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Cooling of Granules in Vibrating, Suspended Bed: Engineering Simulation

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Abstract

Here we suggest an algorithm for calculation of the process parameters and design of a vertical cooler with inclined, gas-permeable blades and with a vibrating, suspended layer of granules on them (Vibrating Fluidized Bed—VFB). The algorithm is based on the use of the equations of heat and material balance, taking into account the influx of moisture into the layer with cold air and dust—as a carryover. Mode entrainment of dust particles and moisture from the VFB is described by using empirical formulas and π-theorem. To calculate the cooling time of granules a model of the dynamics of a variable mass VFB was built, which linked the geometrical and physical process parameters to a single dependency. An example showed that mass flow of granules of 248 kg/h and a volume flow of air of 646 m^3/h with temperature of 30˚C to cool the zeolite granules from 110˚C to 42˚C for 49 s required a vertical apparatus of rectangular shape with four chambers and with volume of 0.2 m^3. A comparative analysis of technological parameters of the projected cooler with the parameters of typical industrial apparatuses showed that for all indicators: the cooling time of granules, the flow rate of gas (air) and the heat flow, a 4-chambered, vertical apparatus of rectangular shape with VFB was the most effective.

Keywords

Cooling of Granules, Vibrating Fluidized Bed, Mathematical Model, Calculation Algorithm

1. Introduction

Currently, a widely used method of materials production in several industries is in the form of granules. The granules come out of the press or granulator at a relatively high temperature of 90˚C - 120˚C and must be cooled to 30˚C - 50˚C—depending on the season and the geographic position of the enterprise. The temperature and humidity of the granules at the output must be reduced to the limits for the conservation of their properties and

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persistent storage. For cooling the granules two basic types of coolers are used: refrigerating vertical columns and horizontal conveyor coolers. The former have a preferential distribution [1] [2]. Cooling of the granules should be done with cold air so that different-sized granules can be distributed uniformly and cooled to the required temperature. It is also necessary to determine the thermal parameters of air and dust at the exit for further treatment in the air-cyclone. To solve the first problem we used a vertical apparatus (**Figure 1**) with a vibrating, suspended layer of granules (Vibrating Fluidized Bed—VFB). This unit was used by us previously [3] to dry heat-sensitive materials. In such an apparatus the hot granules fall from the granulator with a predetermined flow rate, initial temperature and initial humidity fall into the chambers of the cooler with gas-permeable, vibrating blades and asymmetrically arranged side entrances of air. Cold air is fed through the blades and side entrances

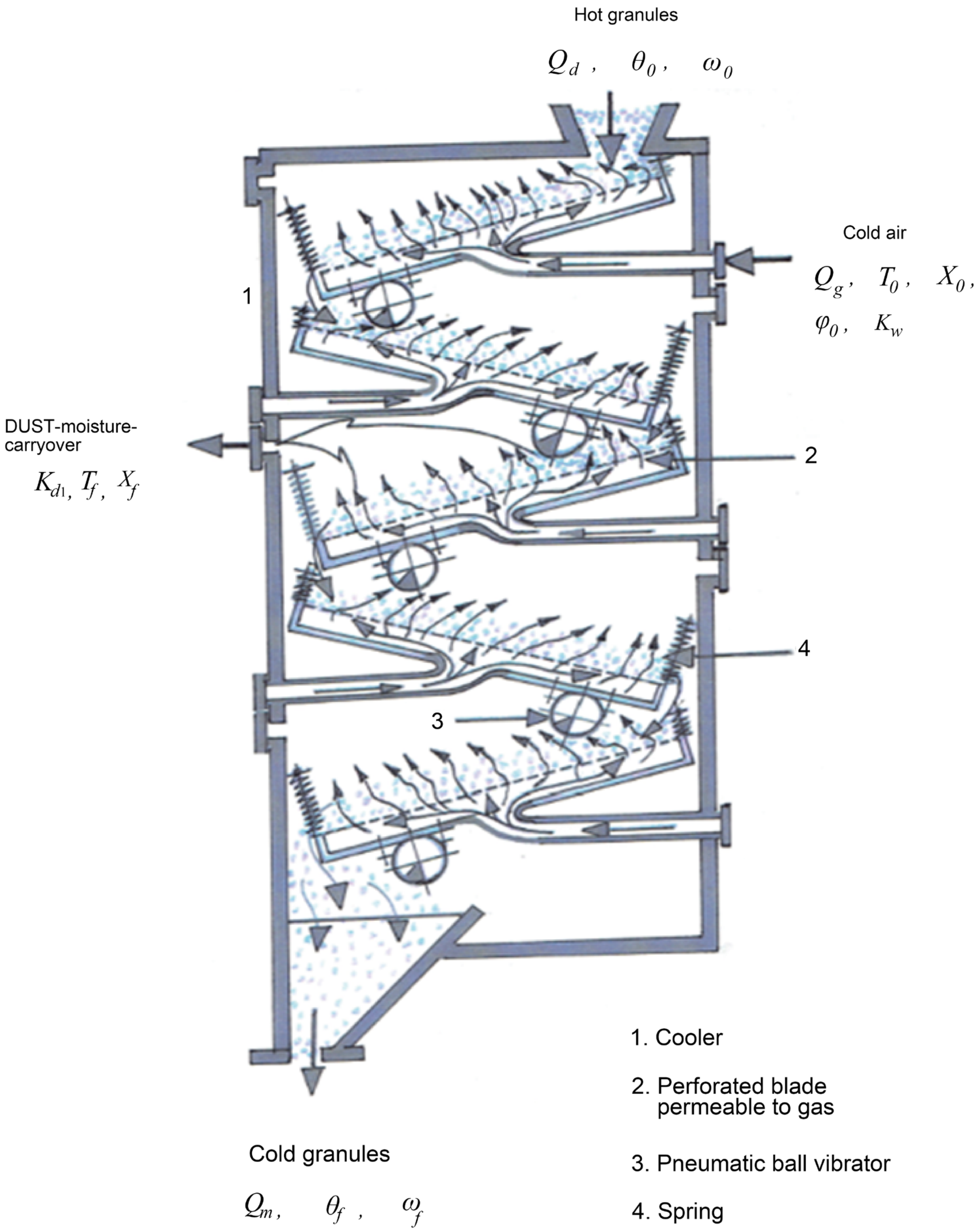

Figure 1. Cooler of granules with vibrating fluidized bed.

towards the falling granules, forming at the same time the VFB. The layer flows from one chamber of the cooler to another, thereby cooling the granules proceeds in continuous mode. In the apparatus, due to vibrations, there is an intensive heat exchange of granules with a stream of cold air and short-term contact with each other. Dust particles, together with droplets of moisture that are formed, are removed from the apparatus through openings in the side walls of the cooler (**Figure 1**). The granules are cooled to a final temperature and humidity using parameters set by users. Cooling of the granules in a vibrating fluidized bed is a complicated deterministic and stochastic process with the fluctuations of the basic physical parameters. Various mathematical models of the processes occurring in the fluidized bed can be found in reviews [4]-[6]. Various heat and mass transfer correlations for fluidized beds are widely represented in [7]. Synchronous vibrations of the blade and the regime of fluidization affect the final temperature and moisture content of granules and any other process parameters of the cooling process.

