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Presentation of Design Equations for Array of Circumferential Slot on Cylindrical Waveguide

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Abstract

In this paper the design equations for an array of circumferential slots on a cylindrical waveguide are obtained, following the procedure introduced by Elliott for slots on rectangular waveguides. The minimization of the error function is achieved for optimization of slot array parameters. The optimization of slot parameters is not goal of this paper but a numerical example are presented as illustrations of the proposed synthesis method. The results of array designs by the method of the least squares are verified by two computer simulation softwares, namely CST and HFSS.

Keywords

Array, Slot Arrays, Cylindrical Waveguide Slot Array, Cylindrical Waveguide, Circumferential Slot Array, Design Equations

1. Introduction

Train communication systems have received a considerable interest over the years, with tunnel connectivity providing an ongoing challenge due to the hostile environmental characteristics. In recent times, security aspects have come to the forefront with high-definition closed-circuit television monitoring being considered, together with possible remote train control and passenger emergency assistance networks [1] [2] [3] [4] [5]. The antenna applied for the broadcasting station of the ultra high frequency television (UHF TV) requires either unidirectional or omnidirectional beam with sufficient gain and high power handling [6].

The first study of radiation from an aperture on an infinite metallic plane was reported by Silver and Saunders in 1950, who derived a formula for the generated external field [7]. Bailin derived formulas for the radiation from axial and circumferential rectangular slots on a conducting circular cylinder in 1955 [8] and compared his results with measurement data. Golden, *et al.* investigated some approximate techniques for the determination of mutual couplings among slots on cylindrical surfaces in 1974 [9]. We follow the general method introduced by Elliott [10] for the evaluation of scattering from an aperture on the surface of a cylindrical waveguide, which is believed to be unprecedented for the circumferential slots in a circular cylindrical surface. Consequently, our main task is to derive two design equations which is done by assuming that the radius of the cylindrical surface is large, providing the possibility of assuming the slots to be located on a flat ground plane. This assumption may lead to some design approximations, which may then be rectified by a full-wave simulation by available computer softwares [11]-[14].

The TM_{01} mode is assumed in the cylindrical waveguide, where the electric field is radial in its cross-section. Consequently, the radiation from the circumferential slots on its surface is omni-directional and independent of the azimuthal angle, which is desired for cylindrical slot arrays.

2. Development of Design Procedure

2.1. First Design Equation

The configuration of circumferential rectangular slots on a cylindrical waveguide is shown in Figure 1, together with the related dimensions which the ***thickness*** parameter is the thickness of wall of waveguide, R is radius of waveguide, $R\alpha$ is length of slot, d is spacing between slots, W is width of slot and φ_0 is the angle offset of slot on circumference of cylindrical waveguide.

The backward (B) and forward (C) scattered field amplitudes are given by [10]:

$$B_{mn} = \frac{\int_{\text{slot}} \overline{E_s} \times \overline{H}_{tC} \cdot \overline{\mathrm{ds}}}{2\int_S \overline{E}_{at} \times \overline{H}_{at} \cdot \overline{u}_z \mathrm{ds}} \tag{1}$$

$$C_{mn} = \frac{\int_{\text{slot}} \overline{E_s} \times \overline{H}_{tB} \cdot \overline{\mathrm{ds}}}{2\int_S \overline{E}_{at} \times \overline{H}_{at} \cdot \overline{u}_z \mathrm{ds}} \tag{2}$$

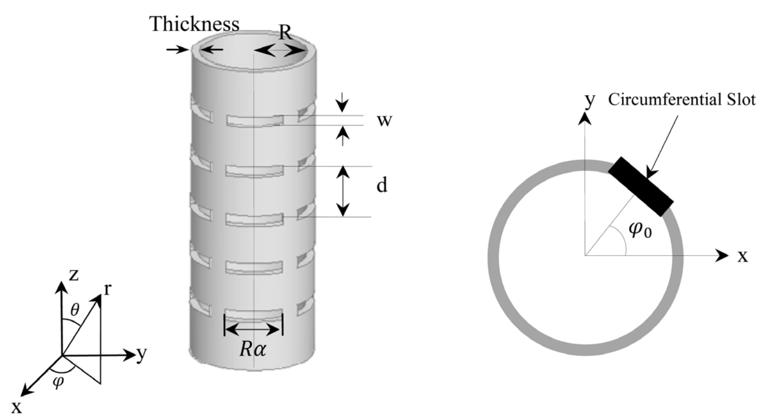


Figure 1. An array of circumferential slots on a circular cylindrical surface.

