

# Welfare Improvement and the Extension of the Income Gap under Monopoly

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## Abstract

This study constructs a model of a monopoly where investors are also actors, and shows that, in contrast to traditional models, this model admits the welfare improvement caused by monopoly. This study also reveals that if a huge income gap exists in the initial stage, then monopoly exacerbates the expansion of the income gap caused by market trades. Moreover, we show that this exacerbation occurs in general situations under some additional (but natural) assumptions.

## Keywords

Monopoly, Income Gap, Total Surplus

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## 1. Introduction

Economics traditionally considers a monopoly to be bad for an economy. The most famous research indicating that monopolies are bad is the classical partial equilibrium analysis performed by Hicks [1]. This research indicates that a monopoly lowers the total surplus, and thus, the economy with a monopoly is not Pareto efficient. The result of this research is summarized in most of the textbook in microeconomics, e.g. Varian (1992), Okuno (2008) or Mas-Colell, Whinston, and Green (1995) [2]-[4].

This research focuses on monopoly from a fresh perspective. The traditional monopoly model includes two characters: the monopolistic firm and the consumer. However, a real monopolistic situation necessarily involves a third character, namely, the **investor**. Under capitalism, investors are also consumers. Therefore, in our model, consumers invest in the monopolistic firm, which distributes its profit into its investors.

We formalize the above circumstance in a model, and analyze its model. We find that the total surplus of an economy may improve under a monopoly, which contradicts the traditional rationale for monopolies being bad. Meanwhile, in such a case the income gap often is expanded by market trade. If the initial income gap is suffi-

ciently large, then a monopoly exacerbates this expansion of the income gap. The reason for this is as follows. Consider there are two consumers, where one is poor and another is rich. Both consumers invest in a firm that sells their own products and transfers its margin to investors in the form of dividends. However, the poor consumer has only limited ability to invest, and thus receives only a small share of the margin on product sales. The bulk of the margin is expropriated by the rich consumer. In this scenario, monopoly exacerbates this expansion of the income gap by enlarging firm's profit.

This is the case in which the initial income gap is very high. In the case where the initial income gap is not so high, under certain assumptions monopoly also exacerbate the expansion of the income gap. Although these assumptions are not clear in the theoretical sense, we believe that these assumptions are intuitively natural.

In Section 2, we introduce our model and show the results. Section 3 is the conclusion.

## 2. The Model

We construct two models, named model 1 and model 2, to compare the competitive case with the monopolistic case. Model 1 corresponds with the competitive case, while model 2 corresponds with the monopolistic case. Both models consist of two consumers and one firm. Both consumers have a utility function  $u_i(c_i) + m_i$ , where  $c_i$  denotes private consumption and  $m_i$  denotes the amount of money. We assume that

$u' > 0, u'' < 0, \lim_{c \downarrow 0} u'(c) = +\infty$ , and  $\lim_{c \rightarrow +\infty} u'(c) = 0$ . In the beginning of the model, consumer  $i$  has  $e_i$  units of money and one unit of labor. Without loss of generality, we assume  $0 \leq e_1 < e_2$ . In the first-stage of the model, consumer  $i$  determines the amount of investment  $a_i \in [0, e_i]$  at same time. Then the stock ratio  $\theta_i$  is

defined as  $\frac{a_i}{a_1 + a_2}$  and the capital of the firm  $K$  is defined as  $a_1 + a_2$ . The product function of the firm is

denoted as  $F(K, L)$ . We assume that  $F$  is homogeneous of degree one,

$F(0, 0) = F(K, 0) = F(0, L) = 0, F(K, L) > 0, F_K(K, L) > 0, F_L(K, L) > 0, F_{LL}(K, L) < 0$  for all  $K, L > 0$ , and  $\lim_{L \rightarrow 0} F_L(K, L) = +\infty$  and  $\lim_{L \rightarrow +\infty} F_L(K, L) = 0$  for all  $K > 0$ .

The second-stage is different from each model. In model 1, each consumer and firm participates in the competitive market and the equilibrium arises. In model 2, the firm determines the price of consumption  $p$  monopolistically and the wage  $w$  is determined competitively<sup>1</sup>.

