Journal of Applied Mathematics and Physics, 2016, 4, 1936-1948
http://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

Study of Temperature Behaviour on Thermally Induced Vibration of Non-Homogeneous Trapezoidal Plate with Bi-Linearly Varying Thickness

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How to cite this paper: Kavita, Kumar, S. and Sharma, P. (2016) Study of Temperature Behaviour on Thermally Induced Vibration of Non-Homogeneous Trapezoidal Plate with Bi-Linearly Varying Thickness. *Journal of Applied Mathematics and Physics*, **4**, 1936-1948.
http://dx.doi.org/10.4236/jamp.2016.410196

Received: August 10, 2016
Accepted: October 25, 2016
Published: October 28, 2016

Abstract

The main aim of the present work is to study the linear temperature behaviour of a non-homogeneous trapezoidal plate whose thickness varies linearly in both directions. The temperature behaviour considered linear along the length of the plate. Non-homogeneity in plate arises due to variation in density along the length of the plate. The two-term deflection function with clamped-simply supported-clamped-simply supported boundary condition is taken into consideration. The effect of structural parameters such as taper constants, thermal gradient, non-homogeneity constant and aspect ratio has been studied. Rayleigh-Ritz method is used to solve the governing differential equations and to obtain the fundamental frequencies for the first two modes of vibration. Results are presented in graphical form.

Keywords

Vibration, Trapezoidal Plate, Taper Constants, Thermal Gradient, Aspect Ratio, Non-Homogeneity, Thickness, Density, Frequencies

1. Introduction

Most of the machines and structures work under the control of high temperature. Due to this, system undergoes some vibrations. Vibrations affect the efficiency, strength and durability of the system. The purpose of vibration study is to reduce vibration through proper and accurate design of machines and structures. Therefore, it is necessary for researchers and design engineers to have pre-knowledge of vibrational characteristics of systems before finalizing the design of structures. The vibrational analysis of plates depends on their geometry. In modern technology, plates of different shapes such as rectangular, circular, elliptical, parallelogram etc. are used in engineering applications. Plates with different shapes, boundary conditions at the edges and various complicating effects have often found applications in different structures such as aerospace, machine design, telephone industry, nuclear reactor technology, naval structures and earthquake-resistant structures. Literature shows that the vibration analysis has inspired many researchers to do work in this direction. Out of them few are given under. Gupta and Sharma [1] had analyzed the effect of linear thermal gradient on vibrations of trapezoidal plates whose thickness varied parabolically. Gupta and Sharma [2] had studied the effect of linear temperature behaviour on a non-homogeneous trapezoidal plate of parabolically varying thickness. Leissa [3] provided an appreciable collection of research papers in his monograph on the vibration of plates of different shapes and under different boundary conditions. Singh and Saxena [4] discussed the transverse vibration of triangular plates with variable thickness. Chen *et al.* [5] had worked on the free vibration of cantilevered symmetrically laminated thick trapezoidal plates. Bambill *et al.* [6] studied the transverse vibrations of rectangular, trapezoidal and triangular orthotropic, cantilever plates. Saliba [7] worked on free vibration analysis of simply supported symmetrical trapezoidal plates. Krishnan and Deshpande [8] studied the free vibration of trapezoidal plates. Liew and Lam [9] had studied the vibrational response of symmetrically laminated trapezoidal composite plates with point constraints. Liew and Lim [10] worked on the transverse vibration of symmetric trapezoidal plates of variable thickness. Liew [11] discussed the vibration of symmetrically laminated cantilever trapezoidal composite plates. Klein [12] analyzed the vibration of simply supported isosceles trapezoidal flat plates. Qatu [13] discussed the vibrations of laminated composite completely free triangular and trapezoidal plates. Zamani *et al.* [14] studied the free vibration analysis of moderately thick trapezoidal symmetrically laminated plates with various combinations of boundary conditions. Manna [15] calculated the free vibration of tapered isotropic rectangular plates with linearly varying thickness by using a high-order triangular element. Bhardwaj *et al.* [16] had studied the transverse vibrations of clamped and simply-supported circular plates with two dimensional thickness variations. Mirza and Bijlani [17] discussed the vibration of triangular plates of variable thickness. Gupta *et al.* [18] worked on vibration of non-homogeneous circular mindlin plates with variable thickness. Narita *et al.* *[19]* observed the transverse vibration of clamped trapezoidal plates having rectangular orthotropy. Zhou and Zheng [20] worked on the vibration of skew plates by the MLS-Ritz method. Quintana and Nallim [21] presented a variational approach to free vibration analysis of shear deformable polygonal plates with variable thickness. Korobko and Chernyaev [22] determinated the maximum deflection in transverse bending of parallelogram plates using the conformal radiuses ratio.

After a careful study of literature, it is recognized that no work has been done on linear density variation with bilinear thickness variation on vibration of heated trapezoidal plate. In this paper, an analysis is presented to study the effect of thermally induced vibration of non-homogeneous trapezoidal plate with bi-linearly varying thickness. To acquire the natural frequencies for the first two modes of vibration, Rayleigh-Ritz's method is used for a non-homogeneous trapezoidal plate whose two sides are clamped and two are simply-supported.

