SJC-Copula have one-to-one correspondence, thus allowing the asymmetry, the tail feature as well as the correlation of the market subject to severe fluctuation to be described.

3. The Construction of the Marginal Distribution Model

The adoption of the marginal distribution is justified by the correlation of financial capital measured by the copula function and the features of the peak of the thick tail and different variance. In this work, we develop the SV-t-EVT model, which allows depicting the marginal distribution of profits of joint capital investments. This is achieved by depicting the fluctuation of profits of individual capital investments, followed by measuring the conditional variance to obtain the individual random disturbance terms after filtration, and finally constructing the model of the upper and lower tail of the random disturbance terms based on the POT pattern of the extreme value theory.

3.1. SV-t Model and Its Filtration of Random Disturbance Terms

SV-t model is an expansion of the basic SV model. It represents the thick tail of the capital profits more realistically and has a greater ability to discern the fluctuation in these profits. Consequently [11], in this work, we adopt SV-t model to depict the fluctuation of profits of individual capital investments, described by the following equation:

$$\begin{cases} y_t = \exp\left(\theta_t/2\right)\varepsilon_t, \varepsilon_t \sim i.i.d\ t\left(0,1,\upsilon\right) \\ \theta_t = \mu + \phi\left(\theta_{t-1} - \mu\right) + \eta_t, \eta_t \sim i.i.d\ N\left(0,\tau^2\right), t = 1,2,\cdots,n \end{cases} \quad (4)$$

where residual term ε_t and η_t are independent; ϕ is a constant parameter, embodying the impact of the current fluctuation on that in the future; when $|\phi| < 1$, the covariance is stable. Compared with the basic SV model, in the SV-t model, ε_t is subjected to the distribution of t with a υ degrees of freedom. This is preferred to standard normal distribution, which reflects the features of financial capital variables—insubordination to the standard distribution of the time series. In order to acquire the samples of a series of random disturbance terms, it is necessary to evaluate the SV-t model parameters. The commonly used methods for evaluating parameters are QML (pseudo maximum likelihood method) and GMM (generalized method of moments), resulting in a greater error in the parameter evaluation for their limits on the condition of the samples. Although the method based on MCMC (Markov Chain Monte Carlo) is highly complex, the parameter estimation is quite accurate [23]. MCMC does not require the analytical expression of posterior density, but provides a method to sample the parameter vector from the posterior distribution. This method sets a Markov Chain, equating its stationary distribution to the posterior density. When the Markov Chain is self-restrained, the value of simulation is supposed to be samples taken from posterior distribution. Therefore, in this work, we apply the MCMC of the Gibbs samples to evaluate parameters by using the SV-t model via the BUGS software.

3.2. Extreme Value Theory and SV-t Marginal Distribution Model

The fluctuation of capital profits is depicted by applying the SV-t model, thereby obtaining a series of random disturbance terms Z_t after filtration. Let $\hat{\mu}_t$ and $\hat{\sigma}_t$ denote the conditional average value and conditional variance of capital profits series, respectively:

$$\left(Z_{t-n+1},\cdots,Z_t\right)=\left(\frac{X_{t-n+1}-\hat{\mu}_{t-n+1}}{\hat{\sigma}_{t-n+1}},\cdots,\frac{X_t-\hat{\mu}_t}{\hat{\sigma}_t}\right) \tag{5}$$

It should be noted that matching the disturbance terms Z_t in standard distribution will underestimate the risks of the tail. Thus, the Pareto (GPD) based on extreme value theory is adopted. EVT can depict the quantile distributed in the tail of the capital profits distribution and the application of risky correlation can improve the accuracy of the analysis [24]. The POT pattern of the EVT can create a model for the samples that are beyond certain maximum threshold value and conduct mathematical analysis of the distribution of losses directly, which can overcome the deficiency of other methods in analyzing the tail distribution.