The Problems of this Report

- Problem 1: An algorithm for calculation and design of the cooler with the vibrating fluidized bed of granules.
- Problem 2: Determining the number of cooling chambers n and technological parameters which ensure cooling hot granules with a predetermined flow rate Q_d, from initial temperature θ_0 and humidity ω_0 to final values of temperature and humidity θ_f and ω_f by a mass influx of moisture into the layer κ_w and the mass flow of dust and moisture from the layer (dust-moisture-carryover) κ_{dw}.
- Problem 3: Comparatively analyzing the efficacy of the investigational cooler with the typical industrial apparatuses which is used to cool the granules in the fluidized bed.

2. Heat and Mass Balance Equations

By using the scheme of convective heat transfer [8]-[10], we write the equations of heat and material balances for continuous cooling of the granules in the cooler with the vibrating fluidized bed (**Figure 1**) in the form of:

$$\begin{aligned} &Q_g c_g \left(T_f - T_0\right) = Q_d \left\{ c_m \left(\theta_0 - \theta_f\right) + \frac{\kappa_w}{Q_d}\left[c_w \left(\theta_0 - \theta_f\right) - q_{hl} + i_{st_f} \right] - \frac{\kappa_{dw}}{Q_d}\left(c_m \theta_f - q_{hl}\right) \right\}, \\ &\kappa_w = \left(Q_m - Q_d\right) + \kappa_{dw} = Q_d \left(\frac{\omega_f - \omega_0}{1 - \omega_f} \right) = Q_g \left(X_f - X_0 \right) = \frac{\alpha_m}{r_{/T_0}} \left(T_f - T_m\right) \cdot S, \end{aligned} \tag{1}$$

where Q_g, Q_d, Q_m, respectively, the mass flow rate of gas (air), hot (dry) and moist granules, kg/s; $q_{hl} = (125.61 \div 251.22)$ kJ/kg—heat loss to the environment; $i_{st} = (2491 + 1.97 \cdot T)$ kJ/kg—enthalpy superheated steam at a temperature of air—T.

$$\begin{aligned} &r = r_0 + \left(c_{st} - c_w\right)_{/T} \cdot T, \quad r_0 = 2500.5 \text{ kJ/kg}, \\ &r = \left(597.2 - 0.545 \cdot T - 0.00038 \cdot T^2\right) \cdot 4.187 \text{ kJ/kg}, \quad \left(0 \le T \le 120^\circ\text{C}\right), \quad c_m = c_d + c_{w/T} \cdot \omega, \\ &\alpha_m = 0.15 \frac{\lambda_{g/T_0}}{d_{m0}} \text{Re}_{m0}^{0.35} \cdot \text{Ar}_{m0}^{0.25}, \quad S = \frac{\kappa_w \cdot r_{/T_0}}{\alpha_m \left(T_f - T_m\right)} = \frac{Q_d \left(\omega_f - \omega_0\right) \cdot r_{/T_0}}{\alpha_m \left(1 - \omega_f\right)\left(T_f - T_m\right)} \end{aligned} \tag{2}$$

there S, α_m—respectively, heat transfer surface and the heat transfer coefficient from the layer of hot granules to the cooling air. From Equation (1) the formulas for the final temperature θ_f and final moisture ω_f of cooling granules follows:

$$\begin{aligned} &\theta_f = \theta_0 \left[1 - \frac{\kappa_{dw} \cdot c_m}{\left(Q_d + \kappa_{dw}\right) c_m + \kappa_w c_w} \right] - \left[\frac{Q_g c_g \left(T_f - T_o\right) + Q_{hl} - \kappa_w i_{st_f}}{\left(Q_d + \kappa_{dw}\right) c_m + \kappa_w c_w} \right], \\ &\omega_f = \omega_0 + \frac{\left(1 - \omega_0\right)}{Q_d + \kappa_w}, \end{aligned} \tag{3}$$

where $Q_{hl} = q_{hl}\left(\kappa_w - \kappa_{dw}\right)$ kJ/s, heat loss in the cooler.

Method of Intervals for Calculating of Parameters

To create an algorithm for calculation of the final values θ_f, ω_f in (3) and determination of the required geometry of the device it is advisable to use the method of the intervals, considered in [11]. According to this method, parameters θ_f and ω_f can be represented as finite values of temperature θ_i and humidity ω_i of the granules at the outlet of the chambers of the cooler, *i.e.* in the form:

$$\begin{gathered}\theta_{fi} = \theta_{fi-1}\left[1 - \frac{\kappa_{dw_i} \cdot c_m}{\left(Q_d + \kappa_{dw_i}\right)c_m + \kappa_{w_i} \cdot c_w}\right] - \left[\frac{Q_g c_g \left(T_{fi} - T_0\right) + Q_{hl_i} - \kappa_{w_i} \cdot i_{st_{fi}}}{\left(Q_d + \kappa_{dw_i}\right)c_m + \kappa_{w_i} \cdot c_w}\right], \\ \omega_{fi} = \omega_{fi-1} + \frac{\left(1-\omega_{fi-1}\right)\cdot \kappa_{w_i}}{Q_d + \kappa_{w_i}}, \quad i = 1,2,3,\cdots,n \quad \theta_n = \theta_f, T_n = T_f, \omega_n = \omega_f.\end{gathered} \tag{4}$$

3. Regime of Fluidization

Mode of fluidization (by the criteria Re_{m0} and Ar_{m0}), the velocity U_g, and mass flow rate of air Q_g, passing through the gas-permeable, perforated blades towards the layer of cooled granules, we determine by empirical relationships, mentioned in [7] [8] [12]:

$$\begin{gathered}\mathrm{Re}_{m0} = 0.24\sqrt{\mathrm{Ar}_{m0}}, \quad \mathrm{Ar}_{m0} = \frac{d_{m0}^3\left(\rho_{m0} - \rho_{g/T_0}\right)g}{\left(\nu_g^2 \cdot \rho_g\right)_{/T_0}}, \quad U_g = \frac{\mathrm{Re}_{m0}\cdot \nu_{g/T_0}}{d_{m0}}, \\ Q_g = \rho_{g/T_0}\cdot U_g \cdot \sigma_1 = \kappa_w \Big/ \left(X_f - X_0\right), \quad \sigma_1 = l\delta\left(1-\varphi_{gr}\right)\end{gathered} \tag{5}$$