where subscripts B and C represent the amplitudes of backward and forward waves, t indicates the tangential field in the cross-section, S indicates the cross-section of uniform waveguide, and "slot" shows the slot surface area.

The field components of TM_{01} mode in the cylindrical waveguide are:

$$E_z = J_0(h\rho)e^{\pm j\beta_{01}z} \tag{3}$$

$$E_\rho = \pm\frac{j\beta_{01}}{h}J_0'(h\rho)e^{\pm j\beta_{01}z} \tag{4}$$

$$H_\varphi = -\frac{j\omega\epsilon}{h}J_0'(h\rho)e^{\pm j\beta_{01}z} \tag{5}$$

$$E_\varphi = H_\rho = H_z = 0 \tag{6}$$

where β_{01} is the phase constant and $h = \frac{2.405}{a}$ is cut-off number with a being the radius of waveguide.

The tangential electric field on the n'th aperture is:

$$E_z = \frac{V_n}{W}\cos\left(\frac{\pi(\varphi-\varphi_0)}{\alpha_n}\right)\begin{cases} z_n - \frac{W}{2} < z < z_n + \frac{W}{2} \\ \varphi_0 - \frac{\alpha_n}{2} < \varphi < \varphi_0 + \frac{\alpha_n}{2} \end{cases} \quad W \to 0 \tag{7}$$

$$\bar{E}_s = E_z\bar{u}_z,\ E_\varphi = 0 \tag{8}$$

where α_n is the angle of n'th slot. The other field component are:

$$\bar{H}_{tB} = H_\varphi\bar{u}_\varphi \tag{9}$$

$$\bar{H}_{tC} = H_\varphi\bar{u}_\varphi \tag{10}$$

$$\bar{H}_{at} = H_\varphi\bar{u}_\varphi \tag{11}$$

$$\bar{E}_{at} = E_\rho\bar{u}_\rho \tag{12}$$

These field components are substituted in Equations (1) and (2) to obtain:

$$C_{01} = B_{01} = \frac{jV_n hJ_1(ha)}{a\beta_{01}\pi^2}\frac{\frac{\alpha_n}{\pi}}{\left[J_1^2(ha) - J_0(ha)J_2(ha)\right]} \tag{13}$$

Observe that the forward and backward traveling wave amplitudes are equal. Therefore the transmission line equivalent circuit consists of a parallel admittance.

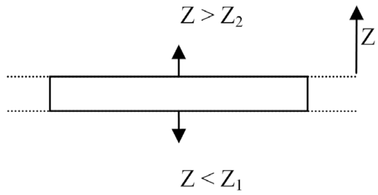
The first design equation is then derived. The reflected power from the aperture is:

$$\begin{aligned} P_{ref} &= \frac{1}{2}Re\int_S\left(\bar{E}_t \times \bar{H}_t^*\right)\cdot\overline{\mathrm{d}s} \\ &= \frac{1}{2}Re\int_{\rho=0}^{a}\int_{\varphi=0}^{2\pi}\left(B_{01}E_{01,t} \times B_{01}^*H_{01,t}^*\right)\cdot\overline{u_z}\rho'\mathrm{d}\rho'\mathrm{d}\varphi' \\ &= \frac{\pi\beta_{01}\omega\epsilon}{h^2}\frac{a^2}{2}\left[J_1^2(ha) - J_0(ha)J_2(ha)\right]B_{01}B_{01}^* \end{aligned} \tag{14}$$

where

$$\bar{E}_t = \begin{cases} B_{01}\bar{E}_{at} & z < z_1 \\ C_{01}\bar{E}_{at} & z > z_2 \end{cases} \tag{15}$$