2. Thickness and Density

As depicted in **Figure 1** a symmetric, non-homogeneous trapezoidal plate has been considered. Thickness varies bilinearly along length and width of the plate as

$$h(\xi) = h_0\left[1-(1-\beta_1)\left(\xi+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)\right] \tag{1}$$

where $h_0 = h$ at $\xi = \eta = -1/2$ and β_1, β_2 are taper constants.

The density is one of the most important aspects of any design. Due to variation in density, non-homogeneity occurs in plate's material which varies linearly along the length of the plate. So, it can be considered as

$$\rho = \rho_0\left[1-(1-\beta)\left(\xi+\frac{1}{2}\right)\right] \tag{2}$$

where $\rho_0 = \rho$ is the mass density at $\xi = -1/2$ and β is non-homogeneity constant.

The temperature of the trapezoidal plate varies linearly along the length of the plate as

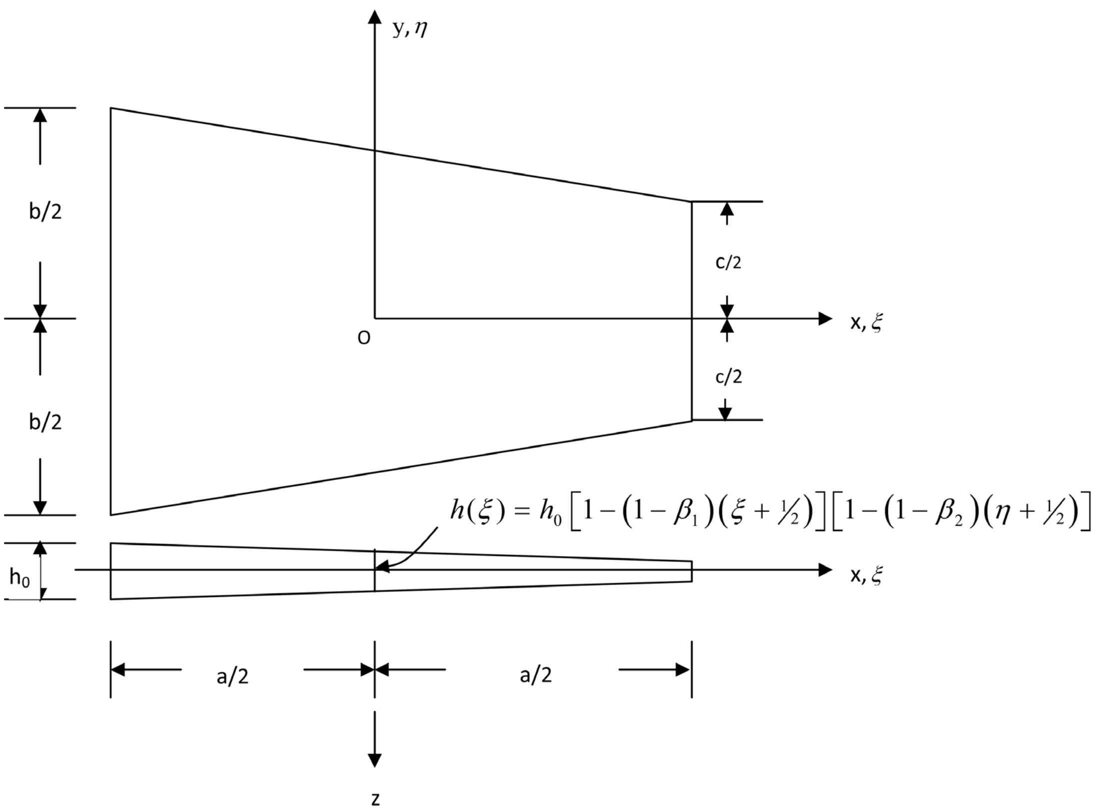


Figure 1. Geometry of the trapezoidal plate.

$$\tau = \tau_0\left(\frac{1}{2} - \xi\right) \tag{3}$$

where τ denotes the excess above the reference temperature at a distance $\xi = x/a$ and τ_0 denotes the temperature excess above the reference temperature at the end $\xi = -1/2$ ..

For most of the structural materials the temperature dependence of the modulus of elasticity is given by Nowacki [23] as

$$E = E_0\left(1 - \gamma\tau\right) \tag{4}$$

where E_0 is Young's modulus value at reference temperature $\tau = 0$ and γ is the slope of variation of E and τ.

By the use of Equation (3) in Equation (4), one obtains

$$E = E_0\left(1 - \alpha\left(\frac{1}{2} - \xi\right)\right) \tag{5}$$

where $\alpha = \gamma\tau_0\left(0 \le \alpha \le 1\right)$ known as thermal gradient.