For the pressure drop $\Delta p_1 = p_1 - p_s$ between inlet and outlet from the enter layer of granules, and between additional side entries into the bed, $\Delta p_2 = p_2 - p_s$ exit from it (**Figure 1**). Taking into account the permeability K_{gr} and porosity ε_{gr} of grid of blades we have:

$$\Delta p_1 = \Delta p_2 + \frac{\nu_{g/T_0}}{K_{gr}}\cdot\frac{Q_1}{l\cdot\varepsilon_{gr}}, \quad Q_1 = Q_g, \quad \Delta p_2 = \frac{1}{2}\zeta_R \rho_{g/T_0}\cdot\left(\frac{Q_2}{\sigma_2}\right)^2, \quad \sigma_2 = \delta H_b\left(1-\varphi_{gr}\right) \tag{6}$$

4. Dust-Moisture-Carryover from the Fluidized Bed of Granules

Speed entrainment of dust particles and moisture from the layer of granules U_{dw} we will determine by the empirical formulas [11] [12]:

$$\begin{gathered}U_{dw} = \frac{\mathrm{Re}_{dw}\cdot\nu_{g/T_0}}{d_{dw}} = \frac{\nu_{g/T_0}}{d_{dw}}\left(\frac{\mathrm{Ar}_{dw}}{18 + 0.61\sqrt{\mathrm{Ar}_{dw}}}\right), \quad \mathrm{Ar}_{dw} = \frac{d_{dw}^3\left(\rho_{dw} - \rho_{g/T_0}\right)g}{\left(\nu_g^2\cdot\rho_g\right)_{/T_0}}, \\ \rho_{dw} \simeq \rho_{m0}, \quad d_{dw} = \sqrt[3]{\frac{6\kappa_{dw_{av}}\cdot t_{av}}{\pi\rho_{dw}\cdot n_{dw}}} \simeq d_w = \frac{2\delta_w}{\pi\rho_w U_g^2}.\end{gathered} \tag{7}$$

We imagine the mass flow of dust and moisture carryover from the layer of granules κ_{dw} as a function of the most important parameters of the process:

$$\kappa_{dw} = f\left(d_{mo}, l, \delta, d_0, A, K_{gr}, Q_g, Q_m, k, t_{av}, T_0, \theta_0, \alpha_m, \lambda_{g/T_0}\right) \tag{8}$$

Then, using π-theorem [13], function (8) can be converted to criterial dependence of the type:

$$\kappa_{dw} = \left(\frac{k\cdot A\cdot T_0\cdot K_{gr}\cdot Q_m^2\cdot t_{av}}{d_0\cdot\theta_0\cdot l\cdot\delta\cdot Q_g}\right)\cdot f\left(\frac{\alpha_m\cdot d_{m0}}{\lambda_{g/T_0}}\right) \tag{9}$$

The factor in (9), comprising a heat transfer coefficient α_m, is a specific form as shown in (2). Therefore, the calculated formula for κ_{dw} would be:

$$\kappa_{dw} = 0.15\left(\frac{k\cdot A\cdot T_0\cdot K_{gr}Q_m^2\cdot t_{av}}{d_0\cdot\theta_0\cdot l\cdot\delta\cdot Q_g}\right)\cdot \mathrm{Re}_{m0}^{0.35}\cdot \mathrm{Ar}_{m0}^{0.25} \quad (10)$$

Here t_{av} is the average time of cooling of the layer of hot granules in one chamber of the cooler.

5. Geometrical and Technological Parameters of the Cooling Process the Granules

For the geometric and technological parameters of the cooling process of the granules we will use a formula similar to that of [8].

The height of the humidified layer of granules on the blade H_b and the height of the space occupied by dust and moisture carried away from the bed H_{dw}:

$$H_b = \frac{Q_{m_{av}}\cdot t_{av}}{\rho_{mo}\cdot l\cdot\delta\cdot\varphi_{gr}},\quad H_{dw} = \frac{1.83\cdot\kappa_{dw_{av}}\cdot t_{av}}{\rho_{mo}\cdot l\cdot d_0},\quad t_{av} = \frac{H_b\cdot\rho_{m0}\cdot l\cdot\delta\cdot\varphi_{gr}}{Q_{m_{av}}} = \frac{1}{n}\sum_{i=1}^{n} t_{av_i} \quad (11)$$

The height and volume of chambers of cooler are H_s, τ_s; overall height and overall volume of the device are H_a, τ_a:

$$H_s = H_b + H_{dw} + H_v,\quad H_a = n\cdot H_s,\quad \tau_s = l\cdot\delta\left(H_b + H_{dw}\right),\quad \tau_a = l\cdot\delta\sin\varphi\cdot H_a, \quad (12)$$

where H_v is the height, occupied by the vibrator, disposed under the shoulder blade (**Figure 1**).

The average mass flow rate of cooling granules $Q_{m/av}$ and the total flow rate of the cooling air entering the device $Q_{g\Sigma}$:

$$Q_{m/av} = \frac{1}{n}\sum_{i=1}^{n} Q_{mi},\quad Q_{g\Sigma} = Q_{g/T_0}\cdot n \quad (13)$$

The average heat flow q_{av} and the total amount of heat q_Σ, transferred from the hot granules to the cold air during stay in the device $t_\Sigma = \sum_{i=1}^{n} t_{av_i}$:

$$q_{av} = \frac{1}{n}\left(Q_g\cdot c_g\right)_{/T_0}\sum_{i=1}^{n}\left(T_{fi} - T_0\right),\quad q_\Sigma = q_{av}\cdot t_\Sigma \quad (14)$$

The total mass of moisture, introduced into the layer m_w, the general m_{dw} and specific dust-moisture carry over from layer e_{dw} during cooling t_Σ:

$$m_w = \kappa_{w/av}\cdot t_\Sigma = \left(\frac{1}{n}\sum_{i=1}^{n}\kappa_{w_i}\right)\cdot t_\Sigma,\quad m_{dw} = \kappa_{dw/av}\cdot t_\Sigma = \left(\frac{1}{n}\sum_{i=1}^{n}\kappa_{dw_i}\right)\cdot t_\Sigma,\quad e_{dw} = \frac{m_{dw}}{\tau_a} \quad (15)$$

For the relationship of geometrical and physical parameters of the cooling process of granules below we propose a model of a dynamics of the vibrating, suspended bed of variable mass. The model is based on the representation VFB of cooled granules, as a system of material points of variable composition, that move with its center of mass under the influence of external and internal forces (**Figure 2**).

