

Ternary Zero Correlation Zone Sequence Sets for Asynchronous DS-CDMA

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Received 10 August 2014; revised 10 September 2014; accepted 8 October 2014

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Abstract

In this paper we propose a new class of ternary Zero Correlation Zone (ZCZ) sequence sets based on binary ZCZ sequence sets construction. It is shown that the proposed ternary ZCZ sequence sets can reach the upper bound on the ZCZ sequences. The performance of the proposed sequences set in asynchronous Direct Sequence-Code Division Multiple Access (DS-CDMA) system is evaluated. In the simulation we used two types of channels: Additive White Gaussian Noise (AWGN) and frequency non-selective fading with AWGN noise. The proposed ternary ZCZ sequence sets show better results, in term of Bit Error Rate (BER), than Hayashi's ternary ZCZ sequence sets.

Keywords

Hadamard Matrix, Zero Correlation Zone Sequences, Correlation, Asynchronous DS-CDMA, BER

1. Introduction

In Code Division Multiple Access (CDMA) systems, the number of spreading sequences determines the number of users and their correlation properties have a significant effect on anti-interference performance of the system [1]. Different types of codes used in communications systems have been studied in order to reduce Multiple Access Interference (MAI) [2] [3]. For an interference-free communication, spreading codes should have zero auto-correlation and zero cross-correlation functions at out-of-phase state. So, spreading sequences with good correlation properties can be used to improve the performance of CDMA systems [3]. One class of spreading sequences called Zero Correlation Zone (ZCZ) sequences possesses good correlation properties but only in specific zones called Zero Correlation Zone (Z_{CZ}). There are several intensive studies of CDMA systems using ZCZ

sequences sets [1] [3]-[6]. Various classes of ternary ZCZ sequences sets have been constructed [3] [4] and [7]-[13]. Ternary ZCZ sequences have the advantage over binary ZCZ sequences that is, for a given sequence length, the set has longer Z_{CZ} lengths and more sequences, and we may employ such hardware in binary ZCZ sequence sets system [3]. Any ternary ZCZ sequences set TZCZ (L',M',Z'_{CZ}) could be characterized by the sequence length L', the number of sequences M' and the zero correlation zone length Z'_{CZ} . An optimal ZCZ set is the one that provides the maximum number of codes for a given Z'_{CZ} and sequences lengths. The proposed ternary ZCZ sequences set with TZCZ (L',M',Z'_{CZ}) is derived from a binary ZCZ sequence set with BZCZ (L,M,Z_{CZ}) . When compared with previous works on ternary ZCZ sequence sets [3] [4] and [7]-[13], our proposed ZCZ sequence set approaches optimality.

The remainder of the paper is organized as follows.

After a review of preliminary considerations in Section 2, the proposed design for sequence construction is explained in Section 3. Example of new ZCZ sequence sets are presented in Section 4. The properties of the proposed sequence sets are explained in Section 5. In Section 6, we consider the performance of the proposed ternary ZCZ sequence sets compared with those in [3] and [10] for the asynchronous DS-CDMA system in both AWGN and nonselective fading with AWGN noise channels. At the end, we draw the concluding remarks.

2. Preliminaries

2.1. Definition 1

For a pair of sequences X_j and X_v of length L, the aperiodic correlation function (ACF) $\varphi_{(X_j, X_v)}(\tau)$ is defined as follows [14] [15]:

$$\varphi_{\left(X_{j}, X_{\nu}\right)}\left(\tau\right) = \begin{cases} \sum_{i=0}^{L-\tau-1} x_{j,i} x_{\nu,(i+\tau)}, & \text{if } 0 \le \tau < L \\ \sum_{i=0}^{L+\tau-1} x_{j,(i-\tau)} x_{\nu,\tau}, & \text{if } -L < \tau < 0 \\ 0, & \text{if } |\tau| \ge L \end{cases}$$
(1)

The periodic correlation function (PCF) between X_i and X_v at a lag τ is determined by [15] [16]:

$$\forall \tau, \ \theta_{(X_j, X_v)}(\tau) = \varphi_{(X_j, X_v)}(\tau \mod L) + \varphi_{(X_j, X_v)}((\tau \mod L) - L)$$
(2)

$$\forall \tau \ge 0, \ \theta_{\left(X_{j}, X_{\nu}\right)}\left(\tau\right) = \sum_{i=0}^{L-1} x_{j,i} x_{\nu,(i+\tau) \bmod(L)} \quad \text{and} \quad \theta_{\left(X_{j}, X_{\nu}\right)}\left(-\tau\right) = \theta_{\left(X_{\nu}, X_{j}\right)}\left(\tau\right) \tag{3}$$