3. Governing Differential Equations

The governing differential equations of kinetic energy T and strain energy V for a trapezoidal plate are given by [10] as

$$T = \frac{ab}{2}\omega^2 \int_A h(\xi)\rho w^2 \mathrm{d}A \tag{6}$$

and

$$V = \frac{ab}{2}\int_A D(\xi)\left\{\left(\frac{1}{a^2}\frac{\partial^2 w}{\partial \xi^2} + \frac{1}{b^2}\frac{\partial^2 w}{\partial \eta^2}\right)^2 - 2(1-\nu)\left(\frac{1}{a^2 b^2}\frac{\partial^2 w}{\partial \xi^2}\frac{\partial^2 w}{\partial \eta^2} - \left(\frac{1}{ab}\frac{\partial^2 w}{\partial \xi \partial \eta}\right)^2\right)\right\}\mathrm{d}A \tag{7}$$

where ν is the Poisson's ratio; ω is the angular frequency of vibration and A is the area of the plate.

Flexural rigidity of the plate $D(\xi)$ can be expressed as

$$D(\xi) = D_0\left[\left[1 - (1-\beta_1)\left(\xi + \frac{1}{2}\right)\right]\left[1 - (1-\beta_2)\left(\eta + \frac{1}{2}\right)\right]\right]^3 \tag{8}$$

where $\xi = \frac{x}{a}, \eta = \frac{y}{b}$ are non-dimensional variables. Here,

$$D_0 = \frac{E h_0^3}{12\left(1-\nu^2\right)} \tag{9}$$

By using Equation (5) and Equation (9) in Equation (8), the flexural rigidity becomes

$$D(\xi) = \frac{E_0 h_0^3}{12\left(1-\nu^2\right)}\left[\left[1 - (1-\beta_1)\left(\xi + \frac{1}{2}\right)\right]\left[1 - (1-\beta_2)\left(\eta + \frac{1}{2}\right)\right]\right]^3\left(1 - \alpha\left(\frac{1}{2} - \xi\right)\right) \tag{10}$$

Using Equation (1) and Equation (2) in Equation (6), we get

$$T = \frac{ab}{2}\rho_0 h_0 \omega^2 \int_A \left[1 - (1-\beta_1)\left(\xi + \frac{1}{2}\right)\right]\left[1 - (1-\beta_2)\left(\eta + \frac{1}{2}\right)\right] \times \left[1 - (1-\beta)\left(\xi + \frac{1}{2}\right)\right] w^2 \mathrm{d}A \tag{11}$$

Using Equation (10) in Equation (7), we get

$$V=\frac{ab}{2}\frac{E_0h_0^3}{12\left(1-\nu^2\right)}\int_A\left[\left[1-(1-\beta_1))\left(\xi+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)\right]\right]^3\left(1-\alpha\left(\frac{1}{2}-\xi\right)\right)$$
$$\times\left\{\left(\frac{1}{a^2}\frac{\partial^2 w}{\partial\xi^2}+\frac{1}{b^2}\frac{\partial^2 w}{\partial\eta^2}\right)^2-2(1-\nu)\left(\frac{1}{a^2b^2}\frac{\partial^2 w}{\partial\xi^2}\frac{\partial^2 w}{\partial\eta^2}-\left(\frac{1}{ab}\frac{\partial^2 w}{\partial\xi\partial\eta}\right)^2\right)\right\}\mathrm{d}A \tag{12}$$

In the present study the two term deflection function which satisfies the boundary condition can be expressed as

$$w=A_1\left\{\left(\xi+\frac{1}{2}\right)\left(\xi-\frac{1}{2}\right)\right\}^2\left\{\eta-\left(\frac{b-c}{2}\right)\xi+\left(\frac{b+c}{4}\right)\right\}\left\{\eta+\left(\frac{b-c}{2}\right)\xi-\left(\frac{b+c}{4}\right)\right\}$$
$$+A_2\left\{\left(\xi+\frac{1}{2}\right)\left(\xi-\frac{1}{2}\right)\right\}^3\left\{\eta-\left(\frac{b-c}{2}\right)\xi+\left(\frac{b+c}{4}\right)\right\}^2\left\{\eta+\left(\frac{b-c}{2}\right)\xi-\left(\frac{b+c}{4}\right)\right\}^2, \tag{13}$$

where A_1 and A_2 are two unknowns to be evaluated. For the solution of the problem the trapezoidal plate is considered whose two sides are clamped and two are simply supported. Therefore, the boundaries are defined by four straight lines

$$\eta=\frac{c}{4b}-\frac{\xi}{2}+\frac{1}{4}+\frac{c\xi}{2b};$$
$$\eta=-\frac{c}{4b}+\frac{\xi}{2}-\frac{1}{4}-\frac{c\xi}{2b};$$
$$\xi=-\frac{1}{2};$$
$$\xi=\frac{1}{2}. \tag{14}$$

