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Solution of the Fuzzy Equation *A* + *X* = *B* Using the Method of Superimposition

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Abstract

Fuzzy equations were solved by using different standard methods. One of the well-known methods is the method of α-cut. The method of superimposition of sets has been used to define arithmetic operations of fuzzy numbers. In this article, it has been shown that the fuzzy equation $A + X = B$, where *A*, *X*, *B* are fuzzy numbers can be solved by using the method of superimposition of sets. It has also been shown that the method gives same result as the method of α-cut.

Keywords: Fuzzy Number, Possibility Distribution, Probability Distribution, Survival Function, Superimposition of Sets, Superimposition of Intervals, α-Cut Method

1. Introduction

Fuzzy equations were investigated by Dubois and Prade [1]. Sanchez [2] put forward a solution of fuzzy equation by using extended operations. Accordingly various researchers have proposed different methods for solving the fuzzy equations [see e.g. Buckley [3], Wasowski [4], Biacino and Lettieri [5]. After this a lot research papers have appeared proposing solutions of various types of fuzzy equations viz. algebraic fuzzy equations, a system of fuzzy linear equations, simultaneous linear equations with fuzzy coefficients etc. using different methods ([see e.g. Jiang [6], Buckley and Qu [7], Kawaguchi and Da-Te [8], Zhao and Gobind [9], Wang and Ha [10]). Klir and Yuan [11] solved the fuzzy equations $A + X = B$ where *A*, *X* and *B* are fuzzy numbers, by using the method of α-cut.

Mazarbhuiya *et al.* [12] defined the arithmetic operations viz. addition and subtraction of fuzzy numbers with out using the method of α-cuts *i.e.* using a method called superimposition of sets introduced by Baruah [13].

In this article, we would put forward a procedure of solving a fuzzy equation $A + X = B$ without utilising the standard methods. Our method is based on the operation of superimposition of sets. It will be shown in this article that our method for the solution of equation $A + X = B$ gives same result as given by the method of α-cut.

The paper is organised as follows. In Section 2 we discuss about the definitions and notations used in this article. In Section 3, we discuss the solution of fuzzy equation by α-cut method. In Section 4, we discuss about equi-fuzzy interval arithmetic. In Section 5, we discuss our proposed method of solution $A + X = B$. In Section 6, we give brief conclusion of the work and lines for future work.

2. Definitions and Notations

We first review certain standard definitions.

Let *E* be a set, and let *x* be an element in *E*. Then a fuzzy subset *A* of *E* is characterized by

$$A = \{x, A(x); x \in E\}$$

where $A(x)$ is the grade of membership of *x* in *A*. $A(x)$ is commonly called the fuzzy membership function of the fuzzy set *A*. For an ordinary set $A(x)$ is either 0 or 1, while for a fuzzy set $A(x) \in [0,1]$. A fuzzy set *A* is said be normal if its membership function $A(x)$ is unity for at least one $x \in E$. An α-cut ${}^{\alpha}A$ of a fuzzy set *A* is an ordinary set of elements with membership not less than α for $0 \le \alpha \le 1$. This means

$${}^{\alpha}A = \{x \in E; A(x) \ge \alpha\}$$

A fuzzy set is said to be convex if all its α-cuts are convex sets (see *e.g.* [14]). A fuzzy number is a convex normal fuzzy set A defined on the real line such that $A(x)$ is piecewise continuous.

The support of a fuzzy set A is denoted by sup $p(A)$ and is defined as the set of elements with membership nonzero *i.e.*,

$$\sup p(A) = \{x \in E; A(x) > 0\}$$

A fuzzy number A, denoted by a triad $[a,b,c]$ such that $A(a) = 0 = A(c)$ and $A(b) = 1$, where $A(x)$ for $x \in [a,b]$ is called the left reference function and for $x \in [b,c]$ is called right reference function. The left reference function is right continuous monotone and non-decreasing where as the right reference function is left continuous, monotone and non-increasing. The above definition of a fuzzy number is called L-R fuzzy number [15].