Let us assume that the distribution function of random disturbance terms Z_t is described by the following equation: $F(Z)=P(Z\le z)$. The random variation Z surpasses the distribution F_u of certain threshold value u, where F is the distribution function of Z. Generally, the distribution function F_u is called the distribution function of conditional extreme loss, and is described by the following equation:

$$F_u(y)=p\left(Z-u\le y\,|\,Z>u\right) \tag{6}$$

where $0\le y\le z_F-u$, $z_F\le\infty$ is the right endpoint of the distribution, and therefore F_u has the following equation:

$$F_u(y)=\frac{F(u+y)-F(u)}{1-F(u)}=\frac{F(z)-F(u)}{1-F(u)} \tag{7}$$

The $F_u(y)$ in Equation (7) is called over threshold value distribution, and there exists a $G_{\xi,\beta}(y)$ for a certain threshold value u which is sufficiently large:

$$F_u(y)\approx G_{\xi,\beta}(y)=\begin{cases}1-\left(1+\xi\dfrac{y}{\beta}\right)^{-\frac{1}{\xi}},\ \xi\ne0\\ 1-\mathrm{e}^{-\frac{y}{\beta}},\ \xi=0\end{cases} \tag{8}$$

where ξ is a shape parameter and β is a scale parameter. When $\xi\ge0$, $y\in\left[x_F,-\sigma/\xi\right]$; when $\xi<0$, $y\in\left[0,-\beta/\xi\right]$. The distribution function $G_{\xi,\beta}(y)$ is called the Generalized Pareto Distribution (GPD) function [25], which can match the tail of the series of capital profits very well, complementing its inadequacy in depicting the series of capital profits. On this basis, we apply EVT to evaluate the distribution of the upper and lower tail of the random disturbance term Z_t and apply the experience-based distribution function to match the random disturbance terms among the upper and lower tail thresholds. This allows obtaining the marginal distribution of random disturbance terms Z_t of the rate of financial capital profits. The marginal distribution model SV-t-EVT is given by:

$$F(Z)=\begin{cases}\frac{N_u^L}{N}\left(1+\xi^L\frac{u^L-z}{\beta^L}\right)^{-1/\xi^L}, z<u^L\\ \Phi(z), u^L\le z\le u^U\\ 1-\frac{N_u^U}{N}\left(1+\xi^U\frac{z-u^U}{\beta^U}\right)^{-1/\xi^U}, z>u^U\end{cases} \tag{9}$$

where ξ^L is the shape parameter of the lower tail; β^L is the scale parameter of the lower tail; u^L is the threshold value of the lower tail; N_u^L is the number of samples of Z, which is lower than the lower tail threshold value; ξ^U is the shape parameter of the upper tail; β^U is the scale parameter of the upper tail; u^U is the threshold value of the upper tail; and N_u^U is the number of samples of Z, which is higher than the threshold value of the upper tail.

3.3. The Construction of Time-Varying Copula-SV-EVT and the Parameter Evaluation

Let f_i denote the marginal density function of financial capital in a particular market at the time *t*. Its corresponding joint function of accumulations is F_i, the parameter of distribution function is θ_i, and the time-varying parameter of SJC Copula is θ_{ct}. The joint expression can be given by:

$$\begin{aligned}&f\left(x_{1t},x_{2t},\cdots;\theta_1,\theta_2,\cdots,\theta_{ct}\right)\\&=c\left(F_1\left(x_{1t};\theta_1\right),F_2\left(x_{2t};\theta_2\right),\cdots,F_N\left(x_{Nt};\theta_2\right),\theta_{ct}\right)\cdot\prod_{i=1}^{N}f_i\left(x_{it};\theta_i\right)\end{aligned} \tag{10}$$

The corresponding log likelihood function of Equation (7) is thus:

$$\begin{aligned}&\ln L\left(x_1,x_2,\cdots x_N;\theta\right)\\&=\sum_{t=1}^{T}\left(\sum_{i=1}^{N}\ln f_n\left(x_{it};\theta_i\right)+\ln\left(c\left(F_1\left(x_{1t};\theta_1\right),F_2\left(x_{2t};\theta_2\right),\cdots,F_N\left(x_{Nt};\theta_N\right);\theta_{ct}\right)\right)\right)\end{aligned} \tag{11}$$

In addition, the parameter variation of time-varying copula function often takes the Maximum Likelihood Evaluation (MLE) and IFM. But it is not easy to get the premium in the parameter evaluation by MLE when the subjects are too much. Moreover, the features of time-varying copula function make the model suitable for multi-step evaluation. In this work, we evaluate the time-varying copula function by applying IFM. More specifically, we first obtain the corresponding θ_i by the maximum log-likelihood estimation (MLE) of the marginal distribution function, which allows us to obtain the θ_{ct} by inserting the θ_i into the log-likelihood function, yielding the maximum likelihood estimation.