### 2.1. The First Model

First, we solve the second-stage. The first-order condition of consumer  $i$  is,

$$\begin{aligned} u_i(c_i) &= p, \\ m_i &= w + e_i - a_i + \theta_i \pi - p c_i. \end{aligned}$$

Hence,

$$c_i = (u')^{-1}(p),$$

and thus,  $c_1 = c_2$  in equilibrium. Meanwhile, the equilibrium condition of this market is

$$c_1 + c_2 = F(K, L),$$

and,

$$L = 2.$$

Hence, the equilibrium price is

$$p^*(K) = u' \left( \frac{F(K, 2)}{2} \right).$$

<sup>1</sup>In the second-stage, we assume that the consumption space of each consumer is  $\mathbb{R}_+ \times \mathbb{R}$ . This assumption is made for the sake of simplicity and is not essential. We note that this setup is introduced in the explanation of the quasi-linear preference in Mas-Colell, Whinston and Green (1995).

Next, the first-order condition of the firm is,

$$pF_L(K, L) = w.$$

Thus, the equilibrium wage is

$$w^*(K) = p^*(K)F_L(K, 2).$$

Then, the profit of firm is<sup>2</sup>

$$\pi_1(K) = p^*(K)F(K, 2) - 2w^*(K) = p^*(K)F_K(K, 2)K,$$

where the subscript 1 represents that it is the profit of the first model. Hence,  $\pi_1(K)$  is positive, and the average profit  $\frac{\pi_1(K)}{K}$  is decreasing.

Therefore, the payoff function of this model  $U_i^1$  is<sup>3</sup>,

$$U_i^1(a_1, a_2) = e_i - a_i + \left(\theta_i - \frac{1}{2}\right)\pi_1(K) + u\left(\frac{F(K, 2)}{2}\right).$$

In the first-stage, consumer  $i$  chooses  $a_i \in [0, e_i]$  simultaneously and the Nash equilibrium arises.

Define

$$g(K) = \frac{\pi_1(K)}{K} - 1,$$

and  $K^*$  as the unique solution of  $g(K) = 0$ . Then,

$$\frac{\partial U_i^1}{\partial a_i} = \left(\theta_i - \frac{1}{2}\right)\left(\pi_1'(K) - \frac{\pi_1(K)}{K}\right) + g(K). \tag{1}$$

Hence,  $\frac{\partial U_1^1}{\partial a_1} + \frac{\partial U_2^1}{\partial a_2} = 2g(a_1 + a_2)$  and thus  $\frac{\partial U_1^1}{\partial a_1} + \frac{\partial U_2^1}{\partial a_2} = 0$  if and only if  $a_1 + a_2 = K^*$ . Note that  $K^*$  is

the social optimal level of capital, since  $U_1^1 + U_2^1 = 2u\left(\frac{F(K, 2)}{2}\right) - K + e_1 + e_2$  for any  $(a_1, a_2)$  and thus

$$\frac{d}{dK}(U_1^1 + U_2^1) = p^*(K)F_K(K, 2) - 1.$$

We show the following proposition:

**Proposition 1:** There exists a Nash equilibrium  $(a_1^*, a_2^*)$ . If  $K^* \leq 2e_1$ , then  $(a_1^*, a_2^*) = \left(\frac{K^*}{2}, \frac{K^*}{2}\right)$  is the unique Nash equilibrium. If not, then for any Nash equilibrium  $(a_1^*, a_2^*)$ ,  $a_1^* = e_1$  and  $a_2^* \in ]e_1, K^* - e_1[$ , and thus,  $a_1^* + a_2^* < K^*$ .

**Proof:** We first suppose  $K^* \leq 2e_1$ . We can easily verify that  $(0, 0)$  is not a Nash equilibrium. Note that  $\pi_1'(K) - \frac{\pi_1(K)}{K}$  is always negative since  $\frac{\pi_1(K)}{K}$  is decreasing. By Equation (1), 1) if  $g(a_1 + a_2) > 0$  and  $a_i \leq a_j$ , then  $a_i < e_1$  and  $\frac{\partial U_i^1}{\partial a_i} > 0$ , which implies that  $(a_1, a_2)$  is not a Nash equilibrium; 2) if

<sup>2</sup>Use the Euler equation  $KF_K + LF_L = F$ .