2.2. Definition 2

A set of *M* sequences $\{X_j\}_{j=0}^{M-1} = \{X_0, X_1, \dots, X_{M-1}\}$ is called zero correlation zone sequence set if the periodic correlation functions satisfy [15] [16]:

$$0 < |\tau| \le Z_{cz}, \ \theta_{(X_j, X_j)}(\tau) = 0 \tag{4}$$

and

$$(j \neq v), \ 0 \leq |\tau| \leq Z_{cz}, \ \theta_{(X_j, X_v)}(\tau) = 0$$

$$(5)$$

3. Proposed Sequence Construction

The construction procedure of the new ternary sequence sets is presented. The construction is accomplished across the following three steps:

Step 1: The j^{th} row of the Hadamard matrix *H* of order *n* is indicated by $h_j = \lfloor h_{j,0}, h_{j,1}, \dots, h_{j,n-1} \rfloor$. A set of 2n sequences d_j , each of length 2n, is constructed as follows [17]:

For $0 \le j < n$

$$d_{j+0} = \left[-h_j, h_j\right] \tag{6}$$

$$d_{j+1} = \begin{bmatrix} h_j, h_j \end{bmatrix} \tag{7}$$

Step 2: For the first stage p = 0, and for a fixed integer value *n*, we can generate, based on the schema for sequence construction in [15], a series of sets $\{B_j\}_{j=0}^{2n-1}$ and $\{T_j\}_{j=0}^{2n-1}$ of 2n sequences as follows: Both sequences sets $\{B_j\}_{j=0}^{2n-1}$ and $\{T_j\}_{j=0}^{2n-1}$ are constructed from the sequences set $\{d_j\}_{j=0}^{2n-1}$. A pair of se-

quences B_{j+0} and B_{j+1} of length $L = (2^{p+2}n)$ are constructed by applying the interleaving operation of a sequence pair (d_{j+0}, d_{j+1}) in Equations (6) and (7) [17] and a pair of sequences T_{j+0} and T_{j+1} are constructed by the interleaving operation of a sequence pair d_{i+0} , d_{i+1} and from padding Z, which are zeros of length K, as follows:

For $0 \le j < n$,

$$B_{j+0} = \left[d_{j+0,0}, d_{j+1,0}, d_{j+0,1}, d_{j+1,1}, \cdots, d_{j+0,2n-1}, d_{j+1,2n-1}\right]$$
(8)

$$B_{j+1} = \left[d_{j+0,0}, -d_{j+1,0}, d_{j+0,1}, -d_{j+1,1}, \cdots, d_{j+0,2n-1}, -d_{j+1,2n-1}\right]$$
(9)

$$T_{j+0} = \left[d_{j+0,0}, Z, d_{j+1,0}, Z, d_{j+0,1}, Z, d_{j+1,1}, \cdots, d_{j+0,2n-1}, Z, d_{j+1,2n-1}, Z \right]$$
(10)

$$T_{j+1} = \left[d_{j+0,0}, Z, -d_{j+1,0}, Z, d_{j+0,1}, Z, -d_{j+1,1}, \cdots, d_{j+0,2n-1}, Z, -d_{j+1,2n-1}, Z \right]$$
(11)

The length of a pair sequences T_{j+0} and T_{j+1} in Equations (10) and (11) is $L' = (2^{p+2}n + 2^{p+K+1}n)$ and the member size of a pair sequence sets $\{T_j\}$ and $\{B_j\}$ is M = M' = 2n. **Step 3:** For p > 0, we may recursively build a new series of sets $\{B_j\}_{j=0}^{2n-1}$ and $\{T_j\}_{j=0}^{2n-1}$ by interleaving actual sets $\{B_j\}_{j=0}^{2n-1}$ and $\{T_j\}_{j=0}^{2n-1}$ respectively. Sets $\{B_j\}_{j=0}^{2n-1}$ and $\{T_j\}_{j=0}^{2n-1}$ are generated as follows: For $0 \le j < n$,

$$B_{j+0} = \begin{bmatrix} B_{j+0,0}, B_{j+1,0}, B_{j+0,1}, B_{j+1,1}, \cdots, B_{j+0,4n-1}, B_{j+1,4n-1} \end{bmatrix}$$
(12)