We would call a fuzzy set $A^{(\lambda)}$ over the support A equi-fuzzy if all elements of $A^{(\lambda)}$ are with membership λ where $0 \le \lambda \le 1$. The operation of superimposition S of equi-fuzzy sets $A^{(\lambda)}$ and $B^{(\mu)}$ is defined as [13]

$$A^{(\lambda)} S B^{(\mu)} = (A - A \cap B)^{(\lambda)} + (A \cap B)^{(\lambda+\mu)} + (B - A \cap B)^{(\mu)}$$

where $\lambda, \mu \ge 0$, $\lambda + \mu \le 1$ and the operation '+' stands for union of disjoint sets, fuzzy or otherwise.

The arithmetic operation using the method of α-cut on two fuzzy numbers A and B is defined by the formula

$$^{\alpha}(A * B) = {}^{\alpha}A * {}^{\alpha}B$$

where $^{\alpha}A$, $^{\alpha}B$ are α-cuts of A and B, $\alpha \in (0,1]$ and * is the arithmetic operation on A and B. In the case of division $0 \notin {}^{\alpha}B$ for any $\alpha \in (0,1]$. The resulting fuzzy number $A * B$ is expressed as

$$A * B = \cup^{\alpha} (A * B) \cdot \alpha \quad \text{(see e.g.[11])} \tag{1}$$

3. Solution of the Fuzzy Equation $A + X = B$ by Using the Method of α-Cut

For any $\alpha \in (0,1]$. Let $^{\alpha}A = [{}^{\alpha}a_1, {}^{\alpha}a_2]$, $^{\alpha}B = [{}^{\alpha}b_1, {}^{\alpha}b_2]$ and $^{\alpha}X = [{}^{\alpha}x_1, {}^{\alpha}x_2]$ denote, respectively, the α-cuts of A, B and X in the given equation (see e.g. Klir and Yuan [11]). Then the given equation has a solution if an only if

1) $^{\alpha}b_1 - {}^{\alpha}a_1 = {}^{\alpha}b_2 - {}^{\alpha}a_2$ for every $\alpha \in (0,1]$ and
2) $\alpha \le \beta \Rightarrow$

$$^{\alpha}b_1 - {}^{\alpha}a_1 \le {}^{\beta}b_1 - {}^{\beta}a_1 \le {}^{\beta}b_2 - {}^{\beta}a_2 \le {}^{\alpha}b_2 - {}^{\alpha}a_2$$

Property 1) ensures that the interval equation

$$^{\alpha}A + {}^{\alpha}X = {}^{\alpha}B$$

has a solution, which is $^{\alpha}X = [{}^{\alpha}b_1 - {}^{\alpha}a_1, {}^{\alpha}b_2 - {}^{\alpha}a_2]$.

Property 2) ensures that the solution of the interval equations for α and β are nested *i.e.* if $\alpha \le \beta$ then $^{\alpha}X \subseteq {}^{\beta}X$. if a solution $^{\alpha}X$ exists for every $\alpha \in (0,1]$ and property 2) is satisfied, then by (2.1) the solution X of the fuzzy equation is

$$X = \bigcup_{\alpha \in [0,1]} {}_{\alpha}X \tag{2}$$

where $_{\alpha}X(x) = \alpha . {}^{\alpha}X(x)$

4. Equi-Fuzzy Interval Arithmetic

The usual interval arithmetic can be generalized for equi-fuzzy intervals. If $A = [a_1, b_1]$ and $B = [a_2, b_2]$, we denote interval addition and interval subtraction as

$$A(+)B = [a_1 + a_2, b_1 + b_2]$$

and $A(-)B = [a_1 - b_2, a_2 - b_1]$

Accordingly,

$$A^{(\lambda)}(+)B^{(\lambda)} = [a_1 + a_2, b_1 + b_2]^{(\lambda)}$$

$$A^{(\lambda)}(-)B^{(\lambda)} = [a_1 - b_2, a_2 - b_1]^{(\lambda)}$$

Let now, $\alpha_{(1)}$, $\alpha_{(2)}$ be the ordered values of α_1, α_2 in ascending magnitude, Then