The equations of the equivalent transmission line are:



$$\begin{cases} V(z) = \overbrace{A\mathrm{e}^{-j\beta z}}^{V^+} + \overbrace{B\mathrm{e}^{j\beta z}}^{V^-} \\ I(z) = \underbrace{AG_0\mathrm{e}^{-j\beta z}}_{I^+} - \underbrace{BG_0\mathrm{e}^{j\beta z}}_{I^-} \end{cases} \tag{16}$$

which give the reflected power:

$$P_{ref,TL} = \frac{1}{2}V^-(z)I^{-*}(z) = -\frac{1}{2}\frac{BB^*}{Z_0} \tag{17}$$

The equality of reflected powers due to the scattered fields and the transmission line leads to the following relation:

$$-\frac{1}{2}\frac{BB^*}{Z_0} = \frac{\pi\beta_{01}\omega\epsilon_0}{h^2}\frac{a^2}{2}\left[J_1^2(ha) - J_0(ha)J_2(ha)\right]B_{01}B_{01}^* \tag{18}$$

However, the scattered voltage amplitudes are [7]:

$$B = C = -\frac{1}{2}\frac{Y_n^a}{G_0}V_n \tag{19}$$

The amplitudes B and B_{01} from Equations (19) and (13) are then substituted in (18) to obtain the first design equation:

$$\frac{Y_n^a}{G_0} = -j\sqrt{\frac{4\omega\epsilon_0 J_1^2(ha)}{G_0\beta_{01}\pi^5\left[J_1^2(ha) - J_0(ha)J_2(ha)\right]}}\frac{V_n^s}{V_n}\alpha_n \tag{20}$$

where the Bessel functions J_0, J_1 and J_2 are calculated for ha = 2.405 for TM_{01} mode.

2.2. Second Design Equation

For the derivation of the second design equation, the procedure described by Elliott ([10], pp: 402-407]) is followed, which for the circumferential slots on cylindrical waveguides gives:

$$\frac{Y_n^a}{G_0} = \frac{\eta^2}{2}G_0|K|^2\frac{\alpha_n^2}{z_n^{d,\alpha}} \tag{21}$$

where $K = -j\sqrt{\frac{4\omega\epsilon_0 J_1^2(ha)}{G_0\beta\pi^5\left[J_1^2(ha) - J_0(ha)J_2(ha)\right]}}$, $\eta = 120\pi$ is the intrinsic impedance of medium and G_0 is characteristics admittance of cylindrical waveguide. Equation (21) can be written as $\frac{Y_n^a}{G_0} = K_1 \frac{\alpha_n^2}{Z_n^{d,a}}$ where $K_1 = \frac{\eta^2}{2} G_0 |K|^2$. $Z_n^{d,a}$ is the active admittance of equivalent dipole that is assumed in the derivation of second design equation. we have $Z_n^{d,a} = Z_{nn} + Z_n^b$ where:

Z_{nn}: Self impedance of circumferential slot, which is equal to $\frac{K_1 \alpha_n^2}{Y_n^{self}/G_0}$

Z_n^b: Mutual impedance between circumferential slots on the cylindrical waveguide, which is equal to $\sum_{\substack{m=1 \\ m \neq n}}^{N} \frac{V_m^s}{V_n^s} Z_{nm}^d$.

Then

$$Z_n^{d,a} = Z_{nn} + Z_n^b = \frac{K_1 \alpha_n^2}{Y_n^{self}/G_0} + \sum_{\substack{m=1 \\ m \neq n}}^{N} \frac{V_m^s}{V_n^s} Z_{nm}^d$$

Z_{nm}^d is mutual impedance between two dipole which may be obtained from the mutual admittance between two slots Y_{nm}^s by the Booker's relation.

$$Z_{nm}^d = \left(\frac{\eta^2}{2}\right) Y_{nm}^s \tag{22}$$

The second design equation is then determined by these relations.

