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Comparative Phenomenological Description of Even-Even Isotopes at Mass Region $A \approx 70$

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Abstract

In the present work the nuclear structure properties and the backbending phenomena of even-even isotopes at $A \approx 70$ mass region are analyzed using two simultaneous theoretical models based on a simple modified version of the collective model predictions besides an improved version of exponential model with the inclusion of pairing correlation. In general, both models successfully describe the backbending phenomena in that region. From the comparison between the predictions of the two proposed models a firm conclusion is obtained concerning the superiority of the simple improved version of the exponential model in describing the forward and down-bending region of the φ-ω^2 plots.

Keywords

Energy Levels, Moment of Inertia, Yrast Bands, Backbending, Cr, Ge, Se and Kr Even Mass Isotopes, Collective Model, Exponential Model

1. Introduction

Lately even-even nuclei at mass region $A \approx 70$ have recently become important testing ground for most of the advanced theories, where the calculated predictions may be compared with the corresponding experimental data. Previous works showed that there is a clear evidence for a major change in the nature of the ground state levels below $I = 18\ \hbar$ in even-even nuclei in that region [1]-[8]. Furthermore, at higher spin values a very regular structure develops. It is simply called the backbending phenomenon which occurs as one plots the moment of

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inertia versus the square of the rotational frequency. These nuclei have several interesting features such as oblate and prolate deformations as well as rapid variations in shape as a function of both spin and mass number. The sudden disappearance of E2 strength at certain spins indicates a shape change that requires the considerations of upper pf configuration [9].

A crossing of any two bands [in the (E, I) plane] means that at certain critical angular momentum $I = I_C$ the energies of the corresponding two states belonging to different bands are approximately equal. In particular, a crossing of any two bands which form a portion of the yrast line leads apparently to a rearrangement in the intrinsic structure in the de-exciting nucleus. Such a rearrangement is sometimes very abrupt.

The band crossing effect looks much more dramatic if the vibrational frequency ω instead of I is used as an independent variable. In such a representation all the important physical quantities as energy, angular momentum, aligned angular momentum and moment of inertia, etc. were discovered experimentally for the first time by Johnson *et al*., [10] and are often called a backbending effect.

Johnson *et al*., [10] chose to represent the excitation energies $E(I)$ of the ground-state levels in terms of a plot between the nuclear moment of inertia φ and the squared rotational frequencies ω^2. Such plots have revealed that in some cases, φ increases so rapidly with I that ω^2 actually decreases as higher spin states are reached, resulting in the appearance of backbending in these plots. That is because the experimental level spacing starts falling below that given by the $I(I + 1)$ rule for $E(I)$.

Well deformed nuclei in their ground states have a moment of inertia which is typically about half of the value expected for rigid rotors. This is interpreted as due to the presence of strong pairing force between the nucleons in the nucleus. With increasing frequency of rotation, the correlations due to the pairing force are reduced as a result of Coriolies anti-pairing effect (the CAP effect) until these correlations disappear at a critical angular momentum. As a result, inertia increases with the rotational angular velocity and is expected to adopt the rigid rotor value at the critical angular momentum. This situation was first predicted by Mottelson and Valatin [11].

The investigation of Lieder *et al*., [12] and Fassler *et al*., [13] indicates that at spin $I \approx 10$ - 14, the ground-state rotational band is crossed and mixed with a second (super) band. After the crossing, the members of the superband become the yarst levels. If the interaction between both bands is strong, the mixing of the wave functions in the crossing region is also strong and the phase transition occurs smoothly; no dramatic irregularities in the behavior of φ and the quadrupole moment Q are observed. If the band interaction V is weak, a sudden change in the intrinsic structure occurs which causes a marked increase of the moment of inertia φ (backbending) and a certain decrease of the transition probabilities $B(E)$ (and also the quadrupole moment Q).

