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2D *J*–INEPT NMR Spectroscopy for CD*n* Groups: A Theoretical Study

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Abstract

2D *J*-INEPT NMR experiment is a combination of heteronusclear 2D *J*-Resolved and INEPT experiments. In this study, 2D *J*-INEPT experiment was analytically investigated by using product operator theory for weakly coupled IS_n (I = 1/2, S = 1; n = 1, 2, 3) spin systems. The obtained theoretical results represent the FID values of CD, CD_2 and CD_3 groups. In order to make Fourier transform of the obtained FID values, a Maple program is used and then simulated spectra for of 2D *J*-INEPT experiment are obtained for CD, CD_2 and CD_3 groups. It is found that 2D *J*-INEPT is a useful experiment for both polarisation transfer and 2D *J*-resolved spectral assignment for CD*n* groups in complex molecules.

1. Introduction

Polarization transfer from high natural abundance nucleus to low natural abundance nucleus is widely used for heteronuclear weakly coupled spin systems in liquid–state NMR experiments [1-3]. The most common examples for the polarization transfer experiments are Distortionless Enhancement by Polarization Transfer (DEPT) and Insensitive Nuclei Enhanced by Polarization Transfer (INEPT). They both are used to increase sensitive enhancement of ^{13}C spectra from spin-1/2 or spin-1 nucleus [4,5]. In order to resolve the chemical shift and spin-spin coupling parameters along the two different axes heteronuclear 2D *J*-Resolved NMR spectroscopy is used. Sometimes, spectral assignments of 2D *J*-Resolved NMR spectra become too difficult due to complex overlapping spectra. In order to overcome this problem, 2D *J*-INEPT experiment [6], which is the combination of 2D *J*-Resolved and INEPT NMR experiments, can be used.

As NMR is a quantum mechanical phenomenon, the product operator theory as a quantum mechanical method is widely used for the analytical description of multipulse NMR experiments on weakly coupled spin systems in liquids having spin-1/2 and spin-1 nuclei [7-18]. Analytical description of polarization transfer in INEPT experiment using product operator formalism has been presented for *IS* and IS_2 (I = 1/2 and S = 1) spin systems [12]. So far, the product operator description of 2D

J-INEPT NMR experiment is not investigated for heteronuclear weakly coupled IS_n (I = 1/2; S = 1; n = 1, 2, 3) spin systems.

In this study, by using product operator formalism, analytical description of 2D *J*-INEPT NMR experiment is presented for heteronuclear weakly coupled IS_n (I = 1/2; S = 1; n = 1, 2, 3) spin systems. Then, using the obtained theoretical results and a Maple program, the simulated spectra of the experiment are obtained for CD, CD_2 and CD_3 groups. Simulated spectra of 2D *J*-INEPT NMR spectroscopy are explained in detail for CD*n* groups. It is shown that this experiment can be used for the polarization transfer and *J*-resolved spectral assignment of CD*n* groups in complex molecules.

2. Theory

The product operator theory is the expansion of the density matrix operator in terms of matrix representation of angular momentum operators for individual spins. For *IS* (I = 1/2, S = 1) spin system, four Cartesian spin angular momentum operators for I = 1/2; E_I, I_x, I_y, I_z and nine Cartesian spin angular momentum operators for S=1; E_S, S_x, S_y, S_z, S_z^2 $\left[S_x, S_z\right]_+$, $\left[S_y, S_z\right]_+$, $\left[S_x, S_y\right]_+$, $\left(S_x^2 - S_y^2\right)$ can be easily found [19]. So, $4 \times 9 = 36$ product operators are obtained with direct products of these angular momentum operators for *IS* (I = 1/2, S = 1)

spin system.

Time dependency of the density matrix is given by [11]

$$\sigma(t) = \exp(-iHt)\sigma(0)\exp(iHt) \qquad (1)$$

where H is the total Hamiltonian which consists of radio frequency (r.f.) pulse, chemical shift and spin–spin coupling Hamiltonians and $\sigma(0)$ is the density matrix at $t = 0$. After employing the Hausdorff formula [11]

$$\exp(-iHt)A\exp(iHt)$$

$$= A - (it)[H,A] + \frac{(it)^2}{2!}\big[H,[H,A]\big] \qquad (2)$$

$$- \frac{(it)^3}{3!}\Big[H,\big[H,[H,A]\big]\Big] + \cdots$$

evolutions of product operators under the r.f. pulse, chemical shift and spin-spin coupling Hamiltonians can easily be obtained [7,11,13,16]. A complete product operator theory for IS ($I = 1/2$, $S = 1$) spin system and its application to some NMR experiments are presented elsewhere [16-18].