$$\begin{aligned}
&\{[a_1, b_1]^{(1/2)} S [a_2, b_2]^{(1/2)} (+) [c_1, d_1]^{(1/2)} S [c_2, d_2]^{(1/2)} \\
&= \left[[a_{(1)}, a_{(2)}]^{(1/2)} + [a_{(2)}, b_{(1)}]^{(1)} + [b_{(1)}, b_{(2)}]^{(1/2)} \right] \\
&(+) \left[[c_{(1)}, c_{(2)}]^{(1/2)} + [c_{(2)}, d_{(1)}]^{(1)} + [d_{(1)}, d_{(2)}]^{(1/2)} \right] \\
&= [a_{(1)} + c_{(1)}, a_{(2)} + c_{(2)}]^{(1/2)} + [a_{(2)} + c_{(2)}, b_{(1)} + d_{(1)}]^{(1)} \\
&+ [b_{(1)} + d_{(1)}, b_{(2)} + d_{(2)}]^{(1/2)}
\end{aligned} \tag{3}$$

where $\bigcap_{i=1}^{2} [a_i, b_i] \ne \phi$, $\bigcap_{i=1}^{2} [c_i, d_i] \ne \phi$

Similarly,

$$\begin{aligned}
&\{[a_1, b_1]^{(1/2)} S [a_2, b_2]^{(1/2)} (-) [c_1, d_1]^{(1/2)} S [c_2, d_2]^{(1/2)} \\
&= \left[[a_{(1)}, a_{(2)}]^{(1/2)} + [a_{(2)}, b_{(1)}]^{(1)} + [b_{(1)}, b_{(2)}]^{(1/2)} \right] \\
&(-) \left[[c_{(1)}, c_{(2)}]^{(1/2)} + [c_{(2)}, d_{(1)}]^{(1)} + [d_{(1)}, d_{(2)}]^{(1/2)} \right] \\
&= [a_{(1)} - d_{(2)}, a_{(2)} - d_{(1)}]^{(1/2)} + [a_{(2)} - d_{(1)}, b_{(1)} - c_{(2)}]^{(1)} \\
&+ [b_{(1)} - c_{(2)}, b_{(2)} - c_{(1)}]^{(1/2)}
\end{aligned} \tag{4}$$

In the next section, we shall use (3) and (4) to find the solution X of the fuzzy equation $A+X=B$.

5. Solution of the Fuzzy Equation $A+X=B$ by Using the Method of Superimposition

Let $a_1, a_2, \cdots, a_n$ are sample realisations from the uniform population $[u_1, v_1]$ and $b_1, b_2, \cdots, b_n$ are sample realisations from the uniform population $[v_1, w_1]$.

We denote $G(a,b)$ as the superimpositions of equi-fuzzy intervals $[a_i, b_i]$; $i=1,2,\cdots,n$ with membership (1/*n*) *i.e.*

$$\begin{aligned} G(a,b) &= [a_1,b_1]^{(1/n)} S [a_2,b_2]^{(1/n)} S \cdots S [a_n,b_n]^{(1/n)} \\ &= \left[a_{(1)}, a_{(2)}\right]^{(1/n)} + \left[a_{(2)}, a_{(3)}\right]^{(2/n)} + \cdots \\ &+ \left[a_{(n-1)}, a_{(n)}\right]^{((n-1)/n)} + \left[a_{(n)}, b_{(1)}\right]^{(1)} \\ &+ \left[b_{(1)}, b_{(2)}\right]^{(1-1/n)} + \ldots + \left[b_{(n-2)}, b_{(n-1)}\right]^{(2/n)} \\ &+ \left[b_{(n-1)}, b_{(n)}\right]^{(1/n)} = H(a,b) \quad \text{(say)} \end{aligned} \tag{5}$$

where $a_{(1)}, a_{(2)}, \cdots, a_{(n)}$ are ordered values of $a_1, a_2, \cdots, a_n$ and $b_{(1)}, b_{(2)}, \cdots, b_{(n)}$ are ordered values of $b_1, b_2, \cdots, b_n$ in ascending magnitude.

