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Extended (G'/G) Method Applied to the Modified Non-Linear Schrodinger Equation in the Case of Ocean Rogue Waves

Atock A. Nwatchok Stéphane[1*], Daika Augustin[2], Mbane Biouélé César[2]

[1]Department of Mathematics and Physics, National Advanced School of Engeneering, University of Yaounde I, Yaounde, Cameroon
[2]Department of Physics, Faculty of Science, University of Yaoundé I, Yaoundé, Cameroon
Email: [*]steph_atock@yahoo.fr

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Abstract

The existence of rogue (or freak) waves is now universally recognized and material proofs on the extent of damage caused by these ocean's phenomena are available. Marine observations as well as laboratory experiments show exactly that rogue waves occur in deep and shallow water. To study the behavior of freak waves in terms of their space and time evolution, that is, their motion and also in terms of mechanical transformations that these systems may suffer in their dealings with other systems, we derive a *modified nonlinear Schrödinger equation* modeling the propagation of rogue waves in deep water in order to seek analytic solutions of this nonlinear partial differential equation by using *generalized extended G'/G-expansion method* with the aid of mathematica. Particular attentions have been paid to the behavior of *rogue wave's amplitude* which highlights rogue wave's destructive power.

Keywords

Deep Water, Generalized Extended G'/G-Expansion Method, Rogue Waves

1. Introduction

During many centuries, rogue waves, unexpectedly high wave, strongly localized in space-time, have been widely reported all over the world. For a long time, they were thought to be a part of marine folklore, but with the development of instrumental measurements their existence has become evident and has been scientifically

[*]Corresponding author.

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proven [1]-[4]. An important milestone in the understanding of rogue wave dynamics occurred in 2001, when two European Space Agency satellites detected more than 10 individual giant waves over 25 m high during only three weeks of monitoring of the world's ocean [5] [6]. This evidence demonstrated that rogue events are not unique and highly improbable but occur regularly in the random wave field. Such extreme events are believed to have caused a number of marine accidents with subsequent pollution of large coastal areas, ship damage and human casualties [7] [8].

The understanding of extreme and rogue waves have significantly advanced recently. A number of extreme and rogue wave studies have been conducted theoretically, numerically, experimentally based on field data [9]-[22]. It has been demonstrated that the contribution of high order nonlinear mechanisms such as the modulational instability of uniform wave packets [23] [24] may give rise to substantially higher waves than that predicted by common second order wave models [25] [26].

Some authors [27] [28] are attempting to discover the probability of their appearances as well as studying the mechanism of their formation. Others have found exact solutions of nonlinear evolution equation (NLEEs) and have explained rogue waves phenomenon [29]-[32].

Recently, Wang *et al.* [33] introduced an expansion technique called the (G'/G)-expansion method and demonstrated that it was a powerful technique for seeking analytic solutions of nonlinear partial differential equations. Bekir [34] and Zedan [35] applied this method to obtain travelling wave solutions of various equations. A generalization of the method was given by Zhang *et al.* [36]. Also, Zhang *et al.* [37] made a further extension of the method for the evolution equations with variable coefficients.

The main aim of this paper is to seek exact solutions of modified nonlinear Schrodinger equation modeling the propagation of rogue waves in deep water with extended G'/G-expansion method. The rest of the paper is organized as follows. In Section 2, we describe the extended (G'/G)-expansion method to seek travelling wave solutions of nonlinear evolution equations and give the main steps of the method. In Section 3, we illustrate the method in detail with the modified non-linear Schrodinger equation in deep water. In Section 4, some conclusions are given.

2. Description of the Extended (G'/G)-Expansion Method

In this section, we describe the main steps of the extended (G'/G)-expansion method for finding travelling wave solutions of nonlinear evolution equations.

Suppose that we have a nonlinear partial differential equation for $u(x,t)$ in the form:

$$P(u,u_x,u_{xx},u_{xt},u_{tt})=0, \tag{1}$$

where P is a polynomial in its arguments. The essence of this approach can be formulated as follows:

Step 1. Find travelling wave solutions of Equation (1) by taking $u(x,t)=U(\xi), \xi=x-ct$ and transform Equation (1) to the ordinary differential equation:

$$Q(U,U',U'',\cdots)=0, \tag{2}$$

where prime denotes the derivative with respect to ξ.