At any time during the experiment, the ensemble averaged expectation value of the spin angular momentum, e.g. for I_y, is

$$\langle I_y \rangle = \mathrm{Tr}\big[I_y\sigma(t)\big] \qquad (3)$$

where $\sigma(t)$ is the density matrix operator calculated from Eq. (6) at any time. As $\langle I_y \rangle$ is proportional to the magnitude of the y-magnetization, it represents the signal detected on y-axis. So, in order to estimate the free induction decay (FID) signal of a multiple-pulse NMR experiment, density matrix operator should be obtained at the end of the experiment.

3. Results and Discussion

In this study, the product operator formalism is used for the analytical description of 2D *J*-INEPT NMR experi-

ment. Pulse sequence of 2D *J*-INEPT, shown in **Figure 1**, is used [6]. The density matrix operator at each stage of the experiment is labelled with numbers. ^{13}C is treated as spin I and D (^{2}H) as spin S. For the analytical descriptions of the experiment, we have written a computer program in Mathematica which is very flexible for the implementation and the evolutions of the product operators under the Hamiltonians [20].

3.1. IS Spin System

σ_0 is the density matrix operator at thermal equilibrium for *IS* spin system. Evolutions of density matrix operator for each labelled point are obtained:

$$\sigma_0 = S_z \qquad (4)$$

$$\sigma_0 \xrightarrow{\;90^\circ_x(S)\;} \sigma_1 = -S_y \qquad (5)$$

$$\sigma_1 \xrightarrow{\;H_J\tau(\tau=1/4J)\;} \sqrt{2}I_zS_x + \frac{1}{\sqrt{2}}S_y \qquad (6)$$

$$\sigma_2 \xrightarrow{\;180^\circ_x(S)/180^\circ_x(I)\;} \sqrt{2}I_zS_x - \frac{1}{\sqrt{2}}S_y \qquad (7)$$

$$\sigma_3 \xrightarrow{\;H_J\tau(\tau=1/4J)\;} -2I_zS_x \qquad (8)$$

At the end of the experiment we obtain

$$\sigma_9 =$$
$$-2I_yS_zc_{2J}c_I + 2I_xS_z^2s_{2J}c_I + 2I_xS_zc_{2J}s_I + 2I_yS_z^2s_{2J}s_I \qquad (9)$$

In Equation (9), $s_{nJ} = \sin(nJ\pi t_1)$, $c_{nJ} = \cos(nJ\pi t_1)$, $s_{nI} = \sin(n\Omega_I t_2)$ and $c_{nI} = \cos(n\Omega_I t_2)$. In density matrix operator theory, only the last term of Equation (9) contributes to the signal as acquisition is taken along y-axes. It is necessary to obtain the $\mathrm{Tr}\big[I_yO\big]$ values of observable product operators indicated by O. For IS_n ($I = 1/2$, $S = 1$; $n = 1, 2, 3$) spin systems, $\mathrm{Tr}\big[I_yO\big]$ values of all the observable product operators can be found elsewhere [16].

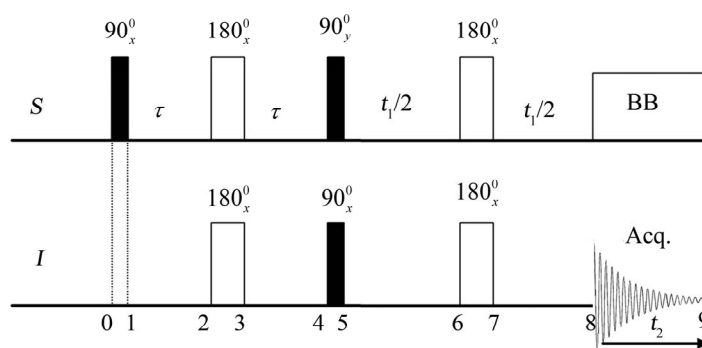Using $\mathrm{Tr}\big[I_yO\big]$ values and making trigonometric

Figure 1. Pulse sequence of 2D *J*-INEPT NMR experiment. Optimum τ value is $(4J)^{-1}$. BB: Broad band decoupling [6].

expansions,

$$M_y(t_1, t_2) \propto \langle I_y \rangle = \mathrm{Tr}[I_y \sigma_9](IS)$$
$$= -\cos[2J\pi t_1 + \Omega_I t_2] + \cos[2J\pi t_1 - \Omega_I t_2] \quad (10)$$

is obtained. This equation shows that the spin–spin coupling information appears on F1 axis and represents two coherences for *I* nucleus with phase of $2\pi J t_1 \pm \Omega_I t_2$. Therefore, they give doublets signals with opposite direction and no signal for central peak. The signals coordinate are $(-J, \Omega_I)$, (Ω_I) and (J, Ω_I) with intensity distribution of $-1{:}0{:}1$, respectively.