$$B_{j+1} = \begin{bmatrix} B_{j+0,0}, -B_{j+1,0}, B_{j+0,1}, -B_{j+1,1}, \cdots, B_{j+0,4n-1}, -B_{j+1,4n-1} \end{bmatrix}$$
(13)

$$T_{j+0} = \left[T_{j+0,0}, T_{j+1,0}, T_{j+0,1}, T_{j+1,1}, \cdots, T_{j+0, (4n(1+K))-1}, T_{j+1, (4n(1+K))-1} \right]$$
(14)

$$T_{j+1} = \left[T_{j+0,0}, -T_{j+1,0}, T_{j+0,1}, -T_{j+1,1}, \cdots, T_{j+0, (4n(1+K))-1}, -T_{j+1, (4n(1+K))-1} \right]$$
(15)

The length of both sequences B_{j+0} and B_{j+1} in Equations (12) and (13) is equal to $(2^{p+2}n)$ [17] and the length of both sequences T_{j+0} and T_{j+1} is equal to $(2^{p+2}n+2^{p+1+K}n)$.

4. Example of Construction

Figure 1 shows the periodic auto-correlation function (PACF) $(\forall j, j = v)$ given in Equation (3) of $B_{0,0}$, and **Figure 2** shows the periodic cross-correlation function (PCCF) $(\forall j, j \neq v)$ given in Equation (3) of B_{0+0} with B_{1+0} .

The PACF and PCCF confirm that $\{B_j\}$ is a ZCZ (32, 4, 4) sequence set. 2) For n = 2 and p = 2, the proposed $\{T_j\}_{j=0}^3$ is generated as follows:

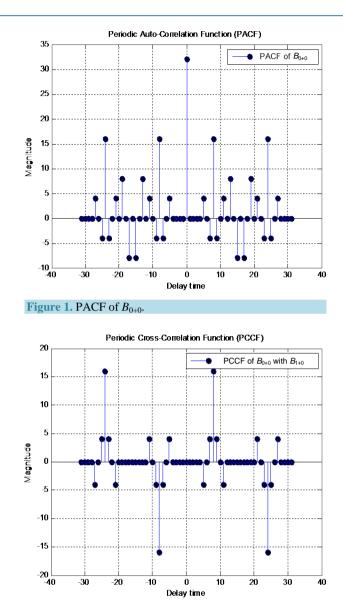
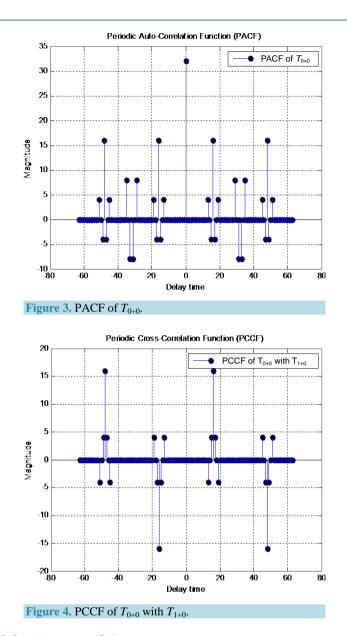




Figure 3 shows the PACF of T_{0+0} , and **Figure 4** shows the PCCF of T_{0+0} with T_{1+0} . The PACF and PCCF confirm that $\{T_j\}$ is a TZCZ (64, 4, 12) sequences set.



5. The Features of the Proposed Sequence

The Binary ZCZ sequence set with BZCZ $(L, M, Z_{CZ}) = (2^{p+2}n, 2n, 2^p)$ is optimal or approach optimal binary ZCZ sequences [17]. The length of T_j in Equations (14) and (15), equals $(L = 2^{p+2}n + 2^{p+1+K}n)$, is twice that of T_j in Equations (10) and (11). Let $F = (2^{p+1})$ and $Z_p = (2^{p+K})$, the proposed ternary ZCZ sequence set with TZCZ $(L', M', Z'_{CZ}) = (2^{p+2}n + 2^{p+K+1}n, 2n, 2^p + 2^{p+K}) = (FM + Z_pM, M, Z_{CZ} + Z_p)$ is derived from the binary ZCZ sequence set BZCZ $(L, M, Z_{CZ}) = (FM, M, Z_{CZ})$. If the obtained ternary ZCZ sequence set is optimal, it satisfies the ratio $M' = L'/(Z'_{CZ} + 1)$ [9] [10].