Several works have confirmed that backbending could be influenced by the ground state band energy spacing and the pairing gap [14]-[17]. Also, the fact that the moment of inertia is almost doubled and is approaching the value of a rigid rotation suggests that the transition is associated with pair correlation [18]. A large amount of works have been done in studying the antialignment effect of pair correlation on the moment of inertia [17] [19].

The backbending mechanism of ^{48}Cr has been studied by Hara *et al.* [5], making use of the projected shell model [20]. The obtained results proved that the backbending in ^{48}Cr is based on band crossing. This result differs from that of Tanaka *et al.* interpretation based on the Cranked Hartee-Fock-Bogoliubov (CHFB), which claims that the backbending in the area under investigation is not due to level crossing mechanism [21].

Furthermore, the pairing force has been considered to have an important role in backbending phenomena but it is not sufficiently outlined [22]. Cranking model analysis of ^{80}Br energy levels reveals the possible existence of neutron alignment at $\omega = 0.7$ MeV [23].

Many attempts have been performed to provide theoretical description of the backbending phenomena. The variable moment of inertia (VMI) gives a very good description of the ground state bands of even-even nuclei up to the point where backbending occurs [24] [25]. Also, several works utilized the band mixing calculations to describe backbending [26] [27].

The lack of clear description concerning the backbending phenomena in the $A \approx 70$ mass region led us to reinvestigate the phenomena applying a simple five parameter formula based on a dynamic version of the unified collective model. Additionally, an improved version of the exponential model with pairing attenuation has been also applied in the present work [28]. It is hoped by such work to have a good description of the backbending regions besides those of low-lying states.

2. The Modified Version of Collective Model Description

Zvonov and Mitroshin [29] have applied a dynamic version of the unified collective model of nuclei as a universal mechanism forming quasirotational bands in spherical, transitional and deformed nuclei. It holds well for the ground state bands in even-even nuclei $40 \le A \le 180$.

In this model, the energy spectrum of vibrational states with $I = \lambda N$ is given by

$$E_N = N\omega^N + \frac{2\lambda+1}{2}\left(\omega^N - \omega^1\right) \tag{1}$$

where λ is a constant depends on the number of phonons "N" and the spin I, for the yrast bands $\lambda = 2$,

$$\omega^N = \omega^1\left(1+2\gamma\left(N-1\right)\right)^{1/2},\ \omega^1 = E_2^+,\ \text{and}\ \gamma = \gamma^-\left(1+\frac{3e^2Z^2}{10\pi R_\circ B_2 E_{2+}^2}\right)$$

where γ^- is a universal constant = 5.5×10^{-2} for ($40 \le A \le 190$) and $B_\lambda = 10\left(\frac{3R_\circ^2 Am}{4\lambda\pi}\right)$

A further improvement of this model is given taking into consideration the possibility that the energy levels of even-even nuclei can be treated as dynamic modes too where the energy E_N can be obtained by the following formula:

$$E_N = A\omega^N + B\left(\omega^N - \omega^1\right) + C\left(\omega^N - \omega^1\right)^2 + D\left(\omega^N - \omega^1\right)^3 + \cdots \tag{2}$$

where A, B, C and D are constants.

The even power terms in the previous expression are comparable to the so-called Harris expansion for rotational spectra [30] [31]. The odd power terms in Equation (2) could be described as the residual interaction coming from band crossing. Furthermore, Equation (2) is equivalent to the extended variable moment of inertia model to high spin given by Anagnostatos [32] based on the article given by Das and Banerjee [33].

In that work the energy of states of an even-even nucleus is in the form:

$$E = C_2\left(\varphi - \varphi_\circ\right)^2 + C_3\left(\varphi - \varphi_\circ\right)^3 + C_4\left(\varphi - \varphi_\circ\right)^4 + \frac{I\left(I+1\right)}{2\varphi^2} \tag{3}$$

where C_2, C_3, C_4 and $\varphi_\circ$ are the four parameters of the model; $\varphi_\circ$ is the moment of inertia of the first excited state (2+).