3.2. IS$_2$ Spin System

For *IS$_2$* spin system, σ_0 is the density matrix operator at thermal equilibrium:

$$\sigma_0 = S_{1z} + S_{2z} \quad (11)$$

The density matrix operator at the end of the experiment is

$$\sigma_9 = I_y (S_{1z}^2 + S_{2z}^2) \cos[2J\pi t_1 - \Omega_I t_2]$$
$$-2I_y S_{1z}^2 S_{2z}^2 \cos[2J\pi t_1 - \Omega_I t_2]$$
$$-I_y (S_{1z}^2 + S_{2z}^2) \cos[2J\pi t_1 + \Omega_I t_2]$$
$$+2I_y S_{1z}^2 S_{2z}^2 \cos[2J\pi t_1 + \Omega_I t_2] \quad (12)$$
$$+I_y S_{1z}^2 S_{2z}^2 \cos[4J\pi t_1 - \Omega_I t_2]$$
$$-I_y S_{1z}^2 S_{2z}^2 \cos[4J\pi t_1 + \Omega_I t_2]$$

Using $\mathrm{Tr}[I_y O]$ values;

$$\mathrm{Tr}[I_y \sigma_9](IS_2)$$
$$= -2\cos[4J\pi t_1 + \Omega_I t_2] - 2\cos[2J\pi t_1 + \Omega_I t_2] \quad (13)$$
$$+2\cos[2J\pi t_1 - \Omega_I t_2] + 2\cos[4J\pi t_1 - \Omega_I t_2]$$

is obtained. This equation represents five signals at the coordinates of $(-2J, \Omega_I)$, $(-J, \Omega_I)$, (Ω_I), (J, Ω_I) and $(2J, \Omega_I)$ with the relative intensities of $-2{:}{-}2{:}0{:}2{:}2$, respectively.

3.3. IS$_3$ Spin System

For *IS$_3$* spin system, applying the same procedure

$$\mathrm{Tr}[I_y \sigma_9](IS_3)$$
$$= -3\cos[6J\pi t_1 + \Omega_I t_2] - 6\cos[4J\pi t_1 + \Omega_I t_2]$$
$$-6\cos[2J\pi t_1 + \Omega_I t_2] + 6\cos[2J\pi t_1 - \Omega_I t_2] \quad (14)$$
$$+6\cos[4J\pi t_1 - \Omega_I t_2] + 3\cos[6J\pi t_1 - \Omega_I t_2]$$

is obtained. As seen in this equation, there exist seven signals at $(-3J, \Omega_I)$, $(-2J, \Omega_I)$, $(-J, \Omega_I)$, (Ω_I), (J, Ω_I), $(2J, \Omega_I)$ and $(3J, \Omega_I)$ coordinates with the relative intensities of $-3{:}{-}6{:}{-}6{:}0{:}6{:}6{:}3$, respectively.

3.4. Simulated Spectra

A computer program was written by Kanters *et. al.* for product operator description of NMR experiments and for the simulations of FID signals [21,22]. This is called Product Operator Formalism (POF.M) using Maple. In this study, in order to obtain the simulated spectra of the FID results, POF.M program is implemented for this experiment. $\mathrm{Tr}[I_y \sigma_9](IS_n)$ values obtained for *IS*, *IS$_2$* and *IS$_3$* spin systems are given in Eqs. (10), (13) and (14), respectively. They represent the FID signals of 2D *J*-INEPT NMR spectroscopy for CD$_n$ groups. By using $\mathrm{Tr}[I_y \sigma_9](IS_n)$ values, simulated spectra of this experiment are obtained and they are given in **Figures 2-4** for CD, CD$_2$ and CD$_3$ groups, respectively. In simulated
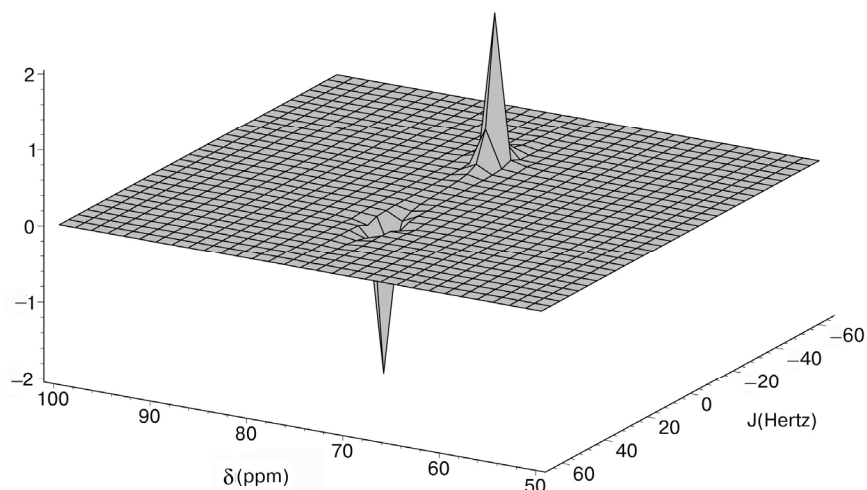