If p = 0 (the first iteration), the obtained sequence set is an optimal ternary ZCZ sequence set.

Proof: Let $R = \frac{M'(Z'_{CZ}+1)}{L'}$, we calculate the following ratio:

$$R = \frac{M'(Z'_{CZ}+1)}{L'} = \frac{2^p + 2^{p+K} + 1}{2^{p+1} + 2^{p+K}}$$

If p=0, $\forall K$, R=1.

If $p \neq 0$ and the number (K) of the padded zeros tends to the infinite, the obtained TZCZ sequences set (see Table 1 for n = 2) is asymptotically optimal.

Proof:
$$R = \frac{M'(Z'_{CZ}+1)}{L'} = \frac{2^K+1}{2^K}$$
 and $\lim_{K \to \infty} (R) = 1$

Noted that for this case, after spreading, the power of the symbol will decrease sharply, it is mandatory to compensate it, but this requirement increases Peak-to-Average Power Ratio (PAPR) and dynamic range of the transmitted signal [3].

From Table 2 we can see that the proposed sequence set in this paper can provide certain benefits. The length of sequences and Z_{CZ} will increase together while the number of sequences remains unchanged.

In the asynchronous DS-CDMA system, the time delay is typically in some chips, and for this, we can increase the Z_{CZ} to reduce MAI, but the member size will be relatively petty.

For a given member size M, we can find various sets of sequences with different lengths L and Z_{CZ} . As an example, assuming that M = 8 and K = 3 (see Table 3 for n = 4), we can draw upper bounds of our code performance and compare it with the Hayashi's approach.

The Hayashi's ternary sequence sets $ZCZ^{-1}((n+1)2^{p+2}, 2n, 2^{p+1}-1)$ in [10] based on Hadamard matrix, the

member size of the sequence set is $\frac{n}{n+1}$ of theoretical upper bound.

It is clear from **Table 3**, the proposed construction is one of the better type in the constructions mentioned in **Table 2**. Compared with the Hayashi's work, our proposed, in all cases, is optimal or approximate optimal ZCZ

Table 1. The parameters of the proposed ternary ZCZ sequence set and the ratio <i>R</i> .															
р	0	0	0	0	0	1	1	1	1	1	2	2	2	2	2
K	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
L'	16	24	40	72	136	32	48	80	144	272	64	96	160	288	544
$Z'_{\rm cz}$	3	5	9	17	33	6	10	18	34	66	12	20	36	68	132
R	1	1	1	1	1	0.88	0.92	0.95	0.97	0.99	0.81	0.88	0.93	0.96	0.98

Table 2	omparison of three types of ZCZ	seguence sets
Labic 2.	omparison of unce types of ZCZ	sequence sets.

Constructed ZCZ sequence sets	Length of sequences	Sequence element	Number size	Z _{cz}	Performance parameter <i>R</i>
ZCZ- $(2^{p+2}n, 2n, 2^p)$ in [17]	$2^{p+2}n$	Binary	2 <i>n</i>	2^{p}	$\frac{\left(2^p+1\right)}{2^{p+1}}$
ZCZ- $((n+1)2^{p+2}, 2n, 2^{p+1}-1)$ in [10]	$2^{p+2}(n+1)$	Ternary	2 <i>n</i>	$2^{p+1} - 1$	$\frac{n}{n+1}$
ZCZ- $(2^{p+2}n+2^{p+K+1}n,2n,2^p+2^{p+K})$ in this paper	$2^{p+2}n+2^{p+K+1}n$	Ternary	2 <i>n</i>	$2^{p} + 2^{p+K}$	$\frac{2^{p}+2^{p+K}+1}{2^{p+1}+2^{p+K}}$

Sequence sets	Hayashi's sequence set				Proposed sequence set				
L	80	160	320	640	80	160	320	640	
Z _{CZ}	7	15	31	63	9	18	36	72	
R	0.8	0.8	0.8	0.8	1	0.950	0.925	0.912	

sequence set. Consequently it has a higher Z_{CZ} and better performance parameter R than that given by Hayashi's construction.