<sup>3</sup>If  $K = 0$ , then no production arises and  $U_i^1 = e_i$ . But we can easily verify that such situation is not a Nash equilibrium, since  $\pi_1(K)$  is positive.

$g(a_1 + a_2) < 0$  and  $a_i \geq a_j$ , then  $a_i > 0$  and  $\frac{\partial U_i^1}{\partial a_i} < 0$ , which implies that  $(a_1, a_2)$  is not a Nash equilibrium; 3) if  $g(a_1 + a_2) = 0$  and  $a_i < a_j$ , then  $a_i < e_1$  and  $\frac{\partial U_i^1}{\partial a_i} > 0$ , which implies that  $(a_1, a_2)$  is not a Nash equilibrium. Hence, there is no Nash equilibrium other than  $\left(\frac{K^*}{2}, \frac{K^*}{2}\right)$ .

To show that  $\left(\frac{K^*}{2}, \frac{K^*}{2}\right)$  is in fact a Nash equilibrium, consider the function  $v(a_1) = U_1^1\left(a_1, \frac{K^*}{2}\right)$ . By Equation (1),  $v'(a_1) > 0$  if  $a_1 < \frac{K^*}{2}$  and  $v'(a_1) < 0$  if  $a_1 > \frac{K^*}{2}$ . Thus,  $a_1 = \frac{K^*}{2}$  is the best response to  $a_2 = \frac{K^*}{2}$ . Likewise, we can show that  $a_2 = \frac{K^*}{2}$  is the best response to  $a_1 = \frac{K^*}{2}$ . This completes the proof of this case.

Next, suppose  $K^* > 2e_1$ . It can easily be verified that there is no Nash equilibrium such that  $a_1 < e_1$ . Next, since the function  $a_2 \mapsto U_2^1(e_1, a_2)$  is continuous on  $[0, e_2]$ , there uniquely exists  $a_2^* \in [0, e_2]$  which attains maximum. If  $a_2^* \leq e_1$ , then  $g(K) > 0$  and thus  $\frac{\partial U_2^1}{\partial a_2} > 0$ , a contradiction. Hence,  $a_2^* > e_1$ . Also, if  $a_2^* + e_1 \geq K^*$ , then  $a_2^* > 0$  and  $\frac{\partial U_2^1}{\partial a_2} < 0$ , a contradiction. Hence,  $a_2^* < K^* - e_1$ . Therefore,  $g(e_1 + a_2^*) > 0$  and thus  $\frac{\partial U_1^1}{\partial a_1} > 0$ , which implies  $(e_1, a_2^*)$  is a Nash equilibrium. This completes the proof.

## 2.2. The Second Model

The demand function of consumer  $i$  on private consumption is simply

$$c_i = (u')^{-1}(p).$$

Hence, the total demand is  $2(u')^{-1}(p)$ . Thus, to sell  $F(K, L)$ , the firm must choose  $p = u'\left(\frac{F(K, L)}{2}\right)$ . Then, the profit function is

$$\pi(K, L) = u'\left(\frac{F(K, L)}{2}\right)F(K, L) - wL.$$

Now, we introduce an assumption.

ASSUMPTION 1: For any  $K > 0$ , there exists  $w^+(K) > 0$  such that  $L = 2$  is a maximum point of  $\pi(K, L)$ .

By first-order condition, we have

$$w^+(K) = u''\left(\frac{F(K, 2)}{2}\right)\frac{F(K, 2)F_L(K, 2)}{2} + u'\left(\frac{F(K, 2)}{2}\right)F_L(K, 2) < w^*(K).$$

Recall that  $p^*(K) = u'\left(\frac{F(K, 2)}{2}\right)$  is the unique value such that  $c_1 + c_2 = F(K, 2)$ . Thus, in equilibrium, the profit of the firm is

$$\pi_2(K) = p^*(K)F(K, 2) - 2w^+(K) = u'\left(\frac{F(K, 2)}{2}\right)F_K(K, 2)K - u''\left(\frac{F(K, 2)}{2}\right)F(K, 2)F_L(K, 2).$$

Then, the payoff function of this model  $U_i^2$  is

$$U_i^2(a_1, a_2) = e_i - a_i + \left(\theta_i - \frac{1}{2}\right)\pi_2(K) + u\left(\frac{F(K, 2)}{2}\right).$$

We want to focus on the case where the equilibrium of the first stage is well-defined. Therefore, we introduce an additional assumption:

ASSUMPTION 2:  $\frac{\pi_2(K)}{K}$  is decreasing in  $K$ .

