

On the Kinetic Energy of the Projection Curve for the 1-Parameter Closed Spatial Homothetic Motion

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Abstract

In this study, the kinetic energy formula of the projection curve under 1-parameter closed homothetic motion is expressed and as a result, theorem is given. Also some special cases are given related with that formula.

Keywords

Kinetic Energy, Homothetic Motion, Kinematics, Projection Curve

1. Introduction

Kinetic energy is the energy that an object possesses because of its motion. The kinetic energy of the object having a linear orbit is called displacement kinetic energy, object with circular orbit called the rotational kinetic energy and also the kinetic energy of the object having high speed called relative kinetic energy.

Kinematics is a branch of mechanics dealing with the reasons that cause the motion to be ignored and how the movement occurs. For this reason, kinematics is generally called as the motion geometry. In kinematics, motion is called open or closed motion, depending on whether the trajectory curves are open or closed. There are many studies on the account of the fields of the trajectory curves. As an example, the Steiner area formula and Holditch theorem can be given [1] [2]. The results obtained in these studies have attracted the attention of many mathematicians and have been a source of inspiration for many studies. Accordingly, H. R. Müller generalized the relationship between the polar moment of inertia and the calculation of the trajectory field [3]. Tutar and Kuruoğlu expressed the Steiner area formula of closed curves for 1-parameterized planar

homothetic motion [4]. Düldül generalized the Holditch theorem for closed space curves [5]. Düldül and Kuruoğlu had calculated the polar moment of inertia for the Holditch type theorem of complex plane trajectory curves during 1-parameter closed homothetic motion [6]. Düldül calculated polar moment of inertia of the projection curve at any point under 1-parameter homothetic motion, then calculated the volume of these points under the 3-parameter homothetic motion. Thus, using the polar moment of inertia and volume formula, similar results to the spatial Holditch-type theorem were obtained [7]. İnan studied the Steiner area formula and polar moment of inertia for closed trajectory curve under homothetic motion in planar kinematics [8]. In addition, the kinetic energy formula for 1-parameter closed planar homothetic and inverse homothetic motion are taken by Tutar and İnan [9] [10]. On the other hand, Dathe and Gezzi calculated the Steiner area formula and the polar moment of inertia for the motion composed of fixed and moving systems between the knee and hip joints during human walking and correlated the polar moment of inertia with the Steiner area formula [11] [12].

In this study, the kinetic energy of the projection curve under 1-parameter closed homothetic space motion is investigated and some results are obtained.

2. Kinetic Energy of the Projection Curve

Let E and E' be moving and fixed planes, O and O' be origin of their coordinate systems and also $B = E/E'$ direct motion with t parameter between these systems. By taking displacement vector $OO' = U'$, total rotation angle $\alpha(t)$,

$$X'(t) = h(t)R(t)X + U'(t)$$

is called the trajectory of the fixed point X , according the moving system. In that equation, h is a homothetic scale of the motion B . Rotation matrix $R(t)$, vector $X'(t)$ and displacement vector $U'(t)$ are the continuous differentiable function of the a reel parameter t .

Motion with a first order in the time derivate can be given

$$E_{kin} = \frac{1}{2}m(\dot{X}'(t))^2$$

which is the integral over the kinetic energy of point with mass $m = 1$.

Let X be a fixed point at moving space and coordinates of the X according to $\{0; e_1, e_2, e_3\}$ system be x_1, x_2, x_3 , and

$$OX = x = x_1e_1 + x_2e_2 + x_3e_3$$

can be written.

If the displacement vector is expressed by

$$OO' = u = u_1e_1 + u_2e_2 + u_3e_3$$

the position vector of the fixed point x , on the 1-parameter closed homothetic spatial motion can be written as following:

$$x' = hx - u \quad (2.1)$$

Fixed point $x \in E$ draws a closed curve while motion. The projection of that closed curve on the plane P , in the direction of the unit e' vector, is also closed curve (**Image 1**).