Here $\bigcap_{i=1}^{n}[a_i, b_i] \neq \varphi$

From (5), we get the membership functions are the combination of empirical probability distribution function and complementary probability distribution function respectively as

$$\Phi_1(x) = \begin{cases} 0, & x < a_{(1)} \\ \dfrac{r-1}{n}, & a_{(r-1)} \le x \le a_{(r)} \\ 1, & x > a_{(n)} \end{cases}$$

and

$$\Phi_2(x) = \begin{cases} 1, & x < b_{(1)} \\ 1 - \dfrac{r-1}{n}, & b_{(r-1)} \le x \le b_{(r)} \\ 0, & x > b_{(n)} \end{cases}$$

It is known that the Glivenko-Cantelli lemma of Order Statistics [16] states that the mathematical expectation of empirical distribution function is the theoretical probability distribution function and that of empirical complementary probability distribution the theoretical survival function. Thus

$$E\left[\Phi_1(x)\right] = P(u_1, x)$$

and

$$E\left[\Phi_2(x)\right] = 1 - P(v_1, x) \tag{6}$$

where

$$P(u_1, x) = \begin{cases} 0, & x < u_1 \\ \dfrac{x-u_1}{v_1-u_1}, & u_1 \le x \le v_1 \\ 1, & x > v_1 \end{cases}$$

is the uniform probability distribution function on $[u_1, v_1]$.

and

$$P(v_1, x) = \begin{cases} 0, & x < v_1 \\ \dfrac{x-v_1}{w_1-v_1}, & v_1 \le x \le w_1 \\ 1, & x > w_1 \end{cases}$$

is the uniform probability distribution function on $[v_1, w_1]$.

From (5) using (6) we get the membership grades in $G(a,b)$ which is nothing but $H(a,b)$ can be estimated by the membership function

$$A(x) = \begin{cases} 0, & x < u_1, x > w_1 \\ \dfrac{x-u_1}{v_1-u_1}, & u_1 \le x \le v_1 \\ 1 - \dfrac{x-v_1}{w_1-v_1}, & v_1 \le x \le w_1 \end{cases} \tag{7}$$

where $A = [u_1, v_1, w_1]$ is a fuzzy number.

Again let $x_1, x_2, \cdots, x_n$ are sample realisations from the uniform population $[u_2, v_2]$ and $y_1, y_2, \cdots, y_n$ are sample realisations from the uniform population $[v_2, w_2]$.

We denote $G(x,y)$ as the superimposition of equi-fuzzy intervals $[x_i, y_i]$; $i=1,2,\cdots,n$ with membership (1/*n*) *i.e.*

$$\begin{aligned} G(x,y) &= [x_1,y_1]^{(1/n)} S [x_2,y_2]^{(1/n)} S \cdots S [x_n,y_n]^{(1/n)} \\ &= \left[x_{(1)}, x_{(2)}\right]^{(1/n)} + \left[x_{(2)}, x_{(3)}\right]^{(2/n)} + \ldots + \\ &\left[x_{(n-1)}, x_{(n)}\right]^{((n-1)/n)} + \left[x_{(n)}, y_{(1)}\right]^{(1)} + \left[y_{(1)}, y_{(2)}\right]^{(1-1/n)} \\ &+ \ldots + \left[y_{(n-2)}, y_{(n-1)}\right]^{(2/n)} + \left[y_{(n-1)}, y_{(n)}\right]^{(1/n)} \\ &= H(x,y) \end{aligned} \tag{8}$$

where $x_{(1)}, x_{(2)}, \cdots, x_{(n)}$ are the ordered values of $x_1, x_2, \cdots, x_n$ and $y_{(1)}, y_{(2)}, \cdots, y_{(n)}$ are the ordered values of $y_1, y_2, \cdots, y_n$ in ascending order of magnitude and here

$$\bigcap_{i=1}^{n}[x_i, y_i] \neq \phi$$

Here the empirical probability distribution function and empirical complementary distribution function are respectively given by

$$\Phi_3(x)=\begin{cases}0, \ x<x_{(1)}\\ \dfrac{r-1}{n}, \ x_{(r-1)}\le x\le x_{(r)}\\ 1, \ x>x_{(n)}\end{cases}$$

and

$$\Phi_4(x)=\begin{cases}0, \ x<y_{(1)}\\ 1-\dfrac{r-1}{n}, \ y_{(r-1)}\le x\le y_{(r)}\\ 1, \ x>y_{(n)}\end{cases}$$

By Glivenko Cantelli lemma of order statistics, we get

$$E\left[\Phi_3(x)\right]=P(u_2,x)$$

and

$$E\left[\Phi_4(x)\right]=1-P(v_2,x) \tag{9}$$

where

$$P(u_2,x)=\begin{cases}0, \ x<u_2\\ \dfrac{x-u_2}{v_2-u_2}, u_2\le x\le v_2\\ 1, \ x>v_2\end{cases}$$

is the uniform probability distribution function on $[u_2,v_2]$. And

$$P(v_2,x)=\begin{cases}0, \ x<v_2\\ \dfrac{x-v_2}{w_2-v_2}, v_2\le x\le w_2\\ 1, \ x>w_2\end{cases}$$

is the uniform probability distribution function on $[v_2, w_2]$.