6. The Performance of the Asynchronous DS-CDMA System Using the Proposed Ternary ZCZ Sequence Set

In this section, we consider the performance of the proposed ternary ZCZ sequence set used as spreading sequences for the DS-CDMA system shared by M asynchronous users simultaneously. In order to show this performance, the BER of an asynchronous DS-CDMA system over a frequency non-selective fading channel with AWGN noise estimated in [18] [19] is used:

BER =
$$Q\left(\frac{T\sqrt{\frac{P}{2}}}{\sqrt{\frac{N_0T}{4} + \frac{TP^2\gamma^2}{4} + (1+\gamma^2)\sigma_A^2(i)}}\right)$$
 (16)

where *P* is the common received power, *T* is the symbol duration, the *Q* function in Equation (16) is given by $Q(z) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-y^2}{2}} dy \quad [20], \text{ the term } \left(\sigma_n^2 = \frac{N_0 T}{4}\right) \text{ is the variance for the AWGN noise, the term } \left(TP^2\gamma^2/4\right)$

denote the faded component power from the user *i* and $\sigma_A^2(i)$ is the global (non-faded) interference MAI power for the required *i*-th user.

Let $\gamma^2 = 0$, the BER of an asynchronous DS-CDMA system over AWGN channels is:

$$BER = Q\left(\frac{T\sqrt{\frac{P}{2}}}{\sqrt{\frac{N_0T}{4} + \sigma_A^2(i)}}\right)$$
(17)

The MAI variance for the required *i*-th user can be calculated as [18] [19]

$$\sigma_A^2(i) = \frac{TP^2}{12L^3} \left(\sum_{\substack{m=1\\m\neq i}}^M r_{m,i} \right)$$
(18)

where $r_{m,i}$ in Equation (18) is the interference term caused by all other users m except the user *i*. The term $r_{m,i}$ from Equation (1) can be written as [18] [19]:

Let $\varphi(\tau) = \varphi_{(X_m, X_i)}(\tau)$

$$r_{m,i} = 2\sum_{\tau=1-L}^{L-1} \varphi(\tau)^2 + \sum_{\tau=1-L}^{L-1} \varphi(\tau) \varphi(\tau+1)$$
(19)

In **Figure 5** and **Figure 6** we compared the BER performance of the asynchronous DS-CDMA system employing Hayashi's TZCZ (40, 8, 3), Hayashi's TZCZ (20, 8, 1) and the proposed ternary ZCZ sequence sets with parameter TZCZ (32, 8, 3).

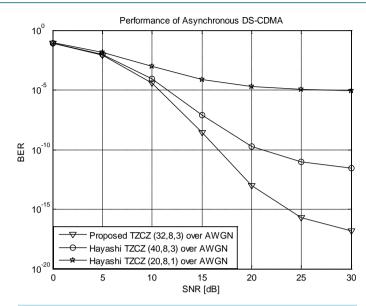
At BER = 10^{-4} in Figure 5, the system using constructed TZCZ can attain 01 dB and 06 dB gains over the same system employing Hayashi's TZCZ (40, 8, 3) and Hayashi's TZCZ (20, 8, 1) in AWGN respectively.

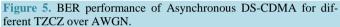
The BER performance, in **Figure 6** was simulated assuming a frequency nonselective fading channel with AWGN noise with the common faded power ratio $\gamma^2 = 0.1$. As we can see in **Figure 6**, the proposed TZCZ sequence sets show better performance than Hayashi's TZCZ sequence sets.

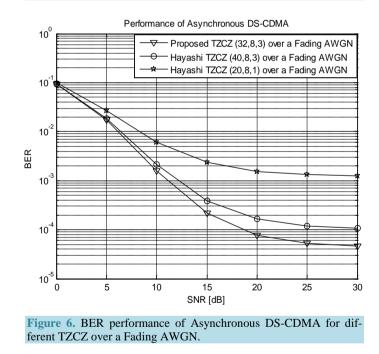
At BER = 0.0015 the system can attain 01 dB and 10 dB gains over the system employing Hayashi's TZCZ (40, 8, 3) and Hayashi's TZCZ (20, 8, 1) respectively. The amelioration over comparable Hayashi's ternary ZCZ sequences is due to the correlation properties of the proposed ternary sequence set.

7. Conclusion

A new construction method to create ternary ZCZ sequences set based on binary ZCZ sequence sets was proposed in this paper. This ternary ZCZ sequences set is either optimal or asymptotically optimal and their con-







struction is more flexible than other ternary ZCZ constructions. The asynchronous DS-CDMA using the proposed ternary ZCZ sequences shows better BER performance in both AWGN and frequency non-selective fading channel with AWGN noise.

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