4. Methodology

For the existing problem, Rayleigh-Ritz's method has been employed. It requires the maximum strain energy must be equal to the maximum kinetic energy. Therefore, it is necessary that the consequent equation must be satisfied

$$\delta\left(V-T\right)=0. \tag{15}$$

Using Equation (14) into Equation (11) and Equation (12), we obtain

$$T=\frac{ab}{2}\rho_0h_0\omega^2\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\frac{c}{4b}+\frac{\xi}{2}-\frac{1}{4}-\frac{c\xi}{2b}}^{\frac{c}{4b}-\frac{\xi}{2}+\frac{1}{4}+\frac{c\xi}{2b}}\left[1-(1-\beta_1)\left(\xi+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)\right]$$
$$\times\left[1-(1-\beta)\left(\xi+\frac{1}{2}\right)\right]w^2\mathrm{d}\eta\mathrm{d}\xi \tag{16}$$

And

$$V=\frac{ab}{2}\frac{E_0h_0^3}{12\left(1-\nu^2\right)}\int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\frac{c}{4b}+\frac{\xi}{2}-\frac{1}{4}-\frac{c\xi}{2b}}^{\frac{c}{4b}-\frac{\xi}{2}+\frac{1}{4}+\frac{c\xi}{2b}}\left[\left[1-(1-\beta_1)\left(\xi+\frac{1}{2}\right)\right]\left[1-(1-\beta_2)\left(\eta+\frac{1}{2}\right)\right]\right]^3\left(1-\alpha\left(\frac{1}{2}-\xi\right)\right)$$
$$\times\left\{\left(\frac{1}{a^2}\frac{\partial^2 w}{\partial\xi^2}+\frac{1}{b^2}\frac{\partial^2 w}{\partial\eta^2}\right)^2-2(1-\nu)\left(\frac{1}{a^2b^2}\frac{\partial^2 w}{\partial\xi^2}\frac{\partial^2 w}{\partial\eta^2}-\left(\frac{1}{ab}\frac{\partial^2 w}{\partial\xi\partial\eta}\right)^2\right)\right\}\mathrm{d}\eta\mathrm{d}\xi. \tag{17}$$

Using Equation (16) and Equation (17) into Equation (15), we get

$$\delta\left(V_1-\lambda^2T_1\right)=0. \tag{18}$$

where

$$\begin{aligned}T_1 &= \int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\frac{c}{4b}+\frac{\xi}{2}-\frac{1}{4}-\frac{c\xi}{2b}}^{\frac{c}{4b}-\frac{\xi}{2}+\frac{1}{4}+\frac{c\xi}{2b}}\left[1-\left(1-\beta_1\right)\left(\xi+\frac{1}{2}\right)\right]\left[1-\left(1-\beta_2\right)\left(\eta+\frac{1}{2}\right)\right]\\ &\times\left[1-\left(1-\beta\right)\left(\xi+\frac{1}{2}\right)\right]w^2\mathrm{d}\eta\mathrm{d}\xi,\end{aligned} \tag{19}$$

$$\begin{aligned}V_1 &= \int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\frac{c}{4b}+\frac{\xi}{2}-\frac{1}{4}-\frac{c\xi}{2b}}^{\frac{c}{4b}-\frac{\xi}{2}+\frac{1}{4}+\frac{c\xi}{2b}}\left[\left[1-\left(1-\beta_1\right)\left(\xi+\frac{1}{2}\right)\right]\left[1-\left(1-\beta_2\right)\left(\eta+\frac{1}{2}\right)\right]\right]^3\left(1-\alpha\left(\frac{1}{2}-\xi\right)\right)\\ &\times\left\{\left(\frac{1}{a^2}\frac{\partial^2 w}{\partial\xi^2}+\frac{1}{b^2}\frac{\partial^2 w}{\partial\eta^2}\right)^2-2\left(1-\nu\right)\left(\frac{1}{a^2b^2}\frac{\partial^2 w}{\partial\xi^2}\frac{\partial^2 w}{\partial\eta^2}-\left(\frac{1}{ab}\frac{\partial^2 w}{\partial\xi\partial\eta}\right)^2\right)\right\}\mathrm{d}\eta\mathrm{d}\xi,\end{aligned} \tag{20}$$

And

$$\lambda^2=\frac{12\omega^2\rho_0a^4\left(1-\nu^2\right)}{E_0{h_0}^2}. \tag{21}$$

is a frequency parameter.

The unknowns A_1 and A_2 in Equation (18) arises due to the substitution of the deflection function w given by Equation (13). From Equation (18) these two constants can be determined, as follows

$$\begin{aligned}\frac{\partial}{\partial A_1}\left(V_1-\lambda^2T_1\right)&=0,\\ \frac{\partial}{\partial A_2}\left(V_1-\lambda^2T_1\right)&=0,\end{aligned} \tag{22}$$

On simplifying (22), we get

$$b_{m1}A_1+b_{m2}A_2=0,\quad m=1,2 \tag{23}$$

where b_{m1},b_{m2} $(m = 1, 2)$ involve parametric constants and the frequency parameter.