Here, we provide a sufficient condition of ASSUMPTION 2 to show this assumption is not too strong.

**Proposition 2:** Suppose that ASSUMPTION 1 holds. Then, ASSUMPTION 2 holds if  $-u''(c)c$  is decreasing in  $c$ .

**Proof:** By ASSUMPTION 1 and the second-order necessary condition, we have

$$4u'F_{LL} + 4u''F_L^2 + u'''FF_L^2 + 2u''FF_{LL} \leq 0.$$

Meanwhile, since  $-u''(c)c$  is decreasing, we have

$$-u'' - cu''' \leq 0.$$

By homogeneity of degree one on  $F$ ,

$$KF_K = F - LF_L.$$

Further, both  $F_K$  and  $F_L$  are homogeneous of degree zero<sup>4</sup>. Therefore,

$$KF_{LK} = -LF_{LL},$$

and thus,

$$K^2F_{KK} = L^2F_{LL}.$$

Hence,

$$\begin{aligned} \frac{d}{dK}\left(\frac{\pi_2(K)}{K}\right) &= \frac{d}{dK}\left(u'F_K - \frac{u''FF_L}{K}\right) \\ &= u'F_{KK} + \frac{u''F_K^2}{2} - \frac{u'''F_KFF_L}{2K} - \frac{u''F_KF_L}{K} - \frac{u''FF_{LK}}{K} + \frac{u''FF_L}{K^2} \\ &= \frac{1}{K^2}\left[4u'F_{LL} + \frac{u''(F - 2F_L)^2}{2} - \frac{u'''F^2F_L}{2} + u''FF_L^2 + 2u''F_L^2 + 2u''FF_{LL}\right] \\ &\leq \frac{1}{K^2}\left[\frac{u''F^2}{2} - u''FF_L + (FF_L)\left(-u'' - \frac{u'''F}{2}\right)\right] \leq \frac{u''FF_K}{2K} < 0, \end{aligned}$$

and thus, ASSUMPTION 2 holds. This completes the proof.

It can be easily verified that  $-u''(c)c$  is decreasing for any  $u$  that has constant or decreasing relative risk aversion. Hence, ASSUMPTION 2 is not too strong<sup>5</sup>.

Define

$$h(K) = p^*(K)F_K(K, 2) + \frac{\pi_2(K)}{K} - 2,$$

and  $K^+$  as the unique solution of  $h(K) = 0$ . If such  $K^+$  does not exist, then let  $K^+ = +\infty$ . Then,

<sup>4</sup>For example, to differentiate  $F(aK, aL) = aF(K, L)$  with respect to  $K$ , we have  $aF_K(aK, aL) = aF_K(K, L)$  and thus  $F_K(aK, aL) = F_K(K, L)$ .

<sup>5</sup>Actually, we think that there may exist a weaker condition than ASSUMPTION 2 ensuring the following Propositions. However, since ASSUMPTION 2 itself is not too strong, we satiate this assumption, at least in this paper.

$\frac{\partial U_1^2}{\partial a_1} + \frac{\partial U_2^2}{\partial a_2} = h(a_1 + a_2)$  and thus  $\frac{\partial U_1^2}{\partial a_1} + \frac{\partial U_2^2}{\partial a_2} = 0$  if and only if  $a_1 + a_2 = K^+$ . Note that  $K^+$  is well-defined under ASSUMPTION 2. Since  $h(K) > 2g(K)$  for all  $K$ , we have  $K^+ > K^*$ .

We will show the following proposition:

**Proposition 3:** Under ASSUMPTIONS 1-2, there exists a Nash equilibrium  $(a_1^+, a_2^+)$ . If  $K^+ \leq 2e_1$ , then  $(a_1^+, a_2^+) = \left(\frac{K^+}{2}, \frac{K^+}{2}\right)$  is the unique Nash equilibrium. If not, then for any Nash equilibrium  $(a_1^+, a_2^+)$ ,  $a_1^+ = e_1$  and  $a_2^+ \in ]e_1, K^+ - e_1[$ .