In sum, connecting Equations (9), (10), and (11) provides the pattern for measuring risk correlation based on the time-varying copula function, random fluctuation and EVT:

$$\begin{cases} f\left(x_{1t}, x_{2t}, \cdots; \theta_1, \theta_2, \cdots, \theta_{ct}\right) \\ = c\left(F_1\left(x_{1t}; \theta_1\right), F_2\left(x_{2t}; \theta_2\right), \cdots, F_N\left(x_{Nt}; \theta_2\right), \theta_{ct}\right) \cdot \prod_{i=1}^{N} f_i\left(x_{it}; \theta_i\right) \\ F\left(Z\right) = \begin{cases} \frac{N_u^L}{N}\left(1+\xi^L \frac{u^L - z}{\beta^L}\right)^{-1/\xi^L}, & z < u^L \\ \Phi\left(z\right), u^L \le z \le u^U \\ 1 - \frac{N_u^U}{N}\left(1+\xi^U \frac{z - u^U}{\beta^U}\right)^{-1/\xi^U}, & z > u^U \end{cases} \\ \theta_i = \arg\max l^c\left(\theta_i\right) = \arg\max \sum_{t=1}^{T} \ln f_{it}\left(x_i, \theta_i\right) \\ \theta_{ct} = \arg\max l^c\left(\theta_{ct}\right) = \arg\max \sum_{t=1}^{T} \ln\left(c\left(F_1\left(x_{1t}, \theta_1\right), \cdots, F_N\left(x_{nt}, \theta_n\right); \theta_{ct}\right)\right) \end{cases} \quad (12)$$

4. Empirical Analysis

Hong Kong, one of the most important financial centers in Asia, owing to its highly unrestricted and internationalized financial capital market, has always been the most important link between the Chinese capital market and the international capital market. This is evident in the Red Chip and Share H comprising about 50% of the Hong Kong stock market for years. The close connection between stock markets makes the impact of risk contagion all the more obvious. In addition, since Chinese QDII fund managers invest into the HK stock market, the risk correlation and contagion between these two markets influence the allocation of investment portfolios. Consequently, in this work, we focus on the risk correlation and contagion between HK Stocks and A Share of Shanghai and Shenzhen Exchange. Comparative analysis is also performed to highlight the practical utility of the model developed in the present study.

4.1. The Data Sources and Descriptive Statistics of Samples

In this work, Hushen 300 Index and HSI (Hang Seng Index) are treated as representative of Chinese stock market and HK stocks, respectively. In the following analyses, we use the daily closing prices and examine the variations in the correlation between the Chinese and HK stock market by adopting four types of copula models. Considering that the statistics of Hushen 300 Index were not available to the public prior to 2005, our research period starts from January 4th, 2005. On the other hand, China is the emerging market under a highly regulated environment [26], and especially after 2013, the Chinese A-share market is very sensitive to the changeable regulation. Since the global finance crisis, the A-share market performed worse than before. And to get out of the downturn, China has suspended the initial public offering for 15 months and restarted it with a restricted 44% underpricing rate in the stock market since 2013. Thus, in order to keep the data consistent and eliminate the interference of regulations in the Chinese stock market, our data sample pertains to the period from January 4th, 2005 to April 11th, 2013, with samples of a single trade day excluded. And we collected the data from WIND and CSMAR database and process the data analysis by using the MATLAB. The profit rate is defined as:

$$X_t = 100\left(\ln p_t - \ln p_{t-1}\right) \tag{13}$$

As can be seen from Table 1, the fluctuation range and fluctuation features of Hushen 300 Index are markedly different from those of HSI. In addition, the profits of the two indices do not comply with normal distribution hypothesis (for example, J-B is very noticeable), as they both display left tendency (skewness < 0), and are characterized by leptokurtosis (leptokurtosis > 3). In addition, as is shown by the Ljung-Box statistics $Q(k)$ in the table, over a longer period, the original hypothesis that profits series of the two indices do not exist is unjustified. Namely, there exists a correlative series in the profit series and $Q^2(k)$ proves that there exists conditional heteroscedasticity in the two series. Moreover, according to Table 1, the roots of unity of the lag intervals of optimal test for Endogenous of Augmented Dickey-Fuller by the minimum AIC norm indicate that the hypothesis which exists a root of unity in the two series is unjustified. Therefore, the two series are both stationary sequence of time and are qualified to further analysis and construction of model on statistics directly.