For a non-zero solution, the determinant of co-efficient of Equation (23) must be zero. Thus the frequency Equation for a (C-S-C-S) trapezoidal plate is given by

$$\begin{vmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\end{vmatrix}=0. \tag{24}$$

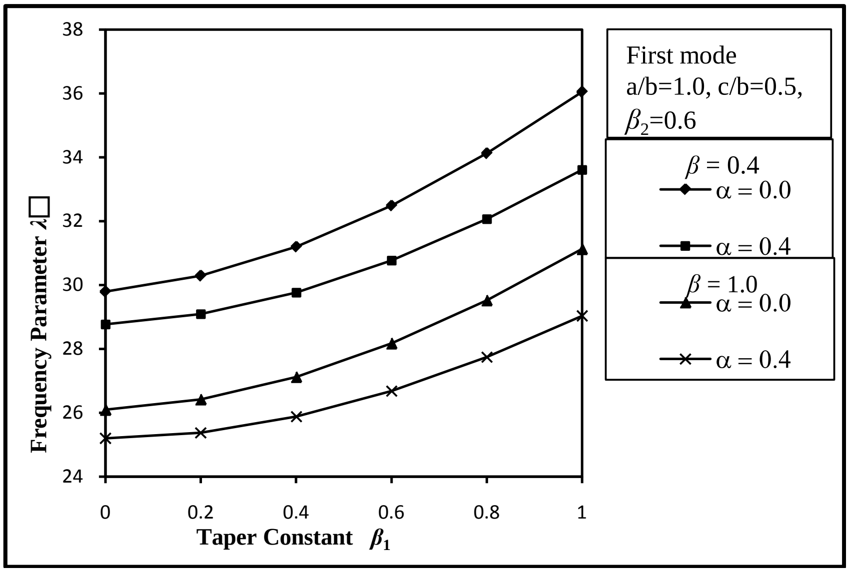
On simplifying Equation (24), a quadratic equation in λ^2 is obtained. Thus, it provides the two values of λ^2 corresponding to the first and second modes of vibration respectively.

5. Results and Discussions

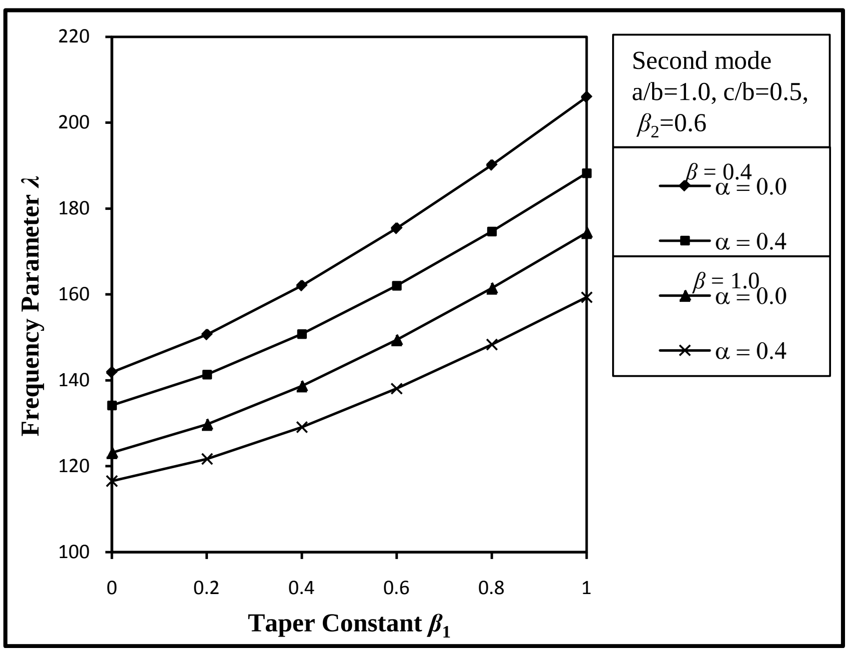
Frequencies for the first two modes of vibration are calculated for non-homogeneous trapezoidal plate whose thickness varies linearly in both directions and density varies linearly in x-direction. Different values of taper constants β_1 & β_2, thermal gradient

α, aspect ratios a/b, c/b and non-homogeneity constant β has been considered. The value of Poisson's ratio ν is taken as 0.33. With the help of graphs all the results have been presented.