**Proof:** It can be verified in the same way as **Proposition 1**.

### 2.3. Example: Improvement of Total Welfare

Suppose  $u(c_i) = 8c_i^{\frac{3}{4}}$ ,  $F(K, L) = K^{\frac{2}{3}}L^{\frac{1}{3}}$ , and  $e_1 = 0, e_2 = 50$ . By easy calculation, we have in model 1,

$$\frac{\partial U_2^1}{\partial a_2}(0, a_2) = -1 + \frac{3\sqrt{2}}{\sqrt{K}},$$

and thus,

$$a_2^* = 18.$$

Therefore, we have

$$p^*(a_2^*) = 2 \cdot 3^{\frac{2}{3}},$$

$$w^*(a_2^*) = 6,$$

$$U_1^1 + U_2^1 = 80.$$

In model 2, we have

$$\frac{\partial U_2^2}{\partial a_2}(0, a_2) = -1 + \frac{25\sqrt{2}}{8\sqrt{K}},$$

and thus,

$$a_2^+ = \frac{625}{32}.$$

Therefore,

$$p^+(a_2^+) = \frac{12}{25^{\frac{1}{3}}},$$

$$w^+(a_2^+) = \frac{75}{16},$$

$$U_1^2 + U_2^2 = \frac{2575}{32} > 80.$$

This example demonstrates that the existence of the case where monopoly improves the total surplus.

### 2.4. Comparative Statics

First, we argue the following result.

**Proposition 4:** Suppose that  $e_1$  is sufficiently low. Define

$$\Delta_j(a_1, a_2) = U_2^j(a_1, a_2) - U_1^j(a_1, a_2), j = 1, 2.$$

Then, we have  $\Delta_1(a_1^*, a_2^*) < \Delta_2(a_1^+, a_2^+)$  for any Nash equilibria  $(a_1^*, a_2^*)$  of model 1 and  $(a_1^+, a_2^+)$  of model 2 with  $a_1^+ \leq a_2^+$ .

**Proof:** It suffices to show that our claim holds if  $e_1 = 0$ , because this model is continuous on parameter  $e_1$ . Thus, we assume  $e_1 = a_1^* = a_1^+ = 0$ . By calculation in subsection 2.2, we have  $w^*(K) > w^+(K)$ , and thus  $\pi_2(K) > \pi_1(K)$  for any  $K > 0$ . Hence, we can easily verify that  $\pi_2(a_2^+) - a_2^+ > \pi_1(a_2^*) - a_2^*$ , and thus

$$\Delta_2(0, a_2^+) - \Delta_1(0, a_2^*) > 0,$$

which completes the proof.

Later, 2) of **Proposition 5** says that under ASSUMPTION 3, the restriction of **proposition 4** is removed. Here we introduce additional assumptions.

ASSUMPTION 3:  $\pi_2(K)$  is increasing in  $K$ .

ASSUMPTION 4:  $w^*(K) - w^+(K)$  is increasing in  $K$ .

**Remark:** ASSUMPTIONS 3-4 are not clear in the theoretical view. However, we think both conditions are natural in the real world. Usually, the bigger the capital obtained, the richer the firm becomes. Also, if the monopolistic power of the firm becomes strong, then we can expect wages to decrease. Note that by definition,  $w^*(K) - w^+(K)$  is always positive.

**Proposition 5:** Suppose ASSUMPTIONS 1-2 hold, and choose any Nash equilibria  $(a_1^*, a_2^*)$  of model 1 and  $(a_1^+, a_2^+)$  of model 2. Then,

- 1)  $\Delta_1 = \Delta_2$  if  $K^+ \leq 2e_1$ ;
- 2) Under ASSUMPTION 3,  $\Delta_2 \geq \Delta_1$  if  $a_2^* < a_2^+$ , and  $\Delta_2 > \Delta_1$  if  $a_2^+ > e_1$ ;
- 3) Under ASSUMPTIONS 3-4,  $a_2^* \leq a_2^+$  and  $\Delta_2 \geq \Delta_1$ . Further,  $\Delta_2 > \Delta_1$  if  $a_2^* < e_2$  and  $a_2^+ > e_1$ .