In a very pronounced description A. Bohr and B. Mottelson [34] have stated a familiar expression obtained by quantizing the classical Hamiltonian for a symmetric top in the following form:

$$E_{rot} = \left(\frac{\hbar^2}{2\varphi}\right)\left[I\left(I+1\right)\right] \tag{4}$$

where φ is the effective moment of inertia. For sufficiently small values of I, one can employ an expression in powers of $I\left(I+1\right)$ for purely rotational motion as follows:

$$E_{rot}\left(I\left(I+1\right)\right) = AI\left(I+1\right) + BI^2\left(I+1\right)^2 + CI^3\left(I+1\right)^3 + DI^4\left(I+1\right)^4 + \cdots \tag{5}$$

where A is the inertial parameter, while $B, C, D, \cdots$. Are corresponding higher-order inertial parameters. In many cases, the precision of the energy measurements makes possible a determination of higher order terms in the expansion in powers $I\left(I+1\right)$. If the energy is expressed as power series in the rotational frequencies (ω) rather than in the angular momentum (I), it is found that a greater simplicity and improvement in the rate of convergence could be obtained [34]. Furthermore, the dependence of the moment of inertia on the collective parameters also gives rise to a coupling between the rotational motion and the vibrational excitation associated with the oscillations in these parameters. As a consequence, there is mainly a competition between combinations of rotational and vibrational motions inside the nucleus.

Based on the present proposed dynamic version (Equation (2)), the aforementioned discussion concerning the rotational-vibrational motion and the previously predicted model given by Anangnostatos [32] (Equation (3)), an improved relation could be stated by adding a term ($FI\left(I+1\right)$) representing the rotational contribution to the nuclear motion as follows:

$$E_N = A\omega^N + B\left(\omega^N - \omega^1\right) + C\left(\omega^N - \omega^1\right)^2 + D\left(\omega^N - \omega^1\right)^3 + FI\left(I+1\right) \tag{6}$$

where F is the inertial parameter and in the same time measures the weighted magnitude of the rotational contribution.

3. The Improved Exponential Model Description

Sood and Jain [35] have previously developed an exponential model based on the exponential dependence of the nuclear moment of inertia on pairing correlation [18]. They gave the following relation:

$$E(I)=\frac{\hbar^2}{2\phi_\circ}I(I+1)\mathrm{e}^{\Delta_\circ\left(1-\frac{I}{IC}\right)^{1/2}} \tag{7}$$

For medium light nuclei, Ic can take values smaller than 18 ħ because the backbending phenomenon in that region ($A \approx 70$) lies at spin $I \approx 10\ \hbar$ [4]. These works led us to use a suitable Ic values to represent both the variation of the moment of inertia and the pairing correlation and to give the model the ability to describe well the φ-ω^2 plot region, in particular the forward and down-bending regions.

The modified version of the exponential model with pairing attenuation has the following form [7] [28]:

$$E(I)=\frac{\hbar^2}{2\varphi_\circ}I(I+1)\mathrm{e}^{\Delta_\circ\left(1-\frac{I}{IC}\right)^{1/\upsilon}} \tag{8}$$

where $\varphi_\circ$, $\Delta_\circ$ and υ are the free parameters of the model, which are adjusted to give a least-square fit to the experimental data. This approach is supported by Ma and Rasmussen suggestion that there is an exponential dependence of the moment of inertia of the parameter υ for a wide range of υ values [36].

4. Investigation of Backbending via the Applied Models Predictions

The anomalous behavior, *i.e.* backbending of several medium light even-even nuclei (Cr, Ge, Se and Kr), has been studied using the modified version of the collective model and the improved exponential model. The predictions of the applied models compared with the corresponding experimental results [37] are given in **Table 1**.