4.2. The Evaluation of Marginal Distribution

In the evaluation of marginal distribution, the SV-t model was evaluated first in order to obtain the standard residual sequence on the basis of the evaluated parameters. Next, the tail data of the standard residual by GDP of the EVT were matched with the results presented in Table 2.

Table 1. Descriptive statistics for Hushen 300 index and HIS.

Index	Average	Standard deviation	Skewness	Kurtosis	J-B	Q (20)	Q^2 (20)	ADF
Hushen 300 Index	0.0239	0.8715	−0.3270	5.3630	429.0***	32.86***	458.3***	−40.51***
HSI	0.0091	0.0339	−0.0520	11.50	5157***	29.11***	2093***	−42.85***

Notes: ***indicates statistical significance at the 1% level.

Table 2. The evaluated results of marginal distribution.

	Hushen 300 Index	HSI			Hushen 300 Index	HSI
μ	−0.9963	−1.359		β^U	0.5393	0.5412
ϕ	0.9916	0.9853		ξ^U	−0.0252	0.0586
τ	0.09456	0.1202		u^L	−1.3560	−1.3025
υ (degree of free distribution of t)	9.32	15.95		u^U	1.4209	1.3420
β^L	0.7045	0.6463	K-S Test	Statistical Magnitude	0.0152	0.0222
ξ^L	−0.0536	−0.1154		Probability Value	0.9891	0.7932

As is shown by Table 2, μ indicates the fluctuation range of the index for each of the two markets. In terms of absolute value, the HK index is a little higher than that of the Chinese stock market. The ϕ values for the two markets are close to 1, indicating a strong continuity of fluctuation of these two markets, even though the association with the Chinese stock market is much stronger. The solution parameter τ of the model reflects the fluctuation noise for the two markets. On the other hand, υ shows that the profit rate of the two markets does not follow normal distribution, as it exhibits both a remarkable peak and a thick tail. These characteristics are much more prominent in the HK stocks relative to the Chinese stock market. The tail parameter estimation of the EVT model indicates that the lower tail shape parameter of the Chinese stock market is relative to that of the HK stock market, with the number of −0.0536 and −0.1154 respectively. And the lower tail of the Chinese stock market is thicker. The upper shape parameters ξ of the two markets are −0.0252 and 0.0586, respectively, with the upper tail of the HK stock market being thicker. In addition, according to the concomitant probability of K-S given in Table 2, the new sequences transformed by probability integral into the original sequences comply with even distribution in the [0, 1] range. Moreover, autocorrelation test of these sequences also indicates that the new sequences do not exhibit autocorrelation and are independent. The K-S test and autocorrelation test indicate that the SV-t-EVT model can describe the marginal distribution of Hushen 300 Index and HSI. In order to verify the evaluation of the tail, Figure 1 shows the GPD distribution matching effect of the upper tail overflow data of Hushen 300 index (left) and HSI (right). Hence, the model of marginal distribution based on SV-T-EVT is justified.