**Proof:** If  $K^+ \leq 2e_1$ , then  $\Delta_2 = \Delta_1 = e_2 - e_1$  and 1) holds.

Suppose ASSUMPTION 3 holds. By easy calculation,

$$\Delta_j(a_1, a_2) = (a_2 - a_1) \left( \frac{\pi_j(K)}{K} - 1 \right) + e_2 - e_1,$$

and thus,

$$\frac{\partial \Delta_j}{\partial a_2}(a_1, a_2) = \frac{2a_2}{K} \frac{\pi_j(K)}{K} + \frac{a_2 - a_1}{K} \pi_j'(K) - 1.$$

Recall that  $h(K^+) = 0$ . By ASSUMPTION 2,  $h$  is a decreasing function, and thus  $h(K) > 0$  for any  $K \in [K^*, K^+]$ . Thus, we have

$$p^*(K^+) F_K(K^+, 2) + \frac{\pi_2(K^+)}{K^+} = 2 < p^*(K) F_K(K, 2) + \frac{\pi_2(K)}{K}.$$

Since  $p^*(K^*) F_K(K^*, 2) = \frac{\pi_1(K^*)}{K^*} = 1$  and  $p^*(K) F_K(K, 2)$  is decreasing, we have  $p^*(K) F_K(K, 2) < 1$ , and thus,

$$\frac{\pi_2(K)}{K} > 1.$$

Note that  $\pi_2'(K) \geq 0$  if  $K \leq K^+$  and  $\frac{2a_2}{K} > 1$  if  $a_2 > a_1$ . Hence, for any  $(a_1, a_2)$  such that  $a_1 + a_2 \in [K^*, K^+]$  and  $a_2 > a_1$ ,  $\frac{\partial \Delta_2}{\partial a_2}(a_1, a_2) > 0$ .

If  $a_2^* \leq e_1$ , then  $\Delta_1(a_1^*, a_2^*) = e_2 - e_1 = \Delta_2(e_1, e_1)$ . Hence,  $\Delta_2(a_1^+, a_2^+) \geq \Delta_1(a_1^+, a_2^+) \geq \Delta_1(a_1^*, a_2^*)$  and if  $a_2^+ > e_1$ , then the inequality is strict. If not, then  $a_1^* = a_1^+ = e_1$ , and thus  $\Delta_2(e_1, a_2^+) > \Delta_1(e_1, a_2^*)$  since  $\pi_1(K) < \pi_2(K)$  for any  $K$ . Thus, (2) holds.

Lastly, suppose ASSUMPTIONS 3-4 hold. If  $K^* \leq 2e_1$ , then (1) and (2) imply (3). Otherwise,  $a_1^* = e_1$  and  $a_2^* > e_1$ . Now,

$$\frac{\partial U_2^2}{\partial a_2}(a_1, a_2) - \frac{\partial U_2^1}{\partial a_2}(a_1, a_2) = (2\theta_2 - 1) \left( (w^*)'(K) - (w^+)'(K) \right) + (1 - \theta_2) \frac{\pi_2(K) - \pi_1(K)}{K} > 0,$$

whenever  $a_2 > a_1$ . Hence, for any  $a_2 \in ]e_1, a_2^*]$ ,

$$\begin{aligned} 0 &\leq U_2^1(e_1, a_2^*) - U_2^1(e_1, a_2) = \int_{a_2}^{a_2^*} \frac{\partial U_2^1}{\partial a_2}(e_1, a) da \\ &< \int_{a_2}^{a_2^*} \frac{\partial U_2^2}{\partial a_2}(e_1, a) da = U_2^2(e_1, a_2^*) - U_2^2(e_1, a_2), \end{aligned}$$

and thus,  $a_2^+ \geq a_2^*$ . If  $a_2^* < e_2$ , then

$$\frac{\partial U_2^2}{\partial a_2}(e_1, a_2^*) > \frac{\partial U_2^1}{\partial a_2}(e_1, a_2^*) = 0,$$

and thus,  $a_2^+ > a_2^*$ . Thus, (3) holds.

### 3. Conclusion

We constructed a model of a monopoly with investors, and showed that monopoly did not necessarily decrease total welfare. Meanwhile, under mild assumptions monopoly exacerbated the expansion of the income gap. Therefore, we revealed a new aspect of the negative influence of monopoly.

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