Let show this closed curve with (X'') and projection of the point X on the plane be with X'' . And also plane M and origin of the fixed plane coincide. From **Image 1**, it can be written:

$$O'X = O'X'' + X''X$$

or

$$x'' = x' - \langle e', x' \rangle e' \quad (2.2)$$

By using Equation (2.2),

$$\begin{aligned} \|x''\|^2 &= \|x'\|^2 - 2\langle x', \langle e', x' \rangle e' \rangle + \|\langle e', x' \rangle e'\|^2 \\ &= \|x'\|^2 - 2\langle e', x' \rangle \langle e', x' \rangle + \|\langle e', x' \rangle\|^2 \|e'\|^2 \\ &= \|x'\|^2 - 2\langle e', x' \rangle^2 + \langle e', x' \rangle^2 \\ &= \|x'\|^2 - \langle e', x' \rangle^2 \end{aligned} \quad (2.3)$$

If the Equation (2.3) is derivated, we obtain

$$\dot{x}'' = \dot{x}' - (\langle \dot{e}', x' \rangle + \langle e', \dot{x}' \rangle) e' - \langle e', x' \rangle \dot{e}' = \dot{x}' - \langle e', \dot{x}' \rangle e' \quad (2.4)$$

And if the norm of the Equation (2.4) is determinated, we get

$$\begin{aligned} \|\dot{x}''\|^2 &= \|\dot{x}'\|^2 - 2\langle \dot{x}', \langle e', \dot{x}' \rangle e' \rangle + \langle e', \dot{x}' \rangle^2 \|e'\|^2 \\ &= \|\dot{x}'\|^2 - 2\langle \dot{x}', e' \rangle \langle e', \dot{x}' \rangle + \langle e', \dot{x}' \rangle^2 \\ &= \|\dot{x}'\|^2 - \langle e', \dot{x}' \rangle^2 \end{aligned} \quad (2.5)$$

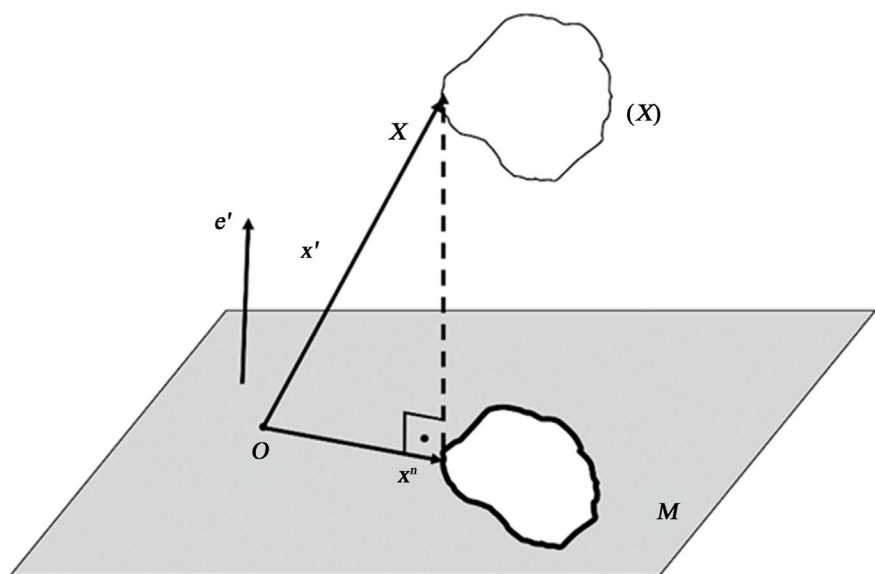


Image 1. The projection of that closed curve on the plane P .

Kinetic energy of the closed projection curve (X^n) according to origin point $O' \in E'$ is

$$E_{kin} = S = \frac{1}{2} \oint \|\dot{x}^n\|^2 dt \quad (2.6)$$

Integration boundary of Equation (2.6) is between 0 and T . If Equation (2.5) is replaced in Equation (2.6) and taking into account Equation (2.1), we obtain,

$$\begin{aligned} E_{kin} = S &= \frac{1}{2} \oint \left(\|\dot{x}'\|^2 - \langle e', \dot{x}' \rangle^2 \right) dt = \frac{1}{2} \oint \left(\|\dot{h}x - \dot{u}\|^2 - \langle e', \dot{h}x - \dot{u} \rangle^2 \right) dt \\ &= \frac{1}{2} \oint \left(\|\dot{h}x\|^2 - 2\dot{h}\langle x, \dot{u} \rangle + \|\dot{u}\|^2 - \left(\langle e', \dot{h}x \rangle - \langle e', \dot{u} \rangle \right)^2 \right) dt \\ &= \frac{1}{2} \oint \left(\|\dot{h}x\|^2 - 2\dot{h}\langle x, \dot{u} \rangle + \|\dot{u}\|^2 - \dot{h}^2 \langle e', x \rangle^2 + 2\dot{h}\langle e', x \rangle \langle e', \dot{u} \rangle - \langle e', \dot{u} \rangle^2 \right) dt \\ 2S &= \oint \dot{h}^2 \|x\|^2 dt - \oint \dot{h}^2 \langle e', x \rangle^2 dt + \oint \left(\dot{h}\langle e', x \rangle \langle e', \dot{u} \rangle - \dot{h}\langle x, \dot{u} \rangle \right) dt \\ &\quad + \oint \left(\|\dot{u}\|^2 - \langle e', \dot{u} \rangle^2 \right) dt \end{aligned} \quad (2.7)$$

