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Bianchi Type-V Cosmological Model with Linear Equation of State in Brans-Dicke Theory of Gravitation

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Abstract

A Bianchi type-V space time is considered with linear equation of state in the scalar tensor theory of gravitation proposed by Brans and Dicke. We use the assumption of constant deceleration parameter and power law relation between scalar field ϕ and scale factor R to find the solutions. Some physical and kinematical properties of the model are also discussed.

Keywords

Bianchi Type-V Model, Brans-Dicke Theory, Linear Equation of State

1. Introduction

Einstein's [1] general theory of relativity has provided a sophisticated theory of gravitation which has been very successful in describing gravitational phenomenon and also served as a basis for models of the universe. The homogeneous isotropic expanding model based on general relativity appears to provide a grand approximation to the observed large-scale properties of the universe. However since Einstein first published his theory of gravitation there have been many criticisms of general relativity because of lack of certain "desirable" features in the theory. For example, Einstein himself pointed out that general relativity does not account satisfactorily for inertial properties of matter *i.e.* Mach's principle is not substantiated by general relativity. So, in recent years, several theories of gravitation have been proposed as alternatives for Einstein's theory. The most important among them are scalar-tensor theories of gravitation formulated by Jordan [2], Brans and Dicks [3], Nordtvedt [4], Ross [5] and Schmidt *et al.* [6].

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Brans and Dicke [3] introduced a scalar-tensor theory of gravitation involving a scalar function in addition to the familiar general relativistic metric tensor. In this theory the scalar field has the dimensions of inverse of the gravitational constant and its role is confined to its effect on gravitational field equations. Several aspects of Brans-Dicke cosmology have been extensively investigated by many authors. The work of Singh and Rai [7] give a detailed survey of Brans-Dicke cosmological models discussed by several authors. Nariai [8], Reddy and Rao [9], Banerjee and Santos [10], Singh *et al.* [11], Ram [12], Ram and Singh [13], Berman *et al.* [14], Reddy *et al.* [15] are some of the authors who have investigated several aspects of this theory. Reddy and Venkateswara Rao [16] and Reddy *et al.* [17] discussed Bianchi type I cosmological models with negative constant declaration parameter in scalar tensor theories formulated by Brans-Dicke theory and Saez-Ballester theory [18].

There has been considerable interest in the study of spatially homogeneous, anisotropic cosmological models. Bianchi type V space-times, being a straight forward generalization of FRW universe with negative curvature are interesting to study because they contain isotropic special cases and allow arbitrarily small anisotropy levels at any instant of cosmic time. Fransworth [19], Collins [20], Maartens and Nel [21], Wainwright *et al.* [22] have studied Bianchi type V cosmological models. Nayak and Sahoo [23] have investigated Bianchi type V models for matter distribution admitting anisotropic pressure and heat flow. Several authors studied Bianchi type V in different context [24]-[37].

Motivated by above discussion, in this paper we have obtained exact solutions of the Einstein field equations of Bianchi type V Cosmological model with linear equation of state in the scalar tensor theory of gravitation proposed by Brans and Dicke.

2. The Metric and Field Equations

We consider Bianchi type-V space time described by the line element

$$\mathrm{d}s^2 = \mathrm{d}t^2 - A^2\mathrm{d}x^2 - \mathrm{e}^{-2\alpha x}\left(B^2\mathrm{d}y^2 + C^2\mathrm{d}z^2\right) \tag{1}$$

where A, B and C are function of cosmic time t.

Brans-Dicke field equations for combined scalar and tensor field are

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) - \phi^{-1}\left(\phi_{;ij} - g_{ij}\phi^{;k}_{;k}\right) \tag{2}$$

and

$$\varphi^{;k}_{;k} = \frac{8\pi}{3+2\omega}T \tag{3}$$

where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is an Einstein tensor, R is the scalar curvature, ω is a dimension less constant and T is the stress energy tensor of the matter, comma and semicolon denote partial and co-variant differentiation respectively.

Also, we have energy-conservation equation

$$T^{ij}_{;j} = 0 \tag{4}$$

are consequences of the field Equations (2) and (3).

The energy momentum tensor

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij} \tag{5}$$

where ρ is the energy density of the perfect fluid and p is its pressure. Here the four velocity vector u_i and x_i satisfy the standard relations

$$u_i u^i = -x_i x^i = 1 \text{ and } u^i x_i = 0 \tag{6}$$

Here we use line are quation of state given by Thirukkanesh and Maharaj [38] for perfect fluid as

$$p = \gamma\rho - \gamma_0 \tag{7}$$

where γ and γ_0 constants.