4.3. Evaluation of Time-Varying Copula Function

On the basis of marginal distribution evaluation, in this work, we apply the

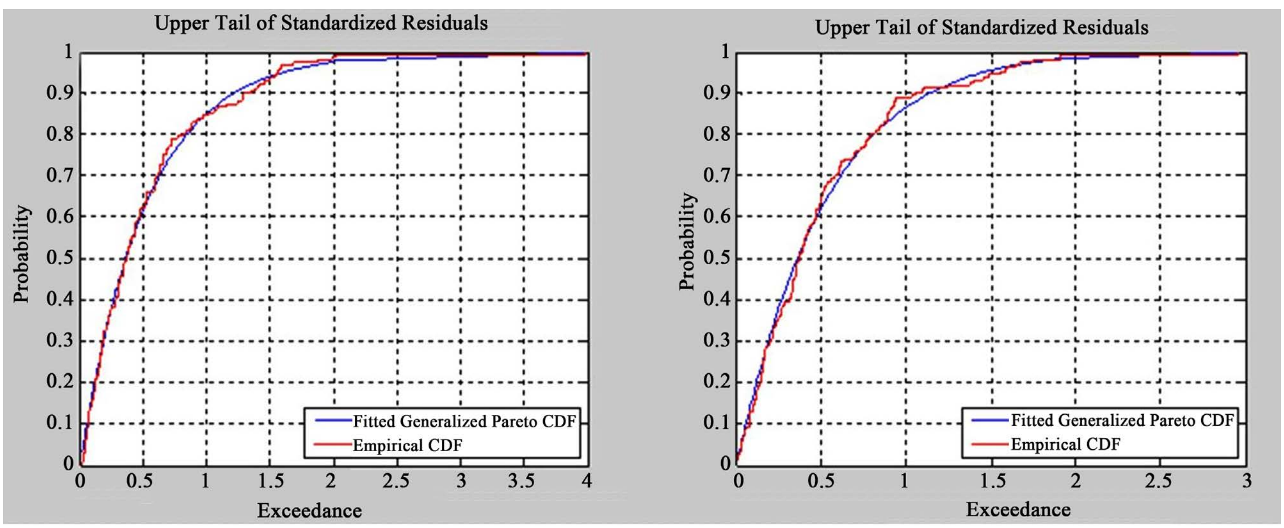

Figure 1. The GDP matching effect of the upper tail overflow data of Hushen 300 Index and HIS.

time-varying SJC-Copula model and evaluation method to analyze the correlation of risks associated with the Hushen 300 Index and HIS. In order to show the dynamics of time-varying SJC-Copula model and conduct a comparative study, we also conduct evaluation of this Copula model of the constant normal correlation, time-varying normal Copula model as well as the constant correlation of SJC-Copula model, as shown in Table 3.

As is shown in Table 3, there exists a proportionate correlation between Hushen 300 Index and the tail of HSI. More specifically, the tail correlation of normal Copula is 0.4287, which is markedly different from the linear correlation of the tail at 0.4520. By using the standard residual sequence of two stock markets, the latter can be obtained with a yield of 5.4%, which is the difference between the preconditions of the normal distribution. In measuring the tail correlation of constant correlation of SJC-Copula, the correlation of the upper tail is 0.1725 and that of the lower tail is 0.2842, indicating that the tail correlation of the bear market is higher than that of the bull market. In addition, whether adopting the maximum likelihood value or sequencing by matching quality of AIC or BIC as a criterion, the time-varying normal Copula and the time-varying SJC-Copula outperform the corresponding normal Copula in the process of simulating the tail correlation. This finding indicates that the dynamic Copula is better suited for describing the correlation among variables. Furthermore, according to the minimum principle of AIC and BIC, the time-varying SJC-Copula performs better in matching the tail correlation of the two stock markets. Consequently, it can be concluded that the measuring of the tail correlation of the two stock markets by time-varying SJC-Copula is effective.

Table 3. Related copula model parameter estimation results.

	δ	ω	α	β	τ^U	τ^L	ω^U
Normal Copula	0.4287	-	-	-	-	-	-
Time-Varying Copula	-	0.0051	0.1041	2.0468	-	-	-
SJC Copula	-	-	-	-	0.1725	0.2842	-
Time-Varying SJC Copula	-	-	-	-	-	-	2.2202

α^U	β^U	ω^L	α^L	β^L	Log likelihood value	AIC	BIC
-	-	-	-	-	−173.8973	−347.7862	−347.7830
-	-	-	-	-	−182.5944	−365.1853	−365.1758
-	-	-	-	-	−173.1515	−346.3007	−346.2943
−12.1143	−4.9450	−1.5583	−1.7649	3.6743	−187.1148	−374.2226	−374.2036

Notes: δ is the parameter of normal copula; τ^U and τ^L are the parameters of the upper and lower tail of SJC copula; ω is constant correlation index; α and β are correlation indices of time-varying SJC copula; and AIC and BIC are criteria of information.