The plots of the calculated data of $2\varphi/\hbar^2$ versus $(\hbar\omega)^2$ for these isotopes are given in **Figure 1**, where the experimental data are also presented. The excitation energy $E(I)$ of the yrast bands, the moment of inertia and the squared rotational frequency ω^2 are deduced by using the well-known relation [28]:

$$\frac{2\varphi}{\hbar^2}=\frac{(4I-2)}{E(I)-E(I-2)} \tag{9}$$

$$(\hbar\omega)^2=\left(I^2-I+1\right)\left[\frac{E(I)-E(I-2)}{2I-1}\right]^2 \tag{10}$$

The calculated parameters are given in **Table 2** where the root mean square deviation (σ) values of fitting procedure are also included. The mean square deviation is given by:

$$\sigma=\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(1-\frac{E_{cal}}{E_{exp}}\right)^2} \tag{11}$$

In **Figure 1**, the experimental data show a clear evidence of backbending phenomenon in all the presented nuclei at $I = 8 - 12\ \hbar$. It is clear from the same figure that the predictions of both the improved exponential and the dynamic collective models describe very well the ground-state levels in Cr, Ge, Se and Kr even mass isotopes up to high spins. Furthermore, the predictions of the applied improved exponential model reproduce very well the backbending phenomenon in those nuclei and its application improves as the atomic mass number increases. This result may give an indication that the pairing force contribution to the backbending phenomenon increases as the atomic mass number increases. Another noticeable success of the model (IEM) is shown in the same figure concerning ^{68}Ge, ^{72}Se and ^{78}Kr and ^{80}Kr where the forward and down-bending regions are very well described by its predictions.

Table 1. Experimental and calculated level energies (keV) of ground-state bands in Cr, Ge, Se and Kr even-even nuclei using a dynamic version (DVM), an improved dynamic version (IDVM) of the collective model and a rather improved exponential model (IEM).

I		2^+	4^+	6^+	8^+	10^+	12^+	14^+	16^+	18^+	20^+	22^+	24^+	26^+	28^+
$^{50}_{24}$Cr	EXP	783.3	1881	3164	4745	6341	7613								
	DVM	783.3	1801	3205	4764	6304	7686								
	IDVM	785.3	1879	3173	4728	6325	7569								
	IEM	662.3	1848	3272	4767	6241	7653								
$^{64}_{32}$Ge	EXP	901.7	2053	3407	5175										
	DVM	902	2049	3377	5081										
	IDVM	893	2039	3367	5071										
	IEM	893	2067	3454	5184										
$^{68}_{32}$Ge	EXP	1016	2268	3696	4837	5961	7320								
	DVM	975.4	2346	3631	4920	6280	7765								
	IDVM	1019	2281	3668	4836	5886	7206								
	IEM	1017	2336	3601	4809	6047	7423								
$^{72}_{34}$Se	Eexp	862.1	1637	2467	3425	4504	5710	7038	8495	10,095	11,832	13,742	15,896	18,216	20,798
	DVM	862.1	1688	2567	3539	4635	5878	7286	8873	10,650	12,624	14,803	17,192	19,795	22,614.7
	IDVM	876.6	1609	2483	3495	4646	5947	7408	9046	10,876	12,915	15,182	17,693	20,465	23,516.9
	IEM	519.6	1394	2418	3511	4650	5843	7114	8492	10,012	11,711	13,626	15,790	18,232	20,969.3
$^{74}_{34}$Se	Eexp	634.7	1363	2231	3198	4256	5443	6736	8117	9680.5	11,360	13,202			
	DVM	634.8	1405	2250	3179	4198	5312	6524	7837	9252.1	10,770	12,393			
	IDVM	633.2	1367	2219	3168	4205	5331	6548	7866	9294.8	10,846	12,534			
	IEM	433.5	1212	2172	3231	4356	5539	6791	8134	9596.5	11,214	13,028			
$^{76}_{34}$Se	Eexp	559.1	1331	2262	3270	4300	5433								
	DVM	559.5	1329	2265	3266	4299	5430								
	IDVM	559.5	1329	2265	3266	4299	5430								
	IEM	530.2	1350	2272	3254	4301	5435								
$^{78}_{34}$Se	Eexp	613.7	1503	2547	3585	4625	5784								
	DVM	613.7	1541	2542	3607	4728	5898								
	IDVM	614.1	1502	2535	3547	4539	5625								
	IEM	597.6	1524	2538	3573	4639	5779								
$^{74}_{36}$Kr	Eexp	455.6	1013	1781	2748	3892	5180	6516	7858	9305.9	10881				
	DVM	456.3	1062	1563	2043	2567	3186	3938	4856	5963.4	7281.6				
	IDVM	466.7	985.4	1785	2785	3932	5191	6539	7966	9466.1	11,038				
	IEM	317.5	948.8	1800	2810	3939	5162	6467	7851	9315.9	10,868				
$^{78}_{36}$Kr	Eexp	455	1119	1978	2994	4106	5218	6480	7938	9570	11,314	13,159	15,163	17,297	
	DVM	455	1208	2054	3008	4081	5283	6619	8093	9709.6	11,470	13,377	15,431	17,632	
	IDVM	429.3	1166	2009	2964	4038	5236	6563	8023	9621.1	11,361	13,248	15,283	17,472	
	IEM	412.6	1124	1996	2979	4060	5243	6541	7966	9532.2	11,253	13,140	15,203	17,451	
$^{80}_{36}$Kr	Eexp	616.6	1436	2392	3410	4378	5438	6681	8088	9690.6					
	DVM	616.6	1587	2608	3726	4976	6382	7966	9742	11723					
	IDVM	609.4	1456	2395	3375	4397	5490	6704	8104	9762.2					
	IEM	579.2	1460	2412	3378	4376	5452	6671	8107	9848.3					