In Figure 2(a) and Figure 2(b) these figures show the variation of the frequency parameter λ with the taper constant β_1 (0.0 to 1.0) for the first and second mode, respectively, for



(a)



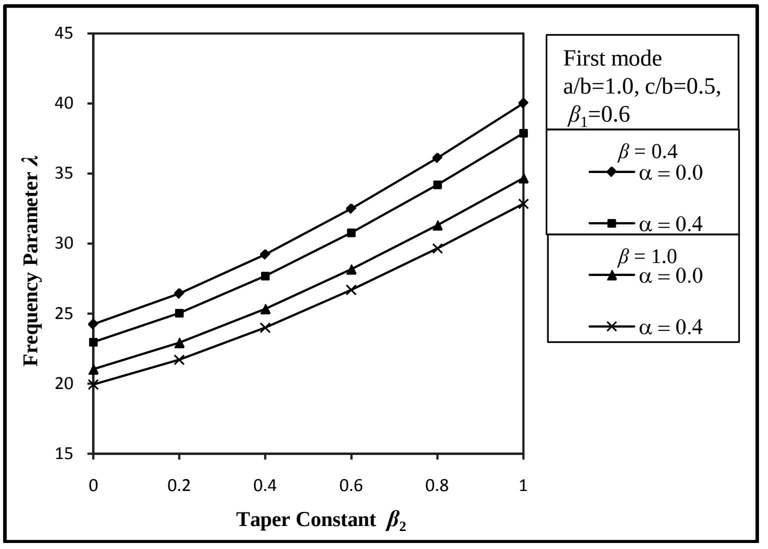
(b)

Figure 2. (a) Variation of frequency parameter λ for different values of taper constant β_1 for the first mode. (b) Variation of frequency parameter λ for different values of taper constant β_1 for the second mode.

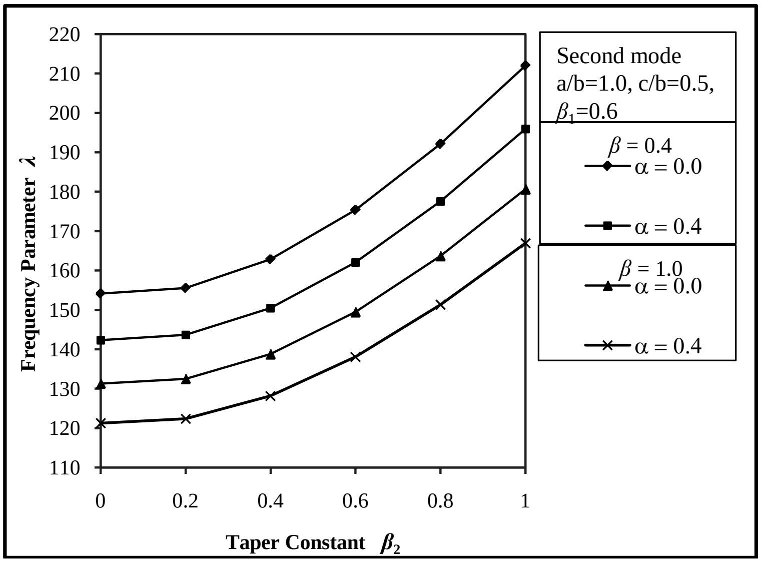
1) $a/b = 1.0, c/b = 0.5$
2) $\beta = 0.4, 1.0$
3) $\alpha = 0.0, 0.4$
4) $\beta_2 = 0.6$

These figures demonstrate that as the taper constant β_1 increases, the frequency parameter λ also increases for both the modes of vibration.

In Figure 3(a) and Figure 3(b), these figures show the variation of the frequency parameter λ with the taper constant β_2 (0.0 to 1.0) for the first and second mode, respectively, for



(a)



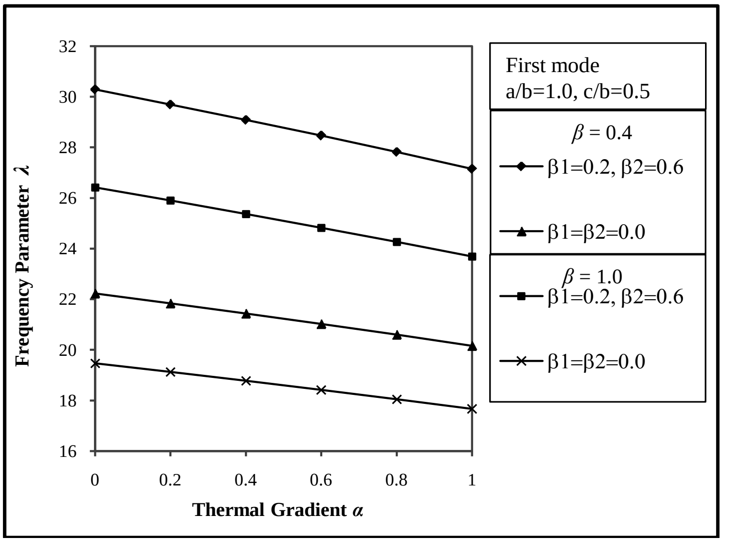
(b)

Figure 3. (a) Variation of frequency parameter λ for different values of taper constant β_2 for the first mode. (b) Variation of frequency parameter λ for different values of taper constant β_2 for the second mode.