Step 2. If possible, integrate Equation (2) term by term one or more times. This yields constant(s) of integration. For simplicity, the integration constant(s) can be set to zero.

Step 3. Introduce the solution $U(\xi)$ of Equation (2) in the finite series form:

$$U(\xi)=\sum_{i=-N}^{N} a_i\left(\frac{G'(\xi)}{G(\xi)}\right)^i, \tag{3}$$

where a_i are real constants with $a_N \neq 0$ to be determine, N is a positive integer to be determined. The function $G(\xi)$ is the solution of auxiliary linear ordinary differential equation:

$$G''(\xi)+\lambda G'(\xi)+\mu G(\xi)=0, \tag{4}$$

where λ and μ are real constants to be determined.

Step 4. Determine N. This, usually, can be accomplished by balancing the linear term(s) of highest order with the highest order nonlinear term(s) in Equation (2).

Step 5. Substitute (3) together with (4) into Equation (2) yields an algebraic set equation involving powers of (G'/G). Equating the coefficients of each power of (G'/G) to zero gives a system of algebraic equations for a_i, λ, μ and c. Then, we solve the system with the aid Mathematica to determine the constants. On the other hand, depending on the sign of the discriminant $\Delta = \lambda^2 - 4\mu$, the solutions of Equation (4) are well known for us. So, we can obtain exact solutions of Equation (1).

3. Application

Deep-water irrotational gravity waves propagating at the surface of an inviscid incompressible fluid are governed at third order in amplitude, by an equation first derived by Zakharov [38]:

$$i\partial_t b(K,t) = \omega(K) b(K,t) + \int T(k,p,q,r) b^*(p,t) b(q,t) b(r,t) \delta(k+p-q-r) \mathrm{d}p\mathrm{d}q\mathrm{d}r, \tag{5}$$

where $\omega(K) = (g|K|)^{\frac{1}{2}}$, and $T(k,p,q,r)$ is Krasitskii's kernel [39].

The modified nonlinear Schrodinger equation [40] obtained from Equation (5) is given by:

$$ia_t + Pa_{zz} + Q|a|^2 a = ir_1 a_{zzz} + ir_2 a^2 a_z^* - ir_3 |a|^2 a_z + r_4 a, \tag{6}$$

where the different coefficient are given by:

$$P = \omega_0 / 8k_0^2 \left(-3\cos(\alpha) + 2\right), \tag{7}$$

$$Q = -\omega_0 k_0^2 / 2, \tag{8}$$

$$r_1 = \omega_0 \cos(\alpha) / 16k_0^3 \left(-5\cos^2(\alpha) - 6\right), \tag{9}$$

$$r_2 = \omega_0 k_0 \cos(\alpha) / 4, \tag{10}$$

$$r_3 = 3\omega_0 k_0 / 2, \tag{11}$$

$$r_4 = k_0 |a|_z^2 \big|_{z=0}, \tag{12}$$

and

$$z = \left(x - \frac{\omega_0 t}{2k_0}\right)\cos(\alpha) + y\sin(\alpha). \tag{13}$$

ω_0 and k_0 are respectively the frequency and the wave number of the carrier wave. Since $a(z,t)$ is a complex function, it can be taken as:

$$a(z,t) = u(z,t) + iv(z,t), \tag{14}$$

an introduce a new variable :

$$\eta = \eta_1 z + \eta_2 t. \tag{15}$$

Replacing Equation (14) into Equation (6), separating the real and imaginary part and using the relation (15) leads to a system of equations:

$$\eta_2 u_\eta + \eta_1^2 P v_{\eta\eta} + Q(u^2+v^2)v = r_1\eta_1^3 u_{\eta\eta\eta} - 2r_2\eta_1 uvv_\eta + (r_2 - r_3)(u^2+v^2)\eta_1 u_\eta \tag{16}$$

$$\eta_2 v_\eta + \eta_1^2 P u_{\eta\eta} - Q(u^2+v^2)u = r_1\eta_1^3 v_{\eta\eta\eta} - 2r_2\eta_1 uvu_\eta - (r_2 - r_3)(u^2+v^2)\eta_1 v_\eta - r_4 u \tag{17}$$