From (8) using (9) we get the membership grades in G(*x*,*y*) which is nothing but $H(x,y)$ can be estimated by the membership function

$$X(x)=\begin{cases}0, x<u_2, x>w_2\\ \dfrac{x-u_2}{v_2-u_2}, u_2\le x\le v_2\\ 1-\dfrac{x-v_2}{w_2-v_2}, v_2\le x\le w_2\end{cases}$$

where $X=[u_2,v_2,w_2]$ is also a fuzzy number.

It was assumed that $\bigcap_{i=1}^{n}[x_i, y_i] \neq \phi$.

Again let $c_1,c_2,\cdots,c_n$ are sample realisations from the uniform population $[u_3,v_3]$ and $d_1,d_2,\cdots,d_n$ are sample realisations from the uniform population $[v_3,w_3]$.

We denote $G(c,d)$ as the superimposition of equi-fuzzy intervals $[c_i,d_i]$; $i=1,2,\cdots,n$ with membership (1/*n*) *i.e.*

$$\begin{aligned}G(c,d)&=\left[c_1,d_1\right]^{(1/n)} S\left[c_2,d_2\right]^{(1/n)} S\cdots S\left[c_n,d_n\right]^{(1/n)}\\ &=\left[c_{(1)},c_{(2)}\right]^{(1/n)}+\left[c_{(2)},c_{(3)}\right]^{(2/n)}+\ldots\\ &+\left[c_{(n-1)},c_{(n)}\right]^{((n-1)/n)}\Big]+\left[c_{(n)},d_{(1)}\right]^{(1)}\\ &+\left[d_{(1)},d_{(2)}\right]^{(1-1/n)}+\ldots+\left[d_{(n-2)},d_{(n-1)}\right]^{(2/n)}\\ &+\left[d_{(n-1)},d_{(n)}\right]^{(1/n)}\\ &=H(c,d)\end{aligned} \tag{10}$$

where $c_{(1)},c_{(2)},\cdots,c_{(n)}$ are the ordered values of $c_1,c_2,\cdots,c_n$ and $d_{(1)},d_{(2)},\cdots,d_{(n)}$ are the ordered values of $d_1,d_2,\cdots,d_n$ in ascending order of magnitude and here $\bigcap_{i=1}^{n}[c_i,d_i]\neq\phi$.

Here the empirical probability distribution function and empirical complementary distribution function are respectively given by

$$\Phi_5(x)=\begin{cases}0, \ x<c_{(1)}\\ \dfrac{r-1}{n}, \ c_{(r-1)}\le x\le c_{(r)}\\ 1, \ x>c_{(n)}\end{cases}$$

and

$$\Phi_6(x)=\begin{cases}0, \ x<d_{(1)}\\ 1-\dfrac{r-1}{n}, \ d_{(r-1)}\le x\le d_{(r)}\\ 1, \ x>d_{(n)}\end{cases}$$

By Glivenko Cantelli lemma of order statistics, we get

$$E\left[\Phi_5(x)\right]=P(u_3,x)$$

and

$$E\left[\Phi_6(x)\right]=1-P(v_3,x) \tag{11}$$

where

$$P(v_3,x)=\begin{cases}0, & x<u_3\\ \dfrac{x-u_3}{v_3-u_3}, & u_3\le x\le v_3\\ 1, & x>v_3\end{cases}$$

is the uniform probability distribution function on $[u_3,v_3]$.
and

$$P(v_3,x)=\begin{cases}0, & x<v_3\\ \dfrac{x-v_3}{w_3-v_3}, & v_3\le x\le w_3\\ 1, & x>w_3\end{cases}$$

is the uniform probability distribution function on $[v_3,w_3]$.