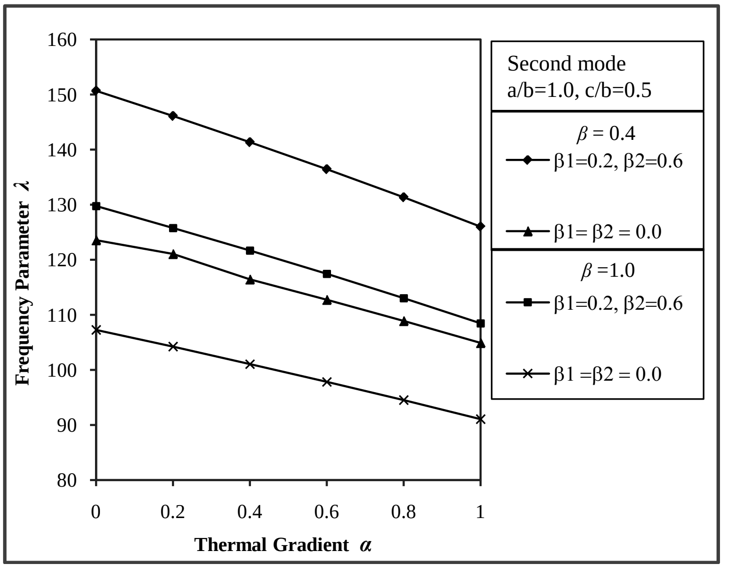
1) $a/b = 1.0$, $c/b = 0.5$
2) $\beta = 0.4, 1.0$
3) $\alpha = 0.0, 0.4$
4) $\beta_1 = 0.6$

These figures explain that as the taper constant β_2 increases, and the frequency parameter λ also increases for both the modes of vibration.

In Figure 4(a) and Figure 4(b) these figures depict the behaviour of the frequency parameter λ with thermal gradient α (varying from 0.0 to 1.0) for the first and second mode, respectively, for



(a)



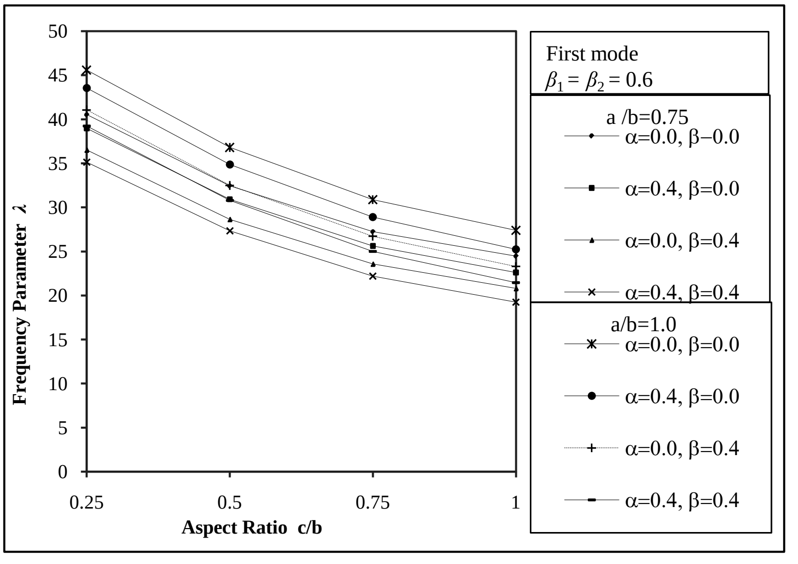
(b)

Figure 4. (a) Variation of frequency parameter λ for different values of thermal gradient α for the first mode. (b) Variation of frequency parameter λ for different values of thermal gradient α for the second mode.

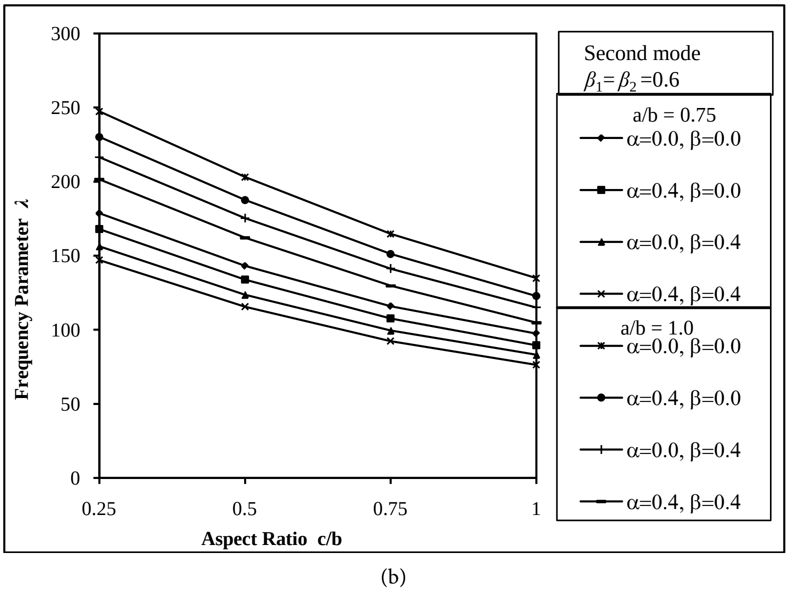
1) $a/b = 1.0, c/b = 0.5$
2) $\beta_1 = 0.2, \beta_2 = 0.6$
3) $\beta_1 = \beta_2 = 0.0$
4) $\beta = 0.4, 1.0$

It is clear from these figures that as the thermal gradient α increases, the frequency parameter λ decreases for both the modes of vibration.