Now we make an ansatz (3) for the solution of Equations (16) and (17). By balancing the terms $u_{\eta\eta\eta}$ and $u^2 v$, $v_{\eta\eta\eta}$ and $v^2 u$ in Equations (16) and (17) yields the leading order $N = 1$. Therefore, we can write the solution of Equation (16) and Equation (17) in an extended symmetric form:

$$u(\eta) = \alpha_{-1} f^{-1} + \alpha_0 + \alpha_1 f, \tag{18}$$

$$v(\eta) = \beta_{-1} f^{-1} + \beta_0 + \beta_1 f, \tag{19}$$

where $f = G'(\eta)/G(\eta)$ and $G(\eta)$ satisfies the second-order ordinary differential Equation (4). By using Equation (4), Equation (18) and Equation (19), we derive:

$$u' = -\alpha_{-1} f' f^{2} + \alpha_1 f', \tag{20}$$

$$\begin{aligned} u'' = {} & \lambda\alpha_{-1} + 3\lambda\mu\alpha_{-1} f^{-2} + 2\mu^2\alpha_{-1} f^{-3} + \left(\lambda^2\alpha_{-1} + 2\mu\alpha_{-1}\right) f^{-1} \\ & + 2\alpha_{-1} f^3 + 2\lambda\alpha_1 f^2 + 2\mu\alpha_1 f - \lambda\alpha_1, \end{aligned} \tag{21}$$

$$\begin{aligned} u''' = {} & \left(\lambda^2\alpha_{-1} + 2\mu\alpha_{-1} - 2\mu^2\alpha_1\right) + \left(6\lambda\mu\alpha_{-1} + \lambda\left(\lambda^2\alpha_{-1} + 2\mu\alpha_{-1}\right)\right) f^{-1} \\ & + \left(6\lambda^2\mu\alpha_{-1} + 6\mu^2\alpha_{-1} + \mu\left(\lambda^2\alpha_{-1} + 2\mu\alpha_{-1}\right)\right) f^{-2} + 12\lambda\mu^2\alpha_{-1} f^{-3} \\ & + 6\mu^3\alpha_{-1} f^{-4} - 10\lambda\alpha_1 f^3 + \left(-4\lambda^2\alpha_1 - 8\mu\alpha_1\right) f^2 - 6\lambda\mu\alpha_1 f, \end{aligned} \tag{22}$$

$$v' = -\beta_{-1} f' f^{-2} + \beta_1 f', \tag{23}$$

$$\begin{aligned} v'' = {} & \lambda\beta_{-1} + 3\lambda\mu\beta_{-1} f^{-2} + 2\mu^2\beta_{-1} f^{-3} + \left(\lambda^2\beta_{-1} + 2\mu\beta_{-1}\right) f^{-1} \\ & + 2\beta_{-1} f^3 + 2\lambda\beta_1 f^2 + 2\mu\beta_1 f - \lambda\beta_1, \end{aligned} \tag{24}$$

$$\begin{aligned} v''' = {} & \left(\lambda^2\beta_{-1} + 2\mu\beta_{-1} - 2\mu^2\beta_1\right) + \left(6\lambda\mu\beta_{-1} + \lambda\left(\lambda^2\beta_{-1} + 2\mu\beta_{-1}\right)\right) f^{-1} \\ & + \left(6\lambda^2\mu\beta_{-1} + 6\mu^2\beta_{-1} + \mu\left(\lambda^2\beta_{-1} + 2\mu\beta_{-1}\right)\right) f^{-2} + 12\lambda\mu^2\beta_{-1} f^{-3} \\ & + 6\mu^3\beta_{-1} f^{-4} - 10\lambda\beta_1 f^3 + \left(-4\lambda^2\beta_1 - 8\mu\beta_1\right) f^2 - 6\lambda\mu\beta_1 f, \end{aligned} \tag{25}$$