From (10) using (11) we get the membership grades in $G(c,d)$ which is nothing but $H(c,d)$ can be estimated by the membership function

$$B(x)=\begin{cases}0, & x<u_3, x>w_3\\ \dfrac{x-u_3}{v_3-u_3}, & u_3\le x\le v_3\\ 1-\dfrac{x-v_3}{w_3-v_3}, & v_3\le x\le w_3\end{cases}$$

where $B=[u_3,v_3,w_3]$ is a fuzzy number.

It was assumed that $\bigcap\limits_{i=1}^{n}[c_i,d_i]\ne\phi$.

The given equation can be written as

$$G(a,b)(+)G(x,y)=G(c,d)$$

Replacing the values of $G(a,b)$, $G(x,y)$ and $G(c,d)$ and using the equi-fuzzy interval arithmetic, we get

$$\begin{aligned}&\left[a_{(1)}+x_{(1)},a_{(2)}+x_{(2)}\right]^{(1/n)}+\left[a_{(2)}+x_{(2)},a_{(3)}+x_{(3)}\right]^{(2/n)}\\&+\ldots+\left[a_{(r-1)}+x_{(r-1)},a_{(r)}+x_{(r)}\right]^{((r-1)/n)}+\ldots\\&+\left[a_{(n-1)}+x_{(n-1)},a_{(n)}+x_{(n)}\right]^{((n-1)/n)}]\\&+\left[a_{(n)}+x_{(n)},b_{(1)}+y_{(1)}\right]^{(1)}+\left[b_{(1)}+y_{(1)},b_{(2)}+y_{(2)}\right]^{(1-1/n)}\\&+\ldots+\left[b_{(n-2)}+y_{(n-2)},b_{(n-1)}+y_{(n-1)}\right]^{(2/n)}\\&+\left[b_{(n-1)}+y_{(n-1)},b_{(n)}+y_{(n)}\right]^{(1/n)}\end{aligned}$$

i.e. $H(a+x,b+y)=H(c,d)$

Using the equality of equi-fuzzy intervals, we get

$a_{(i)}+x_{(i)}=c_{(i)}$ and $b_{(i)}+y_{(i)}=d_{(i)}$; $i=1,2,\cdots,n$.

which gives

$x_{(i)}=c_{(i)}-a_{(i)}$ and $y_{(i)}=d_{(i)}-b_{(i)}$; $i=1,2,\cdots,n$.

This implies

$$\begin{aligned}&\left[x_{(1)},x_{(2)}\right]^{(1/n)}+\left[x_{(2)},x_{(3)}\right]^{(2/n)}+\cdots\\&+\left[x_{(r-1)},x_{(r)}\right]^{((r-1)/n)}+\cdots+\left[x_{(n-1)},x_{(n)}\right]^{((n-1)/n)}\Big]\\&+\left[x_{(n)},y_{(1)}\right]^{(1)}+\left[y_{(1)},y_{(2)}\right]^{(1-1/n)}+\cdots\\&+\left[y_{(n-2)},y_{(n-1)}\right]^{(2/n)}+\left[y_{(n-1)},y_{(n)}\right]^{(1/n)}\\&=\left[c_{(1)}+a_{(1)},c_{(2)}+a_{(2)}\right]^{(1/n)}+\left[c_{(2)}+a_{(2)},c_{(3)}+a_{(3)}\right]^{(2/n)}\\&+\cdots+\left[c_{(r-1)}+a_{(r-1)},c_{(r)}+a_{(r)}\right]^{((r-1)/n)}+\cdots\\&+\left[c_{(n-1)}+a_{(n-1)},c_{(n)}+a_{(n)}\right]^{((n-1)/n)}\Big]\\&+\left[c_{(n)}+a_{(n)},d_{(1)}+b_{(1)}\right]^{(1)}+\left[d_{(1)}+b_{(1)},d_{(2)}+b_{(2)}\right]^{(1-1/n)}\\&+\cdots+\left[d_{(r-1)}-b_{(r-1)},d_{(r)}-b_{(r)}\right]^{((n-r)/n)}+\cdots\\&+\left[d_{(n-1)}-b_{(n-1)},d_{(n)}+b_{(n)}\right]^{(1/n)}\end{aligned} \tag{12}$$