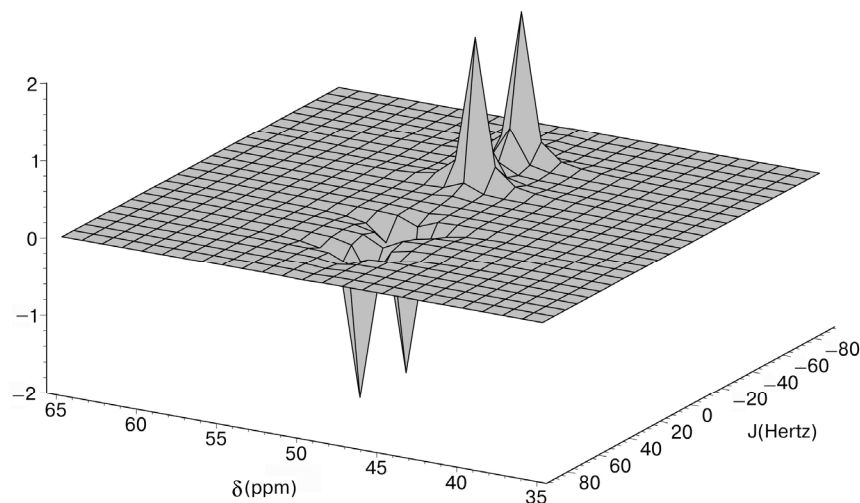Figure 2. Simulated spectrum of 2D *J*-INEPT NMR experiment for CD group.

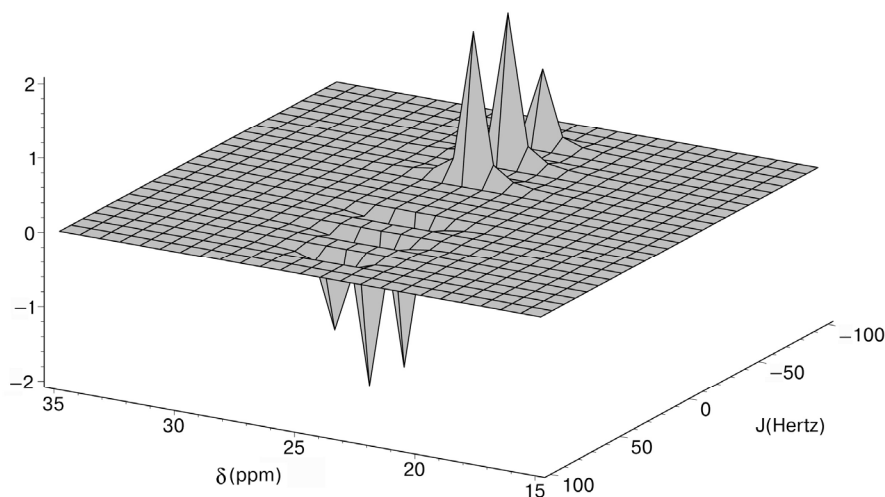**Figure 3. Simulated spectrum of 2D *J*–INEPT NMR experiment for CD$_2$ group.**

Figure 4. Simulated spectrum of 2D *J*-INEPT NMR experiment for CD$_3$ group.

spectra, ^{13}C chemical shift values of CD, CD$_2$ and CD$_3$ groups were assumed to be 75 ppm, 50 ppm and 25 ppm, respectively. Spin-spin coupling constants between ^{13}C and ^{2}D nuclei for all CD, CD$_2$ and CD$_3$ groups were taken as 25 Hz. It can be seen from the theoretical results and the simulated spectra that 2D *J*-INEPT NMR experiment can be used to identify CD, CD$_2$ and CD$_3$ groups from each other and also to determine spin-spin coupling constant between ^{13}C and ^{2}D nuclei in CD$_n$ groups.

4. Conclusions

In this study, by using product operator theory, analytical description of 2D *J*-INEPT NMR experiment is presented for CD$_n$ groups. The obtained theoretical results

represent the FID values of CD$_n$ groups. Then, in order to obtain the simulated spectra for CD, CD$_2$ and CD$_3$ groups, the Fourier transforms of the FID values are performed in Maple. Simulated spectra of 2D *J*-INEPT NMR spectroscopy are explained in detail for CD$_n$ groups. It is shown that, by using 2D *J*-INEPT NMR experiment, CD$_n$ groups can be identified from each other and ^1J(C,D) coupling constants can be determined in deuterated complex molecules.

5. References

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