Substituting these expressions into Equations (16) and (17), we collect and setting all terms of the same power of $G'(\eta)/G(\eta)$ to zero, and then solve the resulting system we obtain:

$$\alpha_0 = \beta_0 = 0, \quad \lambda = \lambda, \quad \mu = \mu, \tag{26}$$

$$\alpha_1^2 = \left(\frac{2r_2^2}{3r_1\eta_1^2\left(r_2 - r_3\right)}\right)^{-1}, \quad \beta_1^2 = \left(\frac{2r_2^2}{3r_1\eta_1^2\left(r_2 - r_3\right)}\right)^{-1}, \tag{27}$$

$$\alpha_{-1}^2 = \left(\frac{2r_2^2}{3\mu^2 r_1\eta_1^2\left(r_2 - r_3\right)}\right)^{-1}, \quad \beta_{-1}^2 = \left(\frac{2r_2^2}{3\mu^2 r_1\eta_1^2\left(r_2 + r_3\right)}\right)^{-1}. \tag{28}$$

The ODE Equation (4) may then be solved exactly and admits the following solutions:

$$G_1(\eta) = c_1 e^{x_1\eta} + c_2 e^{x_2\eta}, \quad \text{when } \lambda^2 - 4\mu \succ 0, \tag{29}$$

$$G_2(\eta) = e^{x_4\eta}\left(c_1\cos\left(x_3\eta\right) + c_2\sin\left(x_3\eta\right)\right), \quad \text{when } \lambda^2 - 4\mu \prec 0, \tag{30}$$

$$G_3(\eta) = \left(c_1 + c_2\eta\right)e^{x_4\eta}, \quad \text{when } \lambda^2 - 4\mu = 0. \tag{31}$$

With:

$$x_1 = \frac{-\lambda - \sqrt{\lambda^2 - 4\mu}}{2}, \quad x_2 = \frac{-\lambda + \sqrt{\lambda^2 - 4\mu}}{2}, \quad x_3 = \frac{\sqrt{4\mu - \lambda^2}}{2} \quad \text{and} \quad x_4 = \frac{-\lambda}{2} \tag{32}$$

where C_1 and C_2 are arbitrary constants, we therefore obtain three categories of travelling wave solutions that propagate in deep water [41]:

First type. Hyperbolic functions travelling wave solutions.

a) if $\lambda^2 - 4\mu \succ 0$ and $c_1 * c_2 \succ 0$, we have:

$$f = \operatorname{thanh}\left[\frac{\sqrt{\lambda^2 - 4\mu}}{2}\eta + \frac{1}{2}\ln\left(\frac{c_2}{c_1}\right)\right] \tag{33}$$

Then:

$$u(\eta)=\frac{\alpha_{-1}}{\left[1-\mathrm{sech}^2\left[\frac{\sqrt{\lambda^2-4\mu}}{2}\eta+\frac{1}{2}\ln\left(\frac{c_2}{c_1}\right)\right]\right]^{1/2}}+\alpha_1\left[1-\mathrm{sech}^2\left[\frac{\sqrt{\lambda^2-4\mu}}{2}\eta+\ln\left(\frac{c_2}{c_1}\right)\right]\right]^{1/2} \quad (34)$$

$$v(\eta)=\frac{\beta_{-1}}{\left[1-\mathrm{sech}^2\left[\frac{\sqrt{\lambda^2-4\mu}}{2}\eta+\frac{1}{2}\ln\left(\frac{c_2}{c_1}\right)\right]\right]^{1/2}}+\beta_1\left[1-\mathrm{sech}^2\left[\frac{\sqrt{\lambda^2-4\mu}}{2}\eta+\ln\left(\frac{c_2}{c_1}\right)\right]\right]^{1/2} \quad (35)$$

If $\alpha_{-1}=0$ Equations (34) and (35) can be expressed in the well-known solitary wave solution of the KdV equation as follows:

$$u(\eta)=\alpha_1+\varsigma_1\mathrm{sech}^2\left[\frac{\sqrt{\lambda^2-4\mu}}{2}\eta+\ln\left(\frac{C_2}{C_1}\right)\right] \quad (36)$$

$$v(\eta)=\beta_1+\xi_1\mathrm{sech}^2\left[\frac{\sqrt{\lambda^2-4\mu}}{2}\eta+\ln\left(\frac{C_2}{C_1}\right)\right] \quad (37)$$