6. Dynamics of the Vibrating Fluidized Bed with Variable Mass: Mathematical Model

Consider the layer of granules, located above the perforated blade, and write the differential equation of motion of the mass center VFB in vector form:

$$m(t)\frac{\mathrm{d}\bar{V}_c}{\mathrm{d}t} = \sum_{k=1}^{8}\bar{F}_k - \frac{\mathrm{d}m_{dw}}{\mathrm{d}t}\left(\bar{U}_{dw} - \bar{V}_c\right) \quad (16)$$

In equation (16) the forces $\bar{F}_k$ are represented by specific formulas:

$$\bar{F}_1 = \Delta p_1\cdot s_1\cdot\bar{i}_z,\quad s_1 = \varphi_{gr}\cdot\frac{Q_{m/av}\cdot t_{av}}{\rho_{m0}\cdot H_b},\quad \bar{F}_2 = \Delta p_2\cdot s_2\cdot\bar{i}_x,\quad s_2 = \varphi_{gr}\cdot H_b\cdot\delta \quad (17)$$

$\bar{F}_1, \bar{F}_2$ are, respectively, the pressure force of the cooling gas on the layer of granules from the gas-permeable blade and from the input side (if any).

$$\bar{F}_3 + \bar{F}_4 = m(t)\bar{g}\left(1 - \frac{\rho_{g/T_0}}{\rho_{m0}}\right) \tag{18}$$

$\bar{F}_3, \bar{F}_4$ are, respectively, the gravity and buoyancy forces.

$$\bar{F}_5 = N \cdot \bar{i}_z, \quad \bar{F}_6 = -f \cdot N \cdot \bar{i}_x \tag{19}$$

$\bar{F}_5, \bar{F}_6$ are the normal reaction of the gas-permeable blade on VFB and the force of friction on the surface of the blade with friction coefficient *f*.

$$\bar{F}_7 = -m(t) \cdot A \cdot k^2 \sin(kt) \cdot \bar{i}_z \tag{20}$$

is the power of the impact of the vibrator on the gas-permeable blade with the layer of granules on it.

$$\bar{F}_8 = -\frac{m(t)}{\rho_{m0}}\left[\varphi_{gr} \cdot \rho_{m0}\left(\bar{g} - \frac{\mathrm{d}\bar{V}_*}{\mathrm{d}t}\right) + 12\zeta_R \cdot \varphi_{gr}\left(1-\varphi_{gr}\right)^2 \cdot \frac{\mu_{g/T_0}}{d_{mo}^2}\left(\frac{1+1.5\eta}{1+\eta}\right)\left(\bar{V}_c - \bar{U}_g\right)\right] \tag{21}$$

is the resistance force layer granules, which takes into account the interaction of discrete granules with the flow of cold air [8].

$\bar{V}_* = \left(1-\varphi_{gr}\right) \cdot \bar{U}_g + \varphi_{gr} \cdot \bar{V}_c$ is the average volumetric rate of the layer of granules.

$\eta = \mu_m / \mu_{g/T_0}$ is the coefficient taking into account the rheological properties of the granules. $\left(\mu_m / \mu_{g/T_0}\right) \to \infty, \left(\rho_{g/T_0} / \rho_{m0}\right) \to 0$, coefficient of resistance ζ_R is:

$$\begin{aligned}&\zeta_R = 2\delta_1 / 3\delta_2, \quad \delta_1 = 3/2 + 5/\left(8\varphi_{gr}\right) - 1/\left(4\varphi_{gr}^{5/3}\right),\\&\delta_2 = 1 - (27/16)\left(\varphi_{gr}^{1/3} - \varphi_{gr} + \varphi_{gr}^{4/3}\right) + (1/4)\varphi_{gr}^{5/3} + (19/32)\varphi_{gr}^2 - (5/32)\varphi_{gr}^{8/3}\end{aligned} \tag{22}$$

$-\frac{\mathrm{d}m_{dw}}{\mathrm{d}t}\left(\bar{U}_{dw} - \bar{V}_c\right) = -\kappa_{dw}\left(\bar{U}_{dw} - \bar{V}_c\right)$ is the "reactive power", that depends on the mass flow of dust and moisture, carried away from the bed of granules κ_{dw}, speed entrainment particles of dust and moisture $\bar{U}_{dw} \le \bar{U}_g$ and speed of the center of mass of the layer $\bar{V}_c$. $m(t) = Q_m \cdot t$ is the variable mass layer of granules on the blade in the cooler chamber. The differential equations of motion of the mass center VFB in projections on the coordinate axes x, z are derived from (16) and are presented below.

For the axis *x*-along the lines of movement of the layer (**Figure 2**):

$$\begin{aligned}m(t)\frac{\mathrm{d}V_{cx}}{\mathrm{d}t} = {}& \Delta p_2 \cdot s_2 + m(t) g \cos\varphi\left(1 - \frac{\rho_{g/T_0}}{\rho_{m0}}\right) - f \cdot N - \frac{m(t)}{\rho_{m0}}\varphi_{gr}\rho_{m0}\left(g\cos\varphi - \varphi_{gr}\frac{\mathrm{d}V_{cx}}{\mathrm{d}t}\right)\\&+ 12\zeta_R\varphi_{gr}\left(1-\varphi_{gr}\right)^2\frac{\mu_{g/T_0}}{d_{m0}^2}\left(\frac{1+1.5\eta}{1+\eta}\right)\left(V_{cx} - U_{gx}\right)\frac{m(t)}{\rho_{m0}} - \kappa_{dw}\left(U_{dw_x} - V_{cx}\right)\end{aligned} \tag{23}$$

For the axis *z*:

$$\begin{aligned}-m(t)\frac{\mathrm{d}V_{cz}}{\mathrm{d}t} = {}& \Delta p_1 \cdot s_1 - m(t) g \sin\varphi\left(1 - \frac{\rho_{g/T_0}}{\rho_{m0}}\right) + N - m(t)k^2 A\sin(kt)\\&- \frac{m(t)}{\rho_{m0}}\left[\varphi_{gr}\rho_{m0}\left(-g\sin\varphi + \varphi_{gr}\frac{\mathrm{d}V_{cz}}{\mathrm{d}t}\right) + 12\zeta_R\varphi_{gr}\left(1-\varphi_{gr}\right)^2\frac{\mu_{g/T_0}}{d_{m0}^2}\left(\frac{1+1.5\eta}{1+\eta}\right)\left(V_{cz} - U_{gz}\right)\right]\\&- \kappa_{dw}\left(U_{dw_z} - V_{cz}\right),\end{aligned} \tag{24}$$

where $U_{gx} = \frac{Q_2}{H_b\delta\left(1-\varphi_{gr}\right)}, U_{dw_x} \simeq 0, U_{gz} \simeq U_g, U_{dw_z} \simeq U_{dw}$