By using Equation (2.7), kinetic energy of the origin point of the projective curve on the plane M is

$$2S_0 = \oint \left(\|\dot{u}\|^2 - \langle e', \dot{u} \rangle^2 \right) dt \quad (2.8)$$

If

$$\sum_{i=1}^3 a_i e_i = e', \quad a_i = a_i(t), \quad \sum_{i=1}^3 a_i^2 = 1$$

are taken, we get

$$\dot{h}\langle e', x \rangle \langle e', \dot{u} \rangle - \dot{h}\langle \dot{u}, x \rangle = \dot{h} \sum_{i=1}^3 \left(-\dot{u}_i + a_i \sum_{j=1}^3 a_j \dot{u}_j \right) x_i \quad (2.9)$$

and also

$$\langle e', x \rangle^2 = \sum_{i,j=1}^3 a_i a_j x_i x_j \quad (2.10)$$

If the Equation (2.8), Equation (2.9) and Equation (2.10) are replaced in Equation (2.7), we obtain,

$$2S = \|x\|^2 \oint \dot{h}^2 dt - \oint \dot{h}^2 \left(\sum_{i,j=1}^3 a_i a_j x_i x_j \right) dt + \oint \dot{h} \sum_{i=1}^3 \left(-\dot{u}_i + a_i \sum_{j=1}^3 a_j \dot{u}_j \right) x_i dt + 2S_0 \quad (2.11)$$

At Equation (2.11),

$$\begin{aligned} \eta &= \oint dt \\ \sigma_{ij} &= \oint a_i a_j dt \\ \tau_i &= \oint \left\{ -\dot{u}_i + a_i \sum_{j=1}^3 a_j \dot{u}_j \right\} dt \end{aligned}$$

By using these assumptions the kinetic energy formula of the fixed $X \in E$ of the closed projective trajectory curve on the plane M is

$$2S = 2S_0 + \eta \sum_{i=1}^3 x_i^2 - \sum_{i,j=1}^3 \sigma_{ij} x_i x_j + \sum_{i=1}^3 \tau_i x_i \quad (2.12)$$

Equation (2.12) is a quadratic equation according to x_i components. As a result of Equation (2.12), the following theorem can be given.

Theorem 1:

The projection of all fixed points of moving space E , which have the equal kinetic energy, are located on the same sphere.

Let take into consideration

$$\eta = \oint \dot{h}^2(t) dt$$

where $\dot{h}^2(t)$ is a continuous function. According to the mean value theorem of integral-calculus, there exists at least a point $t_0 \in [0, T]$ such that the following equation holds:

$$\oint \dot{h}^2(t) dt = \int_0^T \dot{h}^2(t) dt = \dot{h}^2(t_0) T \quad (2.13)$$

If the moving coordinate system are chosen to be $\sigma_{ij} = 0$ for $i \neq j$ and by using to the mean value theorem of integral-calculus to the

$$\sigma_{ii} = \oint \dot{h}^2 a_i^2 dt = \int_0^T \dot{h}^2 a_i^2 dt = \dot{h}^2 a_i^2 T$$

there exists at least a point $t_i \in [0, T]$ such that the following equation holds

$$\sigma_{ii} = \dot{h}^2(t_i) a_i^2 T, i = 1, 2, 3 \quad (2.14)$$

In that case, by using Equation (2.12)

$$\begin{aligned} 2S &= 2S_0 + \dot{h}^2(t_0) T \sum_{i=1}^3 x_i^2 - \sum_{i=1}^3 \dot{h}^2(t_i) a_i^2 T x_i^2 + \sum_{i=1}^3 \tau_i x_i \\ &= 2S_0 + T \sum_{i=1}^3 (\dot{h}^2(t_0) - \dot{h}^2(t_i) a_i^2) x_i^2 + \sum_{i=1}^3 \tau_i x_i \end{aligned} \quad (2.15)$$