4.4. Further Analysis of the Results

In order to further reveal the dynamic correlation of the Chinese and HK stock market, Figure 2 and Figure 3 show the variation tendency of time-varying correlation parameters of normal Copula and SJC-Copula, respectively. Figure 2 presents the constant and time-varying correlation parameters of Chinese and HK shares which obtained by the normal copula function. The tail of China stock and that of HK stock are proportionately correlated to each other, but the intensity of correlation is stronger than fluctuation. In terms of investment portfolio, a suitable combination of weakly correlated assets can decrease the investment risk. The static correlation analysis of the profit rates associated with Chinese and HK stocks indicates that a combination of these stocks can decrease the investment risk effectively. In Figure 2, the time-varying correlation parameter of dynamic copula ranges from 0.1438 to 0.6314. For a wide range of related parameters, the total value of dynamic copula of the time-varying correlation parameter can be greater than or less than the constant, and the maximum value is 0.6314. Furthermore, as is shown in the graph, the correlation of the two markets tends to increase with time.

Figure 3 depicts constant and time-varying tail correlation parameters of Chinese and HK stocks provided by SJC-Copula function. The contrast of the two graphs in Figure 3 indicates that the correlation of the lower tail of the two stock markets is stronger than that of the upper tail, with obvious asymmetry. In the upper tail, the correlation fluctuates around 0.1825 and remains relatively stable during the process of opening-up China's capital market. Absence of upward trend indicates low likelihood of dramatic simultaneous increases in the two market indices. In terms of the lower tail, when both markets are bear, the

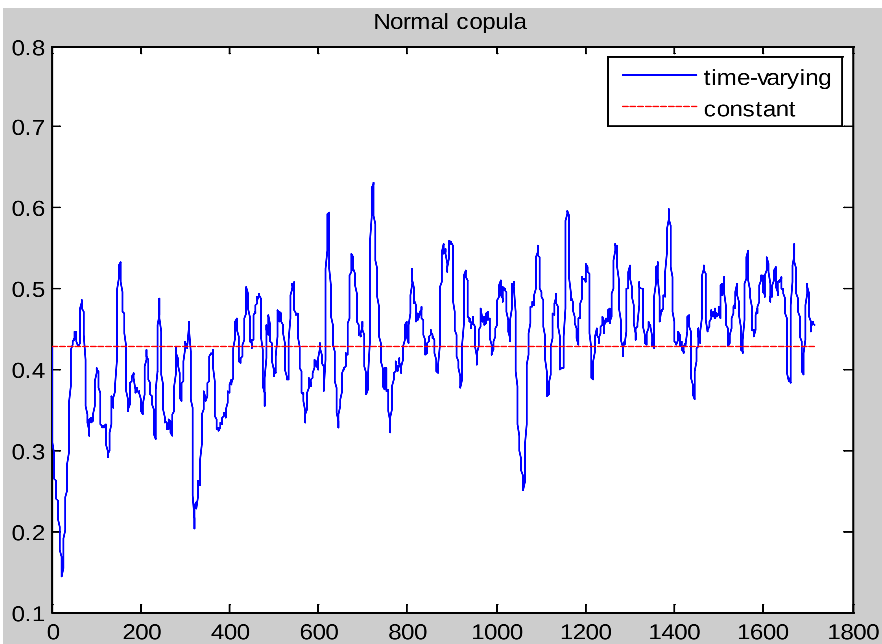

Figure 2. The correlation tendency of normal copula.