Table 2. The fitting parameters of the dynamic version (DVM), the improved dynamic version (IDVM) of the collective model and the improved exponential model (IEM).

Model	DVM (keV)					IDVM (keV)						IEM (MeV)				
Nucleus	A	B	C	D	σ	A	B	C	D	F	σ	$2\varphi_o/\hbar^2$	Ic	Δ_o	υ	σ
$^{50}_{24}$Cr	1	16	0.1950	−0.0005	0.019	47.896	1399	14.05	0.018	−6122.0	0.003	39.803	26	1.7	0.7	0.063
$^{64}_{32}$Ge	1	22	−0.0024	0.0004	0.010	0.99	21.92	−0.002	0.000	0.00	0.013	15.551	18	1.4	0.2	0.009
$^{68}_{32}$Ge	1	25	−0.0196	0.0001	0.041	−48.46	−1433.80	−11.29	−0.012	8376.0	0.009	25.516	20	1.9	0.4	0.018
$^{72}_{34}$Se	1	16	0.0098	0.0001	0.056	−0.321	−26.38	−0.322	0.000	192.2	0.076	38.765	30	1.5	0.4	0.114
$^{74}_{34}$Se	1	20	0.0421	0.0001	0.034	−0.653	−30.36	−0.534	−0.001	174.7	0.028	40.435	28	1.3	0.4	0.102
$^{76}_{34}$Se	1	23	0.0871	0.0000	0.001	2.0331	50.00	0.622	0.000	−96.2	0.001	40.758	50	1.6	0.2	0.022
$^{78}_{34}$Se	1	25	0.0590	0.0000	0.016	−28.4	−841.72	−11.03	−0.019	3007.0	0.014	33.345	20	1.5	0.5	0.012
$^{74}_{36}$Kr	1	15	0.2608	−0.0003	0.298	−1.683	−66.67	−1.067	−0.004	205.7	0.015	47.509	40	1	0.4	0.098
$^{78}_{36}$Kr	1	28	0.0881	0.0004	0.028	0.4291	11.38	−0.157	0.000	39.0	0.022	42.57	80	1.3	0.1	0.027
$^{80}_{36}$Kr	1	27	0.0193	0.0003	0.131	−7.311	−221.57	−3.062	−0.005	852.9	0.008	35.058	20	1.5	0.5	0.021