3. Design of a Linear Traveling Wave Slot Array

Consider the equivalent circuit of the linear traveling wave slot array as shown in Figure 2. The normalized admittance at the n'th slot looking towards the match port is [10]:

$$\frac{Y_n}{G_0} = K_1 \frac{\alpha_n^2}{Z_n^{d,a}} + \frac{(Y_{n-1}/G_0)\cos\beta_{10} d_{n-1} + j\sin\beta_{10} d_{n-1}}{\cos\beta_{10} d_{n-1} + j(Y_{n-1}/G_0)\sin\beta_{10} d_{n-1}} \tag{23}$$

where the second design Equation (21) is used.

The mode voltages at successive junctions are then related by:

$$\begin{aligned} V_n &= V_{n-1}\cos\beta_{01} d_{n-1} + jI_{n-1} Z_0 \sin\beta_{01} d_{n-1} \\ &= V_{n-1}\left[\cos\beta_{01} d_{n-1} + j\frac{Y_{n-1}}{G_0}\sin\beta_{01} d_{n-1}\right] \end{aligned} \tag{24}$$

which may be written for $\frac{V_n}{V_{n-1}}$. This ratio may also be obtained by Equation

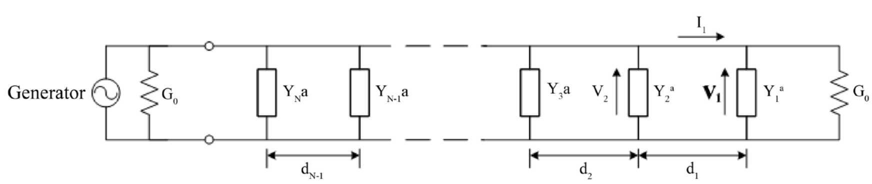


Figure 2. Equivalent circuit of the linear traveling wave slot array.

(18) for n and $n-1$. Equating these two ratios gives:

$$\cos\left(\beta_{01}d_{n-1}\right)+j\frac{Y_{n-1}}{G_0}\sin\left(\beta_{01}d_{n-1}\right)=\frac{V_n^s}{V_{n-1}^s}\cdot\frac{\alpha_n}{\alpha_{n-1}}\cdot\frac{Y_{n-1}^a/G_0}{Y_n^a/G_0} \tag{25}$$

This expression is appropriate for the construction of an error function.

4. Construction of Error Function

The error function consists of three terms:

$$\varepsilon_{\text{Error Function}}=\varepsilon_{\text{Matching}}+\varepsilon_{\text{DesignEqs.}} \tag{26}$$

$$\varepsilon_{\text{Matching}}=W_1\left|\text{Re}\left(\frac{Y_n}{G_0}\right)-1\right|^2+W_2\left|\text{Im}\left(\frac{Y_n}{G_0}\right)\right|^2 \tag{27}$$

$$\varepsilon_{\text{DesignEqs.}}=W_3\left|\sum\nolimits_{n=2}^{N}\left(\cos\left(\beta d_{n-1}\right)+j\frac{Y_{n-1}}{G_0}\sin\left(\beta d_{n-1}\right)\right)-\frac{V_n^s}{V_{n-1}^s}\cdot\frac{\alpha_n}{\alpha_{n-1}}\cdot\frac{Y_{n-1}^{d,a}/G_0}{Y_n^{d,a}/G_0}\right|^2 \tag{28}$$

where

The error function depends on the slot spacings and angular dimensions and will be used for optimizing slot parameters by minimizer algorithms such as gradient conjugate or genetic algorithm.

Modified Taylor Pattern at 5.35 GHz

The cylindrical slot array is designed for 13 slots at the frequency 5.35 GHz. The design parameters of the array are given in Table 1. The pattern of slot array as obtained by the MLS and computer simulations by CST and HFSS are in Figure 3, for comparison. The VSWRs of array at the input port of cylindrical wave-guide are drawn in Figure 4.