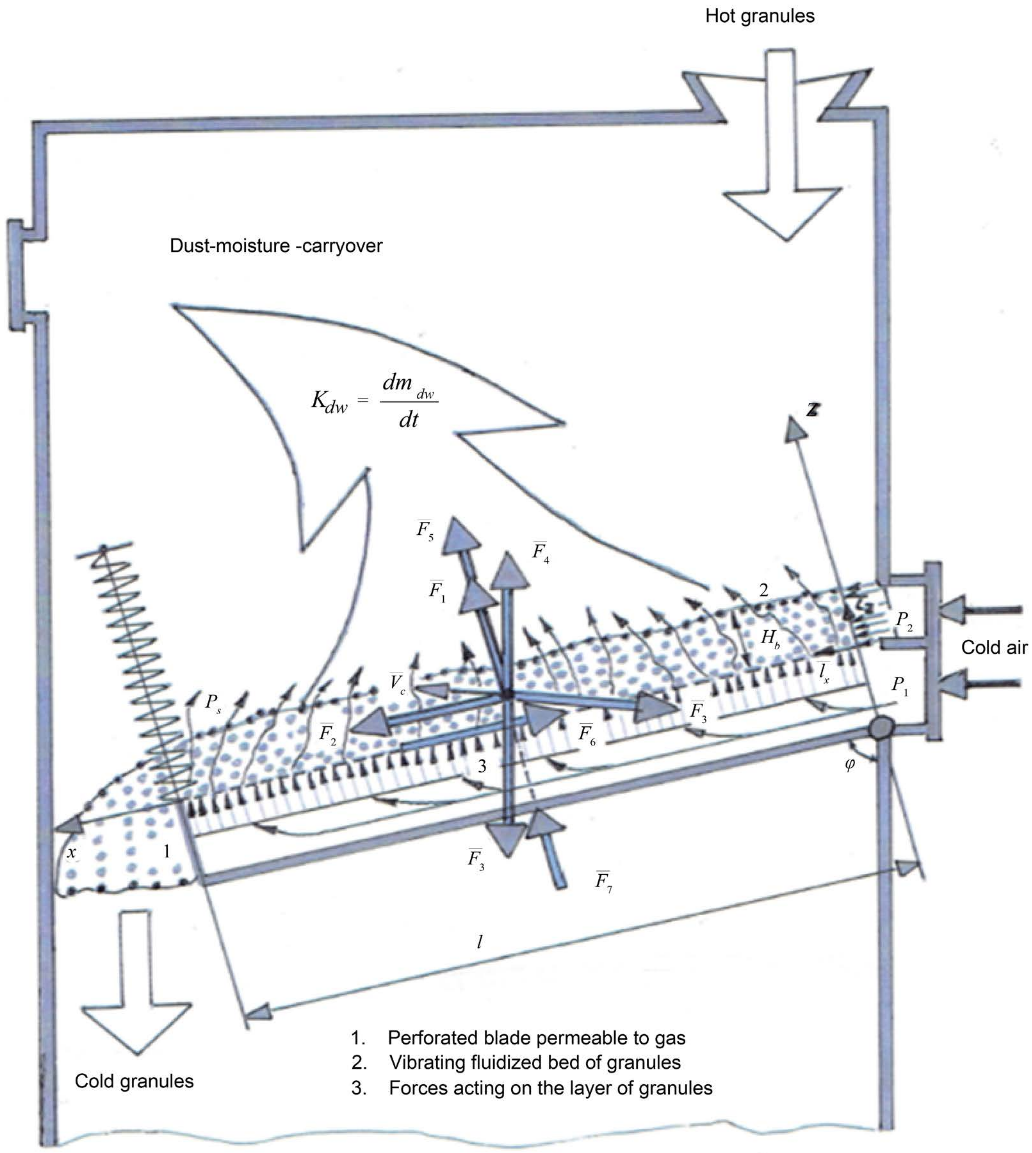

Figure 2. Dynamics of the vibrating fluidized bed with variable mass.

From Equation (24) under $V_{cz} = 0, \frac{\mathrm{d}V_{cz}}{\mathrm{d}t} = 0$, we find the normal reaction *N*:

$$N = m(t)\left[g\sin\varphi\left(1-\frac{\rho_{g/T_0}}{\rho_{m0}}\right)+k^2 A\sin(kt)\right]+\kappa_{dw}\cdot U_{dw}$$
$$-\frac{m(t)}{\rho_{m0}}\left[\varphi_{gr}\rho_{m0}g\sin\varphi+12\zeta_R\varphi_{gr}\left(1-\varphi_{gr}\right)^2\frac{\mu_{g/T_0}}{d_{m0}^2}\left(\frac{1+1.5\eta}{1+\eta}\right)\cdot U_g\right]-\Delta p_1\cdot s_1 \tag{25}$$

Equation (23) after the substitution of (25) can be represented in the form of a linear differential equation:

$$\frac{\mathrm{d}V_{cx}}{\mathrm{d}t}+V_{cx}\cdot P(t)=q(t),\ P(t)=a_1-a_2/t,\quad q(t)=b_1/t-b_2\sin(kt)+b_3, \tag{26}$$

$$
\begin{aligned}
&a_1=\frac{12\zeta_R\varphi_{gr}\left(1-\varphi_{gr}\right)^2\mu_{g/T_0}}{\rho_{m0}d_{m0}^2\left(1-\varphi_{gr}^2\right)}\left(\frac{1+1.5\eta}{1+\eta}\right),\quad a_2=\frac{\kappa_{dw_{av}}}{\left(1-\varphi_{gr}^2\right)Q_{m_{av}}},\\
&b_1=\frac{\Delta p_2\cdot s_2+f\left(\Delta p_1\cdot s_1-\kappa_{dw_{av}}\cdot U_{dw}\right)}{\left(1-\varphi_{gr}^2\right)\cdot Q_{m_{av}}},\quad b_2=\frac{f\cdot A\cdot k^2}{\left(1-\varphi_{gr}^2\right)},\\
&b_3=\frac{1}{\left(1-\varphi_{gr}^2\right)}\left[g\left(1-\frac{\rho_{g/T_0}}{\rho_{m0}}\right)\left(\cos\varphi-f\sin\varphi\right)\right]\\
&\quad+a_1\left(U_{gx}-f\cdot U_g\right)-\frac{\varphi_{gr}g}{\left(1-\varphi_{gr}^2\right)}\left(\cos\varphi+f\sin\varphi\right)
\end{aligned}
\tag{27}
$$

Solution of differential Equation (26) for the velocity of the center of mass V_{cx} and the coordinate of mass center of layer x_c have the form:

$$
\begin{aligned}
&V_{cx}=\frac{1}{\Psi(t)}\left[\int q(t)\cdot\Psi(t)\mathrm{d}t+V_{cx_0}\right],\quad \Psi(t)=\exp\int P(t)\mathrm{d}t,\\
&x_c=\int V_{cx}\mathrm{d}t+x_{c0},\quad V_{cx_0}\simeq\frac{Q_{m_{av}}}{\rho_{mo}\cdot s_2}=V_{av}=\frac{l}{t_{av}},\quad x_c-x_{c0}=l,\quad t=t_c,
\end{aligned}
\tag{28}
$$

where $V_{av}=\left(\sum_{i=1}^{n}V_{av_i}\right)\Big/n$ is the average speed of movement of the granules along the blade.