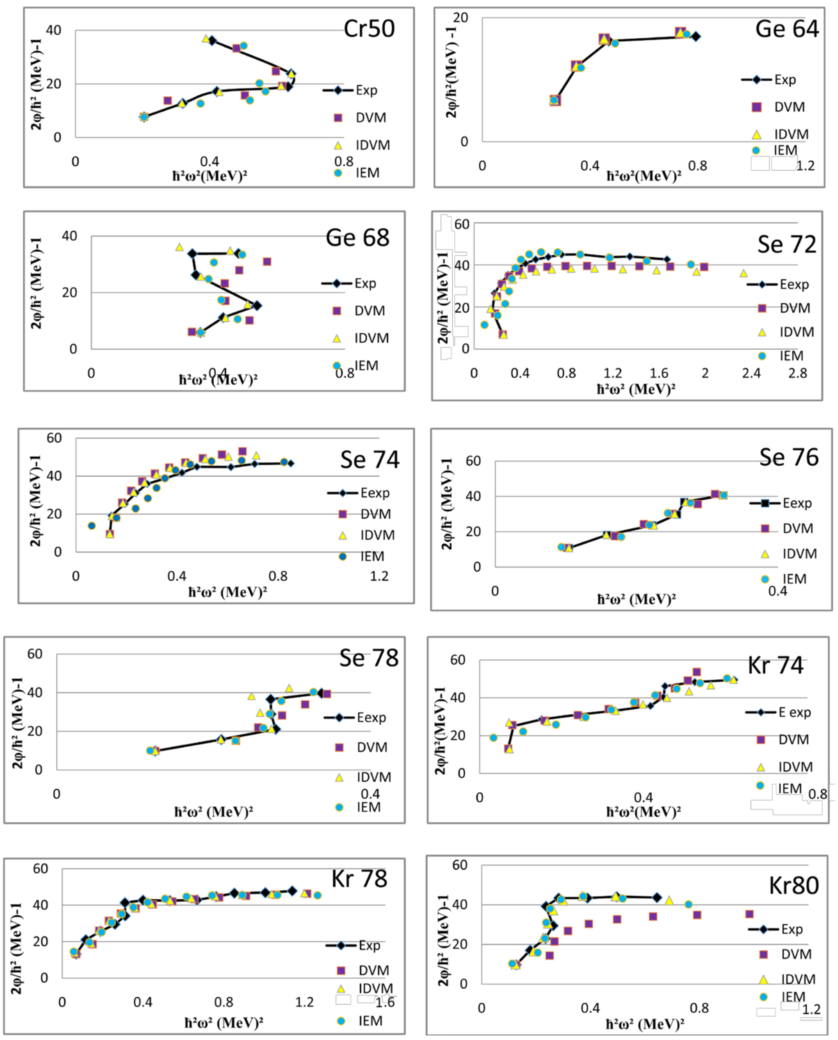

Figure 1. Calculated and experimental moment of inertia $2\varphi/\hbar^2$ vs. $(\hbar\omega)^2$ for yrast band level of some light nuclei at $A \approx 70$.

5. Conclusions

In the persent study, the application of an improved dynamic version of the collective model along with an improved exponential model based on the pairing correlation gives a fairly accurate description of the high spin states in Cr, Ge, Se and Kr. Furthermore, the applied models give overall satisfactory results concering the description of backbending phenomena. The forward and down-bending regions of φ-ω^2 plots are well described by means of the improved exponentional model predictions. In contrary, in acute backbending cases, the improved dynamic version model roughly holds so that further microscopic calculations are needed.

As aconsequance, the appearance of the backbending phenomena in medium light nuclei at low spins (I = 8 − 12 $\hbar$) can be interpreted on the framework of the pairing force which supports the band crossing mechansim in analogy with the earlier calculations [5] based on the projected shell model.

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