5. Conclusion

In this paper the design equations are developed for the traveling wave mode by

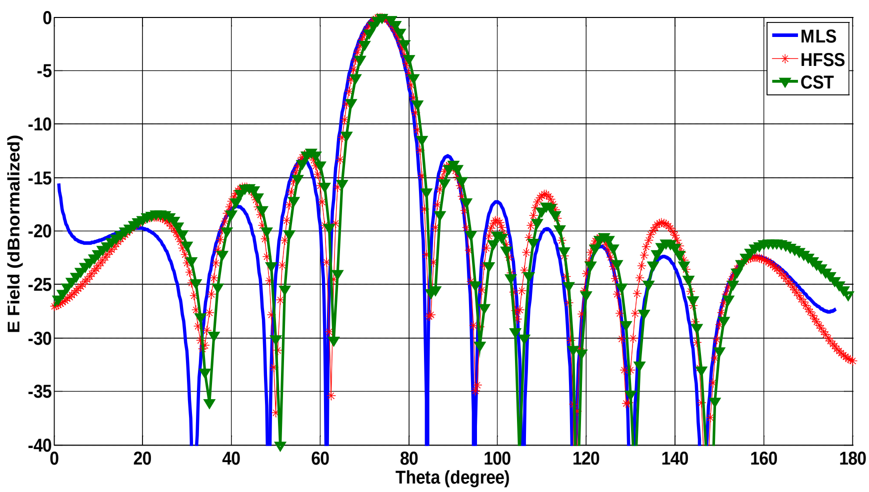


Figure 3. Comparison of patterns by MLS, HFSS and CST at 5.35 GHz.

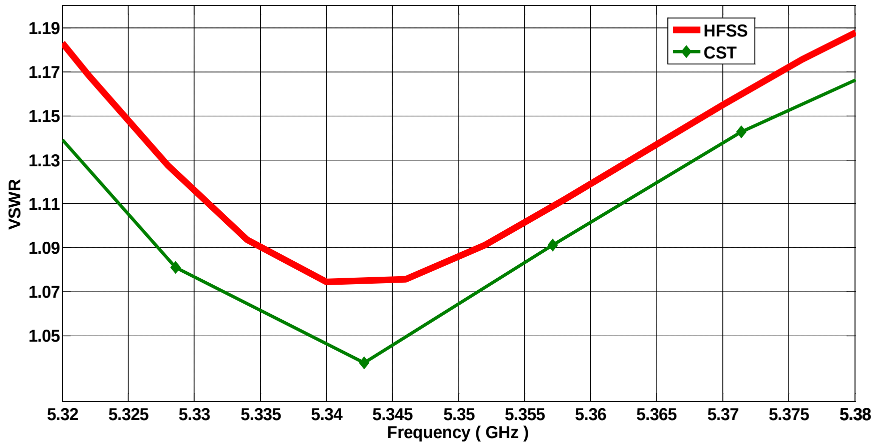


Figure 4. Diagram of VSWR Simulated by HFSS and CST softwares.

Table 1. The parameters and specifications of cylindrical slot array.

Antenna parameter	Number of slots	13
	Slot width: W	5.60 mm
	Thickness of waveguide wall: **Thickness**	0.20 mm
	Radius of cylindrical waveguide: R	22.43 mm
	Operation frequency	5.35 GHz
Optimized parameters	Slot length: $R\alpha$	15.70 mm
	Slot spacing: d	23.05 mm
Characteristics of desired pattern	Modified Taylor pattern with SLL = −13 dB	

employing the equivalent circuits according to the Elliott's method. The geometrical dimensions of the slot array on the cylindrical surface are determined by the minimization of the appropriate error functions. The proposed synthesis method of cylindrical slot array is demonstrated by one design example at 5.35 GHz frequency and is verified by simulation softwares of CST and HFSS. Such arrays are appropriate for various platforms of cylindrical shape, such broadcasting transmitter antennas (TV station).

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