From (28) we obtain the equation to determine the residence time of the granules in the layer on the blade in a chamber of the cooler t_c:

$$
\begin{aligned}
&c_1\cdot t_c^{a_2+1}+c_2\cdot t_c^{a_2}+c_3\cdot t_c^{a_2}\left(1-\mathrm{e}^{-a_1t_c}\right)=l,\\
&c_1=\left[\frac{b_2\cdot k}{a_1^2+k^2}+\xi_t\frac{b_3\cdot l\cdot a_2^2}{\delta\cdot a_1\left(a_2+1\right)\varphi_{gr}}\right],\quad c_2=\frac{b_2\cdot a_1}{\left(a_1^2+k^2\right)\cdot k},\quad c_3=\frac{V_{cx_0}}{a_1}
\end{aligned}
\tag{29}
$$

$\xi_t\simeq1.1$ is the proportionality factor, which is dependent on the accuracy of integration of Equation (28). Equation (29) for cooling time of the granules in the cooling chambers t_c combines in a simple dependence the geometrical and physical parameters. The value for the time of cooling t_c, obtained from the Equation (29), can be compared with the value t_{av} from Equation (11), which uses the initial and averaged data.

Below is a procedure-algorithm for calculating a vertical rectangular cooler with VFB of granules (**Figure 3**).

7. The Algorithm for Calculating the Cooler with Vibrating Fluidized Bed of Granules

The algorithm is based on the above formulas and is reduced to determining the geometrical and technological parameters of the cooler, which provide the specified mode of cooling the granules and dust-moisture carryover from the VFB.

Tables 1-3, referenced in the algorithm, contain the initial data: geometry of permeable to gas, vibrated blade and layer of granules on it; kinematic, heat and mass transfer parameters, involved in the calculations, and the critical parameters-limitations, which are dependent on the requirements by the consumers. One example is cooling of granules of zeolite introduced specific numerical values of design parameters.

Table 4 gives the change cycle of temperature θ_{fi} and humidity of granules ω_{fi} to the required values, allowing definition of the necessary geometry of the cooler. The parameters of the cooling air $X_{fi}=X_{/T_{fi}}$ are determined from tables or $I-X$ diagrams shown, for example, in [14] [15]. The physical parameters- $\kappa_{w_i},\kappa_{dw_i},V_{av_i},t_{av_i},Q_{mi},Q_{hl_i},i_{st_{fi}},\omega_{fi},\theta_{fi}$ are given by formulas: (1), (3), (4), (10), (11) and (28).

Table 5 and **Table 6** show numerical values of the geometrical and technological parameters of the cooling process the granules of zeolite in the vertical, four - chambers cooler with VFB, selected according to the results of calculation.

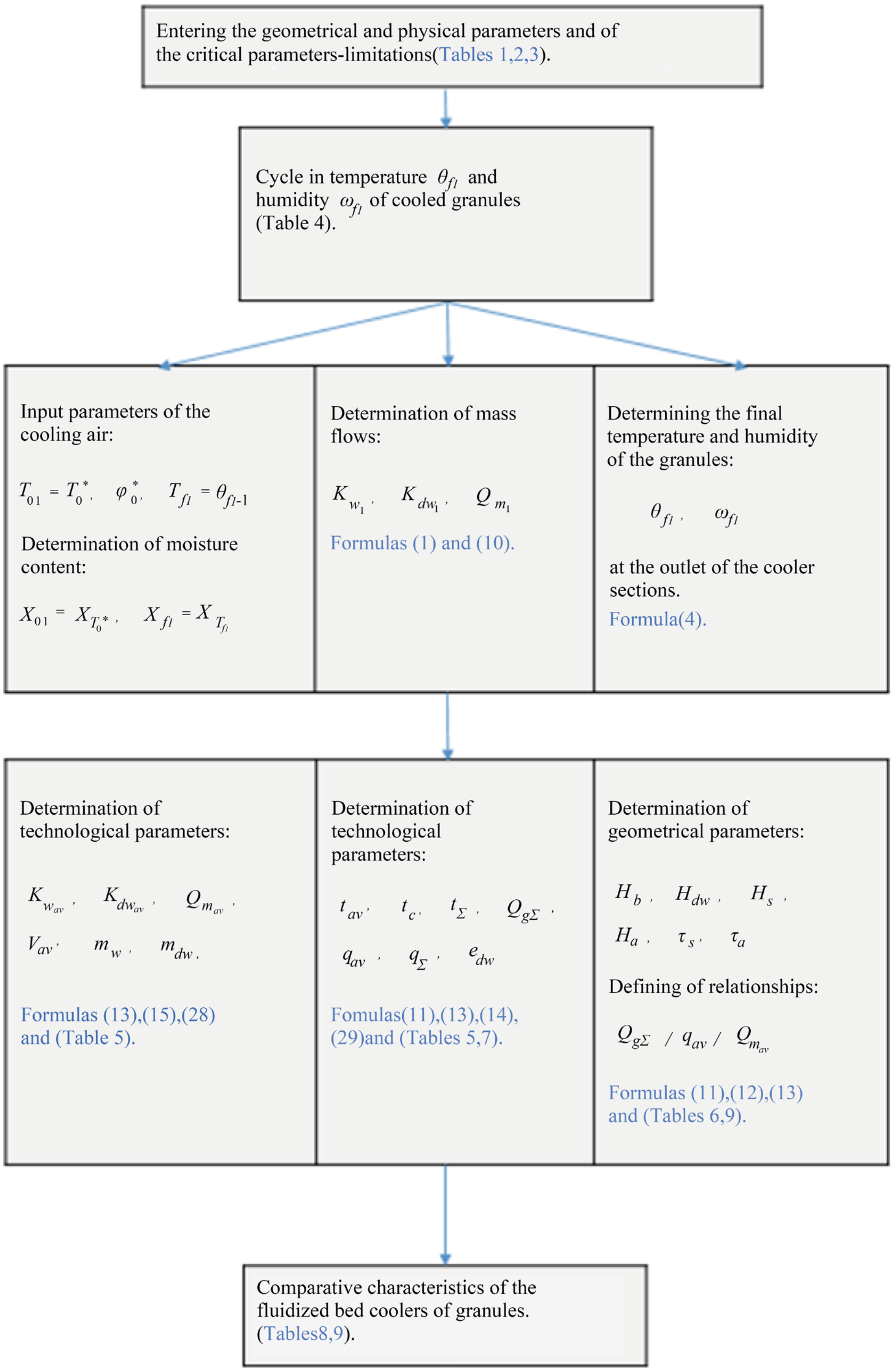

Figure 3. The algorithm for calculating the geometrical and technological parameters of a vertical cooler with a vibrating, fluidized bed of granules.

Table 7 gives the numerical values of the coefficients in Equation (29), designed to determine the residence time of the granules in the cooling chamber. Equation (29) follows from the mathematical model of motion of the layer of granules along the vibrating blade.

Table 8 includes the raw data and critical parameters for comparison of typical industrial coolers with the fluidized bed of granules [1] [3] [16].