b) if $\lambda^2-4\mu\succ 0$ and $c_1*c_2\prec 0$, we have:

$$f=\mathrm{cothanh}\left[\frac{\sqrt{\lambda^2-4\mu}}{2}\eta+\frac{1}{2}\ln\left(\frac{-c_2}{c_1}\right)\right] \quad (38)$$

$$u(\eta)=\frac{\alpha_{-1}}{\mathrm{cothanh}\left[\frac{\sqrt{\lambda^2-4\mu}}{2}\eta+\frac{1}{2}\ln\left(\frac{-c_2}{c_1}\right)\right]}+\alpha_1\mathrm{cothanh}\left[\frac{\sqrt{\lambda^2-4\mu}}{2}\eta+\ln\left(\frac{-c_2}{c_1}\right)\right] \quad (39)$$

$$v(\eta)=\frac{\beta_{-1}}{\mathrm{cothanh}\left[\frac{\sqrt{\lambda^2-4\mu}}{2}\eta+\frac{1}{2}\ln\left(\frac{-c_2}{c_1}\right)\right]}+\beta_1\mathrm{cotanh}\left[\frac{\sqrt{\lambda^2-4\mu}}{2}\eta+\ln\left(\frac{-c_2}{c_1}\right)\right] \quad (40)$$

Second type. Trigonometric functions travelling wave solutions.

c) if $\lambda^2-4\mu\prec 0$ and $c_1*c_2\succ 0$, we have:

$$f=x_4+x_3\tan\left[\frac{\sqrt{4\mu-\lambda^2}}{2}\eta-\arctan\left(\frac{c_2}{c_1}\right)\right] \quad (41)$$

Then:

$$u(\eta)=\frac{\alpha_{-1}}{x_4+x_3\tan\left[\frac{\sqrt{4\mu-\lambda^2}}{2}\eta-\arctan\left(\frac{c_2}{c_1}\right)\right]}+\alpha_1\left(x_4+x_3\tan\left[\frac{\sqrt{4\mu-\lambda^2}}{2}\eta-\arctan\left(\frac{c_2}{c_1}\right)\right]\right) \quad (42)$$

$$v(\eta)=\frac{\beta_{-1}}{x_4+x_3\tan\left[\frac{\sqrt{4\mu-\lambda^2}}{2}\eta-\arctan\left(\frac{c_2}{c_1}\right)\right]}+\beta_1\left(x_4+x_3\tan\left[\frac{\sqrt{4\mu-\lambda^2}}{2}\eta-\arctan\left(\frac{c_2}{c_1}\right)\right]\right) \quad (43)$$

d) if $\lambda^2 - 4\mu \prec 0$ and $c_1 * c_2 \prec 0$, we have:

$$f = x_4 + x_3 \text{cotg}\left[\frac{\sqrt{4\mu-\lambda^2}}{2}\eta + \arctan\left(\frac{c_1}{c_2}\right)\right] \tag{44}$$

Then:

$$u(\eta) = \frac{\alpha_{-1}}{x_4 + x_3 \text{cotg}\left[\frac{\sqrt{4\mu-\lambda^2}}{2}\eta - \arctan\left(\frac{C_1}{C_2}\right)\right]} + \alpha_1\left(x_4 + x_3 \text{cotg}\left[\frac{\sqrt{4\mu-\lambda^2}}{2}\eta - \arctan\left(\frac{C_1}{C_2}\right)\right]\right) \tag{45}$$

$$v(\eta) = \frac{\beta_{-1}}{x_4 + x_3 \text{cotg}\left[\frac{\sqrt{4\mu-\lambda^2}}{2}\eta - \arctan\left(\frac{C_1}{C_2}\right)\right]} + \beta_1\left(x_4 + x_3 \text{cotg}\left[\frac{\sqrt{4\mu-\lambda^2}}{2}\eta - \arctan\left(\frac{C_1}{C_2}\right)\right]\right) \tag{46}$$

Third type. Rational functions travelling wave solutions.