In Figure 5(a) and Figure 5(b) these figures demonstrate the effect of aspect ratio c/b (varying from 0.25 to 1.0) on the frequency parameter λ for the first and second mode, respectively, for



(a)



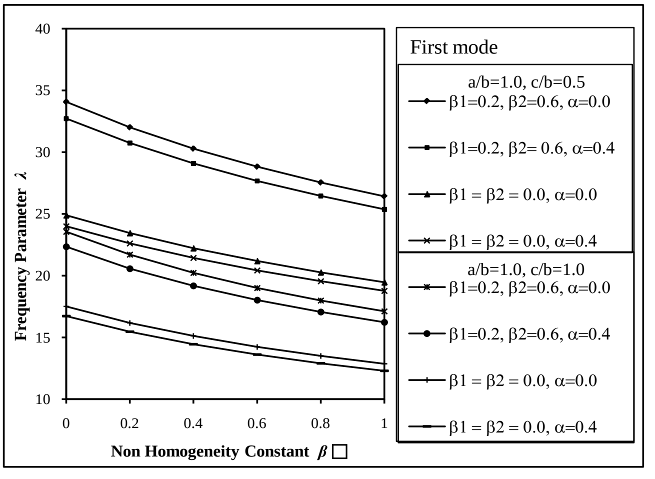
(b)

Figure 5. (a) Variation of frequency parameter λ for different values of aspect ratio c/b for the first mode. (b) Variation of frequency parameter λ for different values of aspect ratio c/b for the second mode.

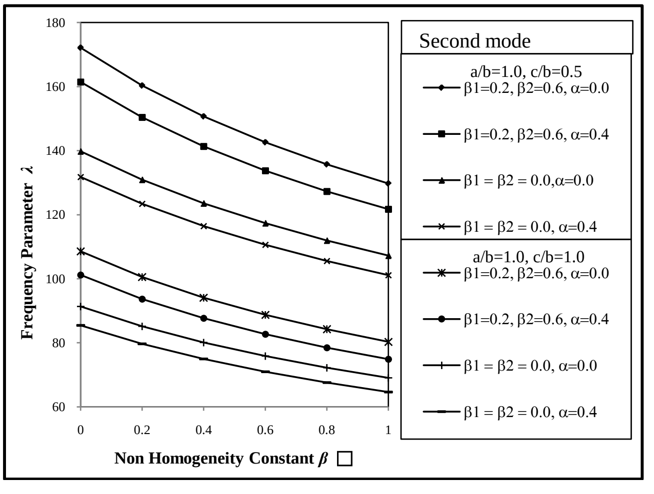
1) $a/b = 0.75, 1.0$
2) $\beta_1 = \beta_2 = 0.6$
a) $\alpha = 0.0, \beta = 0.0$
b) $\alpha = 0.4, \beta = 0.0$
c) $\alpha = 0.0, \beta = 0.4$
d) $\alpha = 0.4, \beta = 0.4$

It is evident from the figures that as aspect ratio c/b increases, the frequency parameter decreases for both the modes of vibration. From Figure 5(a) and Figure 5(b) it is observed that with increase in aspect ratio a/b the frequency increases for both the modes of vibration.

In Figure 6(a) and Figure 6(b) these figures show the effect of non-homogeneity constant β (varying from 0.0 to 1.0) on the frequency parameter λ for the first and second mode, respectively, for



(a)



(b)

Figure 6. (a) Variation of frequency parameter λ for different values of non-homogeneity constant β for the first mode. (b) Variation of frequency parameter λ for different values of non-homogeneity constant β for the second mode.

1) $a/b = 1.0,\ c/b = 0.5, 1.0$
2) $\beta_1 = \beta_2 = 0.0$ and $\beta_1 = 0.2, \beta_2 = 0.6$
3) $\alpha = 0.0, 0.4$

These figures show that as the non-homogeneity constant β increases, the frequency parameter λ decreases for both the modes of vibration.

6. Conclusion

In the present paper, the effect of temperature on the vibration of symmetric, non-homogeneous trapezoidal plate of isotropic material with clamped-simply supported-clamped-simply supported-boundary condition has been studied by using the Rayleigh-Ritz method. Effect of other plate's parameters such as non-homogeneity constant, aspect ratios, taper constants has also been considered. It is obvious from the graphs that by the increase of taper constants, aspect ratio a/b the frequency of both the modes of vibration increases. On the other hand, frequency decreases with increasing values of thermal gradient, aspect ratio c/b and non-homogeneity constant for both the modes of vibration. By the proper selection of various plate parameters such as taper constants, thermal gradient, aspect ratio and non-homogeneity constant, a desired frequency can be attained for the first two modes of vibration which would be helpful for the design engineers.

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