The left side of the identity (12) is $G(x,y)$ whose membership function $X(x)$ is estimated by (9) and from the right side, we get the empirical probability distribution function and survival function as

$$\Phi_7(x)=\begin{cases}0, & x<c_{(1)}-a_{(1)}\\ \dfrac{r-1}{n}, & c_{(r-1)}-a_{(r-1)}\le x\le c_{(r)}-a_{(r)}\\ 1, & x>c_{(n)}-a_{(n)}\end{cases}$$

and

$$\Phi_8(x)=\begin{cases}0, & x<d_{(1)}-b_{(1)}\\ 1-\dfrac{r-1}{n}, & d_{(r-1)}-b_{(r-1)}\le x\le d_{(r)}-b_{(r)}\\ 1, & x>d_{(n)}-b_{(n)}\end{cases}$$

By Glivenko Cantelli Lemma of order Statistics

$$E\left[\Phi_7(x)\right]=P(u_3-u_1,x)$$

and

$$E\left[\Phi_8(x)\right]=1-P(v_3-v_1,x) \tag{13}$$

where

$$P(u_3-u_1,x)=\begin{cases}0, & x<u_3-u_1\\ \dfrac{x-(u_3-u_1)}{(v_3-v_1)-(u_3-u_1)}, & u_3-u_1\le x\le v_3-v_1\\ 1, & x>v_3-v_1\end{cases}$$

is the uniform probability distribution function on $[u_3-u,v_3-v_1]$ and

$$P(u_3-u_1,x)=\begin{cases}0, x<v_3-v_1\\ \dfrac{x-(v_3-v_1)}{(w_3-w_1)-(v_3-v_1)}, v_3-v_1\le x\le w_3\\ 1, x>w_3-w_1\end{cases}$$

is the uniform probability distribution function on $[v_3-v_1, w_3-w_1]$.

From (13), we get the solution of the equation $A+X=B$ as

$$X=[u_3-u_1, v_3-v_1, w_3-w_1] \quad (14)$$

where

$$X(x)=\begin{cases}0, x<u_3-u_1, x>w_3-w_1\\ \dfrac{x-(u_3-u_1)}{(v_3-v_1)-(u_3-u_1)}, u_3-u_1\le x\le v_3-v_1\\ 1-\dfrac{x-(v_3-v_1)}{(w_3-w_1)-(v_3-v_1)}, v_3-v_1\le x\le w_3-w_1\end{cases}$$

Obviously,

$$\begin{aligned}A+X&=[u_1,v_1\cdot w_1]+[u_3-u_1,v_3-v_1,w_3-w_1]\\&=[u_3,v_2\cdot w_3]=B\end{aligned}$$

From the Equation (14), we get $X=[u_3-u_1,v_3-v_1,w_3-w_1]$ is a fuzzy number whose α-cut is given by

$$\begin{aligned}{}^{\alpha}X=\big[&(u_3-u_1)+\alpha((v_3-v_1)-(u_3-u_1))\\&(w_3-w_1)-\alpha((w_3-w_1)-(v_3-v_1))\big]\end{aligned}$$

which is the solution of ${}^{\alpha}A+{}^{\alpha}X={}^{\alpha}B$
Obviously

$$\begin{aligned}X=\bigcup_{\alpha\in[0,1]}\alpha\big[&(u_3-u_1)+\alpha((v_3-v_1)-(u_3-u_1)),\\&(w_3-w_1)-\alpha((w_3-w_1)-(v_3-v_1))\big]\end{aligned}$$

that is similar to the Equation (2).

Thus, we can conclude that the method of superimposition gives the same result as given by the method of α-cut.

6. Conclusion and Lines for Future Works

In this article, we have presented a new method of solving fuzzy equation $A+X=B$. The method is based on the set superimposition operation. The set superimposition method has been used to define the arithmetic operations on fuzzy numbers. It has been found that the arithmetic operation based on set superimposition operation gives the same result as given by other standard method viz. the method of α-cut. In this article, we have shown that our method of solution of fuzzy equation $A+X=B$ gives the similar results as given by other standard methods. In future we would like solve other kind of fuzzy equation namely fuzzy differential equation, fuzzy integral equation etc. using same method.

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