e) if $\lambda^2 - 4\mu = 0$, we have:

$$f = x_4 + \frac{c_2}{c_1 + c_2\eta} \tag{47}$$

Then:

$$u(\eta) = \frac{\alpha_{-1}(c_1 + c_2\eta)}{c_2 + (c_1 + c_2\eta)x_4} + \alpha_1\left(\frac{c_2}{c_1 + c_2\eta} + x_4\right) \tag{48}$$

$$v(\eta) = \frac{\beta_{-1}(c_1 + c_2\eta)}{c_2 + (c_1 + c_2\eta)x_4} + \beta_1\left(\frac{c_2}{c_1 + c_2\eta} + x_4\right) \tag{49}$$

Hyperbolic and trigonometric solutions can also be express in the form [42]-[46]:

$$f = -\frac{\lambda}{2} + \frac{\sqrt{\Delta}}{2} * \frac{K_1 \cosh\left[\frac{\sqrt{\Delta}}{2}z\right] + K_2 \sinh\left[\frac{\sqrt{\Delta}}{2}z\right]}{K_2 \cosh\left[\frac{\sqrt{\Delta}}{2}z\right] + K_1 \sinh\left[\frac{\sqrt{\Delta}}{2}z\right]} \tag{50}$$

with $\Delta = \lambda^2 - 4\mu$

$$f = -\frac{\lambda}{2} + \frac{\sqrt{\Delta}}{2} * \frac{K_1' \cos\left[\frac{\sqrt{\Delta}}{2}z\right] + K_2' \sin\left[\frac{\sqrt{\Delta}}{2}z\right]}{K_2' \cos\left[\frac{\sqrt{\Delta}}{2}z\right] + K_1' \sin\left[\frac{\sqrt{\Delta}}{2}z\right]} \tag{51}$$

with $\Delta = 4\mu - \lambda^2$

We are on deep water, the behavior of the ocean cant not be determine exactly. We don't know the form that wave will takes. All cases are possible. The **Figures 1-3** show the behavior of the exact solution of the modified nonlinear Schrödinger equation modeling the propagation of rogue waves in deep water for certain values of the system parameters. The squared modulus of the amplitude of the wave $|a|^2$ is plots versus the coordinates x ad y for a given value of time.

The snapshot of **Figure 1** is a typical representation of one pulse-type solutions [47], proof that the solutions thus obtained are general and take into account the solutions already existing in the open literature. When the system parameters vary, there is a sudden variation in the amplitude of wave **Figure 2** and **Figure 3**. These results allow us to confirm the fact that the amplitudes of waves may vary in exceptional cases by simply changing a parameter of the system, take us with amplitude of one to over one hundred without any trial. These results re-

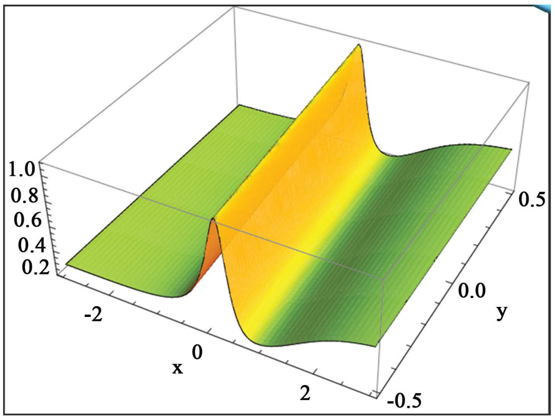

Figure 1. One pulse with low amplitude. $\lambda=1$, $\mu=1$, $K_1=-0.01$, $K_2=0.002$, $\alpha=\pi/8$, $\omega_0=2.5$, $k_0=0.4$.

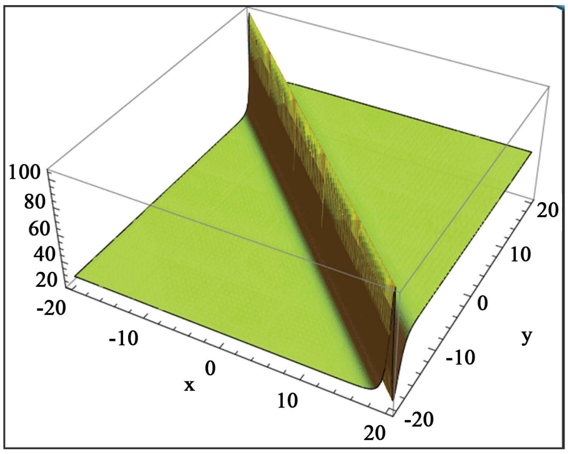

Figure 2. One pulse with large amplitude presented like a barrier. $\lambda=1$, $\mu=1$, $K_1=-0.01$, $K_2=0.002$, $\alpha=\pi/4$, $\omega_0=2.5$, $k_0=0.4$.