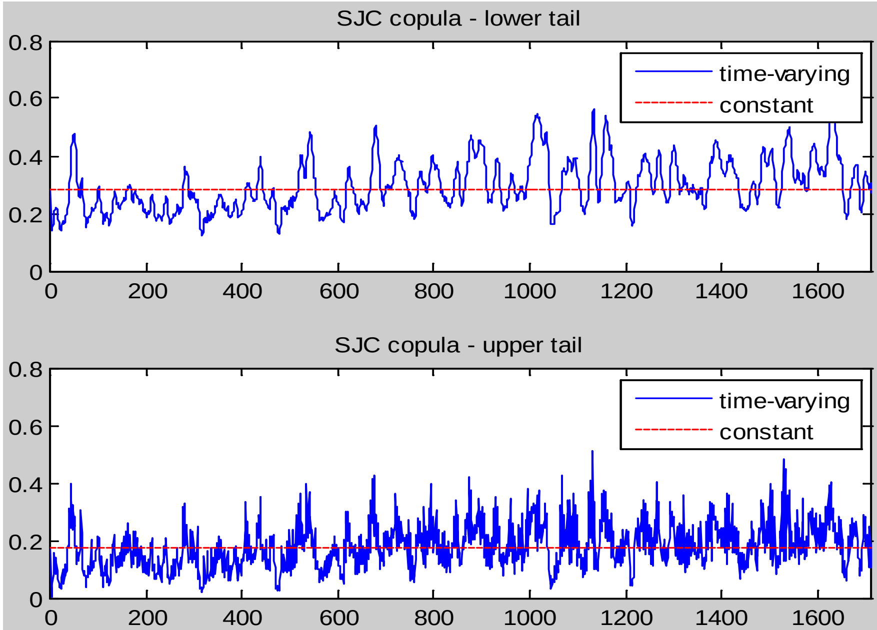

Figure 3. The correlation tendency of SJC-copula.

correlation parameter is about 0.3, which indicates their stronger mutual influence. After the exchange rate revolution in 2005 and the issue of QDII in 2006, the tail correlation of the two markets experienced a significant increase. In addition, during the process of opening-up, the time-variation of the tail correlation of the two markets is obvious with a maximum value of 0.5786. This is a persuasive argument for the competence of SJC-Copula in grasping the dynamic correlation of the Chinese and HK stocks. This is similar to the results yielded by the comparison of extreme likelihood values, AIC and BIC, which clearly demonstrates that, when the value of one market declines, the investors pay more attention to the other market and have a great tendency to follow up. Moreover, with the accelerating pace of China's opening-up to the external investment, the correlation of risks associated with the two markets becomes stronger, making both markets are more likely to suffer sudden shocks simultaneously.

Generally speaking, whether constant correlation or the time-varying copula is used, the SJC-Copula with a consideration of tail correlation outperforms the normal Copula, which indicates that the correlation between China and HK stocks exhibits an obvious asymmetry. This asymmetry can be explained from the perspective of investors, who would be more sensitive to news signifying low profit rather than to positive news. When the stock value begins to decline, investors become very anxious and would immediately take action; on the contrary, when stock market prices are increasing, most investors will take no action.

5. Conclusions

The correlation structure of financial assets is an important element in the analysis of the financial market risks. The analyses of risks that were performed to date tended to focus on the distribution of profits of financial assets, while neglecting the different roles of the risks associated with the market as a whole and those pertaining to the individual shares. In recent years, the correlative structure of assets has started to attract much greater attention. In this paper, we discussed the correlation pattern of the financial market on the basis of time-varying copula. More specifically, we combined the random fluctuation model with EVT to create a new SV-t-EVT model to simulate the actual characteristics of profit sequences of marginal distribution. On this basis, four copula models were constructed to study the correlation between Chinese and HK stocks. As was shown in this work, SJC-Copula outperforms the normal Copula, while the dynamic Copula is better than the stationary one. In addition, the findings confirmed presence of asymmetry in the correlation variation of Chinese and HK stocks, with the lower tail correlation much higher than that of the upper tail. As the effect of bear market is obvious, the Gaussian correlation structure in the traditional sense cannot fully reflect the correlation of the financial assets, in particular the tail correlation.

It is also noteworthy that, in terms of investment portfolio, risk management and assets pricing, the depiction of the correlation of financial assets, and the tail correlation structure in particular, is of great significance. Authors of extant studies on the financial market correlation mainly focused on the linearity and the symmetry, rather than the correlation structure of non-linearity, asymmetry and productivity and its range of correlation. As an analytical correlation and multi-statistics tool, the copula model is suitable for correlation studies, especially for financial time sequences. The dynamic Copula model can often outperform the static Copula model in depicting the correlation structure of the financial capital and financial market. However, empirical studies of dynamic Copula model are inadequate, as without the knowledge of the change of correlation, it may misjudge the correlation structure by choosing a wrong model to simulate. Therefore, *a priori* estimation of the form of the correlation structure is conducive to choosing a suitable copula function. In conclusion, a new dynamic Copula model is established to describe the related structures, *i.e.*, considering the diversified time-varying Copula and the time-variation of the degrees of freedom. While this study did not consider about the interference of the changeable regulations on the market performance in the Chinese stock market after 2013, we will extend the research with consideration of the distinct regulations which highly influenced in the emerging stock market to better study the topic in the future.

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