Table 1. Geometrical parameters used in the algorithm.

The geometrical parameters of a gas-permeable blade and layer of granules on it		
Symbols	**Values**	**Units**
l	0.45	m
δ	0.15	m
H_b	0.01	m
φ	78	degree
ε_{gr}	0.2	dimensionless
K_{gr}	5×10^{-10}	m^2
d_0	3.75×10^{-5}	m
H_s	0.09	m
k	208	s^{-1}
A	1.2×10^{-3}	m
d_{m0}	2.675×10^{-3}	m
φ_{gr}	0.6	dimensionless
s_1	0.0242	m^2
s_2	9×10^{-4}	m^2
σ_1	0.027	m^2
σ_2	6×10^{-4}	m^2
$d_w = d_{dw}$	1.68×10^{-4}	m

Table 2. Physical parameters used in the algorithm.

Initial physical parameters		
Symbols	**Values**	**Units**
ζ_R	75.2	dimensionless
ρ_{m0}	2100	kg/m^3
f	0.2	dimensionless
g	9.81	m/s^2
δ_m	0.7275	N/m
Q_2	0	Kg/s
Δp_2	0	Pa
U_{g0}	0	m/s
Δp_1	16,948	Pa
ρ_{g/T_0}	1.15	Kg/m^3
μ_{g/T_0}	1.7×10^{-5}	Kg/m·s
ν_{g/T_0}	1.48×10^{-5}	m^2/s
c_{g/T_0}	0.962	kJ/kg·˚C
λ_{g/T_0}	2.45×10^{-5}	kW/m·˚C
c_m	1	kJ/kg·˚C
c_w	4.2	kJ/kg·˚C
q_{hl}	125.61	kJ/kg
Ar_{m0}	1,568,827	dimensionless
Re_{m0}	300.6	dimensionless
U_g	1.66	m/s
Q_g	0.0516	kg/s
Ar_{dw}	388.6	dimensionless
U_{dw}	1.14	m/s

Table 3. Critical parameters—limitations used in the algorithm.

Critical parameters—limitations		
Symbols	**Values**	**Units**
$Q_{m0} = Q_d^*$	250	kg/h
θ_0^*	110	˚C
θ_f^*	45	˚C
ω_0^*	0.01	kg/kg
T_0^*	30	˚C
φ_0^*	60	%
$k_{dw}^* \leq 0.01 \cdot Q_d^*$	2.5	kg/h

Table 4. Cycle in temperature θ_{fi} and humidity ω_{fi} of cooled granules.

Chamber 1			Chamber 2		
Symbols	**Values**	**Units**	**Symbols**	**Values**	**Units**
φ_{01}	60	%	φ_{02}	60	%
φ_{f1}	0	%	φ_{f2}	5	%
$T_{f1} = \theta_0$	110	˚C	$T_{f2} = \theta_{f1}$	87.345	˚C
$X_{01} = X_{T_0^*}$	0.0154	kg/kg	$X_{02} = X_{T_0^*}$	0.0154	kg/kg
$X_{f1} = X_{T_{f1}}$	0.0326	kg/kg	$X_{f2} = X_{T2}$	0.02017	kg/kg
k_{w_1}	0.89×10^{-3}	kg/s	k_{w_2}	0.25×10^{-3}	kg/s
V_{av_1}	0.0367	m/s	V_{av_2}	0.0369	m/s
t_{av_1}	12.255	s	t_{av_2}	12.204	s
k_{dw_1}	0.60×10^{-3}	kg/s	k_{dw_2}	0.76×10^{-3}	kg/s
Q_{m1}	0.0697	kg/s	Q_{m2}	0.0689	kg/s
Q_{hl_1}	0.03592	kJ/s	Q_{hl_2}	−0.0646	kJ/s
$i_{st_{f1}}$	2707.7	kJ/kg	$i_{st_{f2}}$	2663	kJ/kg
ω_{f1}	0.0225	kg/kg	ω_{f2}	0.026	kg/kg
θ_{f1}	87.345	˚C	θ_{f2}	56.54	˚C
Chamber 3			**Chamber 4**		
Symbols	**Values**	**Units**	**Symbols**	**Values**	**Units**
φ_{03}	60	%	φ_{04}	60	%
φ_{f3}	20	%	φ_{f4}	30	%
$T_{f3} = \theta_{f2}$	56.54	˚C	$T_{f4} = \theta_{f3}$	49.79	˚C
$X_{03} = X_{T_0^*}$	0.0154	kg/kg	$X_{04} = X_{T_0^*}$	0.0154	kg/kg
$X_{f3} = X_{T_{f3}}$	0.02129	kg/kg	$X_{f4} = X_{T_{f4}}$	0.02198	kg/kg
k_{w_3}	0.30×10^{-3}	kg/s	k_{w_4}	0.34×10^{-3}	kg/s
V_{av_3}	0.0364	m/s	V_{av_4}	0.0362	m/s
t_{av_3}	12.347	s	t_{av_4}	12.416	s
k_{dw_3}	1.16×10^{-3}	kg/s	k_{dw_4}	1.31×10^{-3}	kg/s
Q_{m3}	0.0685	kg/s	Q_{m4}	0.0684	kg/s
Q_{hl_3}	−0.10775	kJ/s	Q_{hl_4}	−0.12144	kJ/s
$i_{st_{f3}}$	2602.4	kJ/kg	$i_{st_{f4}}$	2589.1	kJ/kg
ω_{f3}	0.03024	kg/kg	ω_{f4}	0.035	kg/kg
θ_{f3}	49.79	˚C	θ_{f4}	42.1	˚C

Table 5. The process parameters for cooling granules.

Technological parameters		
Symbols	**Values**	**Units**
$k_{w_{av}}$	0.44×10^{-3}	kg/s
$k_{dw_{av}}$	0.96×10^{-3}	kg/s
V_{av}	0.0366	m/s
m_w	0.022	kg
m_{dw}	0.047	kg
m_m / m_{dw}	46.4	%
t_{av}	12.3	s
t_Σ	49.2	s
$Q_{g\Sigma}$	646	m^3/h
q_{av}	8662	kJ/h
$Q_{m_{av}}$	248	kg/h
q_Σ	118.4	kJ
e_{dw}	0.252	kg/m^3

Table 6. The geometrical parameters of the cooling process the granules.

Geometrical parameters		
Symbols	**Values**	**Units**
H_b	0.01	m
H_{dw}	0.608	m
H_s	0.708	m
H_a	2.832	m
τ_s	0.0408	m^3
τ_a	0.187	m^3

Table 7. The residence time of the granules in the cooling chamber.

Coefficients of Equation (29)		
Symbols	**Values**	**Units**
a_1	0.1532	s^{-1}
a_2	0.0217	dimensionless
b_1	1860.6	m/s
b_2	16.224	m/s^2
b_3	−3.574	m/s^2
c_1	0.0188	m/s
c_2	2.76×10^{-7}	m
c_3	0.239	m
t_c	12	s

Table 8. Baseline and critical values for comparison of the coolers with the fluidized bed of granules.