In the moving coordinate system, we have from (5) and (6)

$$T_1^1 = T_2^2 = T_3^3 = -p, \quad T_0^0 = \rho, \quad T = \rho - 3p \tag{8}$$

Now the Brans-Dicke field Equations (2) and (3) for the metric (1) with the help of (5) to (7) can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = -8\pi\phi^{-1}p \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right) = -8\pi\phi^{-1}p \tag{10}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} + \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) = -8\pi\phi^{-1}p \tag{11}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{3\alpha^2}{A^2} - \frac{\omega\dot{\phi}^2}{2\phi^2} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 8\pi\phi^{-1}\rho \tag{12}$$

$$2\alpha\frac{\dot{A}}{A} - \alpha\frac{\dot{B}}{B} - \alpha\frac{\dot{C}}{C} = 0 \tag{13}$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{8\pi}{3+2\omega}(\rho - 3p) \tag{14}$$

where dot over the field variables denote differentiation with respect to t.

Spatial volume and the scale factor for the metric (1) are defined respectively, by

$$V = R^3 = ABC \tag{15}$$

$$R = (ABC)^{1/3} \tag{16}$$

The physical quantities of observational interest in cosmology are the expansion scalar (θ), shear scalar (σ^2) and the mean anisotropic parameter (A_m) defined as

$$\theta = 3H = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{17}$$

$$2\sigma^2 = \sum_{i=1}^{3} H_i^2 - \frac{\theta^2}{3} \tag{18}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{19}$$

3. Solutions and the Model

From Equations (10) and (11), we get

$$\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{\phi}}{\phi}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0 \tag{20}$$

Integrating Equation (13), we obtain

$$A^2 = kBC \tag{21}$$

where k is a constant of integration. The constant k, without loss of generality, can be chosen as unity so that we have, from Equation (21),

$$A^2 = BC \tag{22}$$

In order to get a deterministic solution, we take the following plausible physical condition:

i) The shear scalar σ is proportional to scalar expansion θ, which leads to the linear relationship between the metric potentials B and C, that is,

$$B = C^n \tag{23}$$

where $n \neq 0$ is a constant.

ii) With the help of special law of variation of Hubble's parameter proposed by Berman [39] yields constant deceleration parameter models of the universe,

$$q = -\frac{R\ddot{R}}{\dot{R}^2} \tag{24}$$

this admits the solution

$$R = (at + b)^{1/1+q} \tag{25}$$

where $a \neq 0$ and b are constants of integration.

This equation implies that the condition for accelerated expansion of universe is $1+q>0$.

Now from Equations (16), (22), (23) and (25), we obtain

$$A = (at + b)^{1/1+q} \tag{26}$$

$$B = (at + b)^{2n/(1+n)(1+q)} \tag{27}$$

$$C = (at + b)^{2/(1+n)(1+q)} \tag{28}$$

Now considering a well-accepted power law relation [40]-[42] between scale factor $R(T)$ and scalar field ϕ of the form

$$\phi = \phi_0 R^m \tag{29}$$

where m is the constant of proportionality.

Using Equation (23) in Equation (20), we obtain

$$n = 1$$

Using this value of n in Equations (26)-(28) and by a suitable choice of coordinates and constants (*i.e.* take $a = 1, b = 0$), the metric (1) can be written as

$$\mathrm{d}s^2 = \mathrm{d}t^2 - t^{\frac{2}{1+q}}\left[\mathrm{d}x^2 + \mathrm{e}^{-2\alpha x}\left(\mathrm{d}y^2 + \mathrm{d}z^2\right)\right] \tag{30}$$

Further we find the volume V, mean Hubble parameter H, expansion scalar θ, shear scalar σ, and mean anisotropic parameter A_m as

$$V = t^{\frac{3}{1+q}} \tag{31}$$

$$H = \frac{1}{(1+q)t} \tag{32}$$

$$\theta = 3H = \frac{3}{(1+q)t} \tag{33}$$

$$\sigma^2 = 0 \tag{34}$$

$$A_m = 0 \tag{35}$$

Using Equations (26)-(29), we get the energy density as

$$\rho = \frac{\phi_0}{8\phi}\left(\frac{6(1+m) - \omega m^2}{2(1+q)^2 t^{2-\frac{m}{1+q}}} - \frac{3m^2}{t^{\frac{2-m}{1+q}}}\right) \tag{36}$$

With the help of Equations (7) and (36), we get the pressure as

$$p=\frac{1}{8\pi}\left\{\gamma\phi_0\left(\frac{6(1+m)-\omega m^2}{2(1+q)^2 t^{2-\frac{m}{1+q}}}-\frac{3m^2}{t^{\frac{2-m}{1+q}}}\right)-8\pi\gamma_0\right\} \tag{37}$$

We observe that the scale factor $R(t)$ with respect to cosmic time grows rapidly as shown in **Figure 1** while **Figure 2** indicates that scalar field $\varphi(t)$ is increasing with evolution of universe. It can be easily seen from **Figure 3** and **Figure 4** that the volume goes on increasing as time increases and the energy density is decreasing function of time.