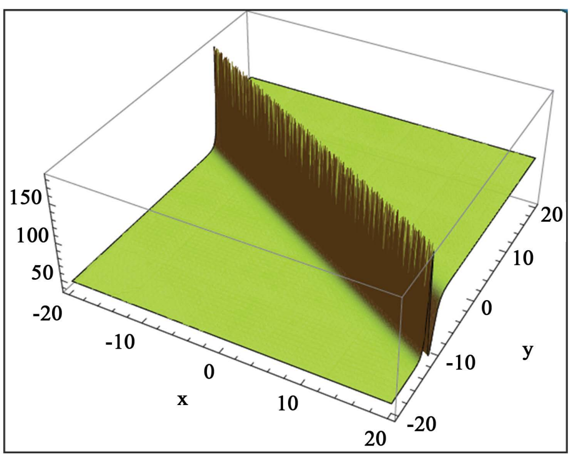

Figure 3. One pulse with large amplitude presented like a giant saw. $\lambda=1$, $\mu=1$, $K_1=-0.01$, $K_2=0.002$, $\alpha=\pi/3$, $\omega_0=2.5$, $k_0=0.4$.

flect very well the situation encountered by sailors in ocean: the free surface of a body of deep water just move from a situation of absolute calm to the appearance of a gigantic wall of water [48]. **Figure 2** and **Figure 3** are as gigantic barrier 2 have a regular summit while 3 has a peak saw tooth which allow us to conclude that in most cases, the tops of the waves is not regular. When these kinds of waves propagate at high speed and collided with a tanker or striking an oil platform, these structures will be send to the mat with frightening speed and efficiency. Freaks waves arise abruptly, when one of the form 2 or 3 surprises ships from below, it behaves like a giant saw, cutting steel look like a knife on the butter or breaks it in two because the cumulative effects of their considerable height and wavelength literally raised the ship from both ends. It central part is then in vacuum, or at least less driven by water and would then be subjected to enormous stresses.

4. Conclusion

In this paper, generalized extended (G'/G)-expansion method is used to obtain the exact solutions of modified nonlinear Schrödinger equation in deep water. Particular attentions have been paid to the amplitude of the found solutions and to the relationship between dynamics of these solutions and some characteristics of extreme abnormal sea wave with abnormal shape. The solutions are expressed in the form of hyperbolic functions, trigonometric functions and rational solutions from which some special solutions including the known solitary wave solution are derived by setting appropriate values for the parameter. Compared with other methodologies mentioned in introduction, this method is direct, concise, elementary and it can be implemented in more complicated nonlinear equations by using symbolic computations. One pulse with large amplitude presented like a barrier or like a giant saw are very dangerous for sailors, offshore oil platforms and coastal structures. The representation on this paper give partially the reasons of the damage caused on the hulls of super tankers when they collide with this crazy waves like that in **Figure 4** and **Figure 5**.

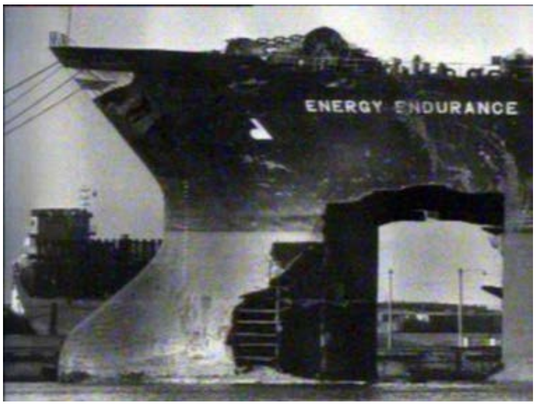

Figure 4. Profile view of damage caused on energy endurance [49].

Figure 5. The WILSTAR Norvegian cargo boat hit by a rogue waves [49].

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