Type of devices, the starting materials				
Type 1—Tower rectangular shape. Material-Ammonium nitrate.		Type 2—Vertical rectangular unit. Material-Urea granules.	Type 3—The test apparatus (Figure 1). Material-Granules of zeolite.	
Symbols	Values	Values	Values	Units
τ_a	11 × 8 × 15	4.65 × 4.65 × 1.5	0.4 × 0.15 × 2.83	m3
d_{gr}	(1/5) × 10^{-3}	(1/4) × 10^{-3}	2.675 × 10^{-3}	m
φ_0	≥60	60	60	%
T_0	25/30	28/30	30	˚C
H_b	0.1/0.15	0.15	0.01	m
θ_0^*	80/110	70/80	110	˚C
θ_f^*	40/50	40/50	42	˚C
n	13.7	8.35	4	-
k_{dw}^*			2.5	kg/h

Table 9 shows the comparative characteristics of the tested vertical, four-chamber device and typical industrial coolers with the fluidized bed of granules (**Type 1** and **Type 2**, **Table 8**). For comparison, it has taken critical process parameters of the cooling process, such as: $t_\Sigma, \kappa_{dw}, Q_{g\Sigma}, q_{av}, Q_{m_{av}}, Q_{g\Sigma}/Q_{m_{av}}, q_{av}/Q_{m_{av}}$.

8. Conclusions

In order to implement the given mode of the cooling granules that takes into account dust-moisture carryover, the most effective shape is the vertical device, with a rectangular shape with sloping, gas-permeable blades and with the vibrating fluidized bed of granules on them (**Table 9**). In the example of cooling the zeolite granules from $\theta_0 = 110^\circ\mathrm{C}$ to $\theta_f = 42^\circ\mathrm{C}$ at the mass flow of granules $Q_{m_{av}} = 248\ \mathrm{kg/h}$ the number of cooling chambers is equal to $n = 4$ (**Table 4**); thus, the cooling time of granules is equal to $t_\Sigma = 0.82\ \mathrm{min}$ (**Table 5** and **Table 9**).

The proposed mathematical model of the dynamics of a vibrating, suspended layer, allows us to estimate the effect of geometrical and physical parameters on the time of cooling the granules in the cooling chamber t_c. This is accomplished by varying the coefficients $C_i\,(i = 1,2,3)$ in Equation (29). In particular, the time t_c decisively influences the coefficients C_1, C_3, associated with the mass flows $Q_{m_{av}}, \kappa_{dw_{av}}$ and vibration parameters k, A (**Table 7**).

A comparative analysis of the technological parameters of the study device (**Figure 1**) and typical representatives of industrial devices (**Table 8** and **Table 9**) was made. It was found that by all most important process parameters: the cooling time $t_\Sigma = 0.82\ \mathrm{min}$, the specific flow rate of air and heat flow: $Q_{g\Sigma}/Q_{m_{av}} = 2.6\ \mathrm{m^3/kg}$, the considered cooler (**Figure 1**) with the vibrating, and suspended bed of granules were the most effective.

The proposed algorithm for calculation of coolers with the VFB is based on the joint use of the equations of heat and mass balances (1) and the mathematical model of dynamics of a vibrating fluidized bed (Equation (16) and Equation (29)).

The algorithm takes into account the massive influx of moisture into the layer of granules κ_w and mass flow of dust and moisture from the layer κ_{dw} and makes it possible to solve design problems of cooling heat-stable and thermally unstable materials.

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Table 9. Process parameters for comparison of the coolers with the fluidized bed of granules.

Type of devices				
Type 1—Tower rectangular shape.		Type 2—Vertical rectangular unit.	Type 3—The test apparatus	
Symbol	**Values**	**Values**	**Values**	**Units**
t_{Σ}	2/3	4/5	0.82	Min
$Q_{g\Sigma}$	(75/80) × 10^3	(22/84) × 10^3	646	m^3/h
q_{av}	2 × 10^6	0.6 × 10^6	8662	kJ/h
$Q_{m_{av}}$	6864	4176	248	kg/h
$Q_{g\Sigma}/Q_{m_{av}}$	11.3	5.6	2.6	m^3/kg
$q_{av}/Q_{m_{av}}$	291	144	35	kJ/kg

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Notations

A : oscillations amplitude of vibrator, m; c : specific heat capacity, kJ/kg C; d : diameter of granules, dust particles and moisture, m; d_0 : diameter of grid holes, m; g : acceleration of gravity, m/s^2; K_{gr} : permeability of grid blade, m^2; k : oscillations frequency of vibrator, Hz; l : length of blades, m; m : mass of granules, m; n : number of cooling chambers; n_{dw} : number particles of dust and moisture; p : pressure of cooling gas, Pa; Δp : pressure drop of the heat carrier (air), Pa; Q : mass flow rate, kg/s, kg/h; $Q_{g\Sigma}$: total flow rate of gas, m^3/h; q : heat flow, kJ/h; r : heat of evaporation, kJ/kg; S : heat exchange surface, m^2; s : sectional area of the granules layer, m^2; t : current cooling time, s, min; T : temperature of the heat carrier (air), C; U_g : velocity of the cooling gas (air), m/s; U_{dw} : velocity of the dust-moisture-carryover, m/s; V : velocity of the mass center of bed of granules, m/s; VFB : vibrating, fluidized bed; X : air humidity, kg of moisture to kg of dry air; x, z : coordinates, m.

Greek symbols: α_m : heat transfer coefficient, kW/m^2·C; δ : width of the blade, m; δ_w : surface tension, N/m; ε_{gr} : porosity of grid; θ : temperature of granules, C; λ : coefficient of the thermal conductivity, kW/m C; μ : dynamic coefficient of viscosity, Pa·s; ν : kinematic coefficient of viscosity, m^2/s; ρ : density, kg/m^3; σ : area of " live " section of bed of granules, m^2; φ_{gr} : concentration of granules in the bed; φ : the blade angle, degree; relative humidity of air, %; ω : absolute humidity of granules, kg of moisture to total weight.

Superscript: $*$: Critical values-limitations.

Subscripts: $1, 2$: gas permeable blade, lateral injection of gas (air); a : apparatus; av : average value; b : bed of granules; c : center of mass, cooling; d : dry (hot) granules; dw : dust-moisture-carryover; g : gas (air); gr : granules, grid; hl : heat loss in the cooler; m : material (granules), wet thermometer; s : sections (chambers) of cooler; st : superheated steam; w : water, humidity; o, f : initial and end values; $\sum$: sum value; $\overline{i_x}, \overline{i_z}$: unit vectors of the axes x, z .

Dimensionless groups-Criteria: Re : Reynolds number; Ar : Archimedes number.