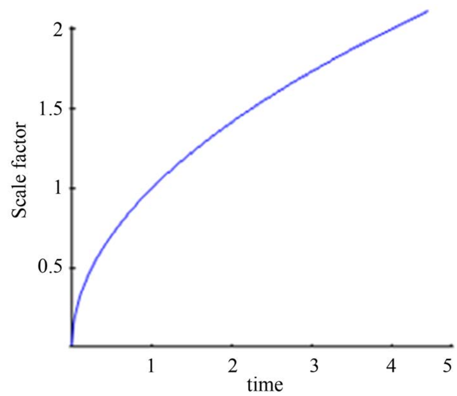

Figure 1. Scale factor versus time.

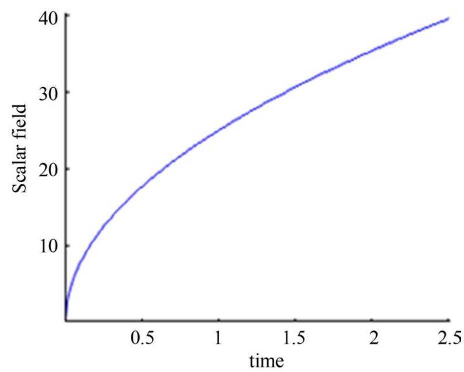

Figure 2. Scalar field versus time.

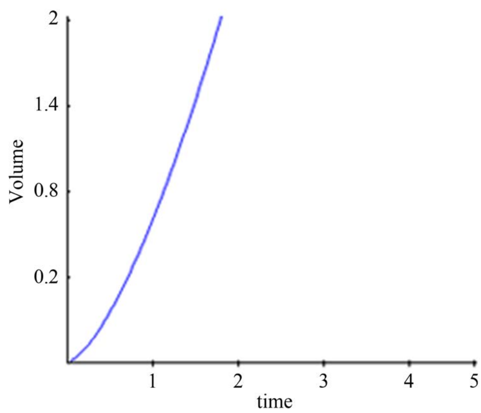

Figure 3. Volume versus time.

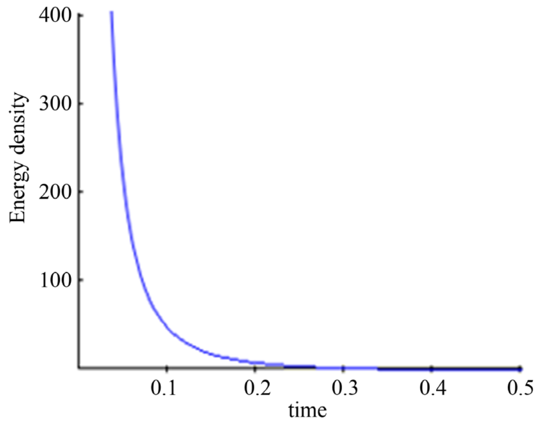

Figure 4. Energy density versus time.

4. Conclusion

In this paper, we have obtained Bianchi type V space-time with linear equation of state for perfect fluid in the Brans-Dicke theory. While solving Brans-Dicke's field equations for Bianchi type V cosmological model, we have used a special law of variation of Hubble parameter proposed by Berman. It can also be observed that θ and H decrease with time and approach zero as $t \to \infty$ while they all become infinitely large as t approaches zero. Also the model is expanding with time since $1+q>0$ and it has initial singularity *i.e.* at $t=0$. It is also interesting to note that the average anisotropic parameter vanishes so that the model does not remain anisotropic throughout the evolution of the universe and the model becomes shear free. In this model the universe starts with finite volume and volume goes on increasing as time t increases. Depending on the values of γ, γ_0 and other constant, the pressure is negative which works as dark energy. Also the generalized equation of state for $\gamma = \frac{1}{3}$ and $\gamma_0 = \frac{4}{3} B_c$ reduces to equation of state for strange quark matter.

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