is obtained. Additionally, since

$$\sum_{i=1}^3 a_i^2 = 1$$

for $\forall t \in [0, T]$;

$$a_i^2(t) \leq 1$$

can be written. Furthermore, if the both side of the Equation (2.12) multiply with $\dot{h}^2(t) T$ and integrate, we get

$$\int_0^T \dot{h}^2(t) a_i^2(t) T dt \leq \int_0^T \dot{h}^2(t) T dt$$

Since $T > 0$,

$$\dot{h}^2(t_i) a_i^2(t_i) T \leq \dot{h}^2(t_0) T$$

or

$$\dot{h}^2(t_i) a_i^2(t_i) T \leq \dot{h}^2(t_0) T \quad (2.16)$$

can be written. Let, L be the maximal value of the $\dot{h}^2(t)$ function at $[0, T]$. In that case, for $\forall t \in [0, T]$,

$$0 \leq \dot{h}^2(t) \leq L$$

and

$$0 \leq \dot{h}^2(t_i) \leq L, i = 1, 2, 3$$

can be written. Since $a_i^2(t) \leq 1$, we get

$$0 \leq \dot{h}^2(t_i) a_i^2(t_i) \leq L$$

By using Equation (2.16),

$$0 \leq \dot{h}^2(t_0) - \dot{h}^2(t_i) a_i^2(t_i) \leq L$$

can be written. Because of the $\dot{h}^2(t)$ is continuous at $[0, T]$, there exists at least a point $t_{ii} \in [0, T]$ such that the following equation holds

$$\dot{h}^2(t_0) - \dot{h}^2(t_i) a_i^2(t_i) = \dot{h}^2(t_{ii}), i = 1, 2, 3$$

Thus,

$$\sum_{i=1}^3 (\dot{h}^2(t_0) - \dot{h}^2(t_i) a_i^2(t_i)) x_i^2 = \sum_{i=1}^3 \dot{h}^2(t_{ii}) x_i^2 \quad (2.17)$$

can be written. Since $0 \leq \dot{h}^2(t) \leq L$, we have

$$0 \leq \dot{h}^2(t_{11}) x_1^2 \leq L x_1^2$$

$$0 \leq \dot{h}^2(t_{22}) x_2^2 \leq L x_2^2$$

$$0 \leq \dot{h}^2(t_{33}) x_3^2 \leq L x_3^2$$

If these equations are arranged,

$$0 \leq \dot{h}^2(t_{11}) x_1^2 + \dot{h}^2(t_{22}) x_2^2 + \dot{h}^2(t_{33}) x_3^2 \leq L(x_1^2 + x_2^2 + x_3^2)$$

can be written. There exists $\exists t_4 \in [0, T]$ such that the following equations hold,

$$0 \leq \frac{\dot{h}^2(t_{11}) x_1^2 + \dot{h}^2(t_{22}) x_2^2 + \dot{h}^2(t_{33}) x_3^2}{x_1^2 + x_2^2 + x_3^2} \leq L$$

or

$$\frac{\dot{h}^2(t_{11}) x_1^2 + \dot{h}^2(t_{22}) x_2^2 + \dot{h}^2(t_{33}) x_3^2}{x_1^2 + x_2^2 + x_3^2} = \dot{h}^2(t_4) \quad (2.18)$$

In this case, we have

$$\sum_{i=1}^3 \dot{h}^2(t_{ii}) x_i^2 = \dot{h}^2(t_4) \sum_{i=1}^3 x_i^2$$

And by using Equation (2.17)

$$\sum_{i=1}^3 (\dot{h}^2(t_0) - \dot{h}^2(t_i) a_i^2(t_i)) x_i^2 = \dot{h}^2(t_4) \sum_{i=1}^3 x_i^2 \quad (2.19)$$

can be written. If Equation (2.19) is replaced in Equation (2.15), kinetic energy formula of the projection curve of fixed point, which is chosen from moving

space, under the 1-parameter closed spatial motion can be written as:

$$S = S_0 + \frac{1}{2}Th^2(t_4) \sum_{i=1}^3 x_i^2 + \frac{1}{2} \sum_{i=1}^3 c_i x_i \quad (2.20)$$

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