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On Slide Mode Control of Chaotic Rikitake Two-Disk Dynamo

—Chaotic Simulations of the Reversals of the Earth's Magnetic Field

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Abstract

The modern nonlinear theory, bifurcation and chaos theory are used in this paper to analyze the dynamics of the Rikitake two-disk dynamo system. The mathematical model of the Rikitake system consists of three nonlinear differential equations, which found to be the same as the mathematical model of the well-known Lorenz system. The study showed that under certain value of control parameter, the system experiences a chaotic behaviour. The experienced chaotic oscillation may simulate the reversal of the Earth's magnetic field. The main objective of this paper is to control the chaotic behaviour in Rikitake system. So, a nonlinear controller based on the slide mode control theory is designed. The study showed that the designed controller was so effective in controlling the unstable chaotic oscillations.

Keywords

Chaos Theory, Nonlinear Control, Magnetic Theory, Sliding-Mode Control Theory

1. Introduction

Figure 1 [1] [2] shows the Rikitake two-disk dynamo. As shown in **Figure 1**, two conducting disks, D_1 (Left) and D_2 (Right) are subjected to a common torque of magnitude *G*. Disks D_1 and D_2 rotate in the same sense. Disk D_1 has an angular velocity, ω_1, around the axis of rotation A_1 and D_2 has an angular velocity, ω_2, around the axis of rotation A_2. Current loop L_1 is coaxial with disk D_2 with axis A_2 and L_1 is below D_2, while current loop L_2 is coaxial with disk D_1 with axis A_1 and L_2 is above D_1. Currents I_1 and I_2 both run upwards in the axial wires A_1 and A_2 respectively. From the configuration of **Figure 1** we see that current I_1 runs radially outward in Disk D_1 while current I_2 runs radially inward in Disk D_2. Loop L_1 is connected to Disk D_1 and its axis of rotation A_1 by conducting brushes, similarly loop L_2 is connected to

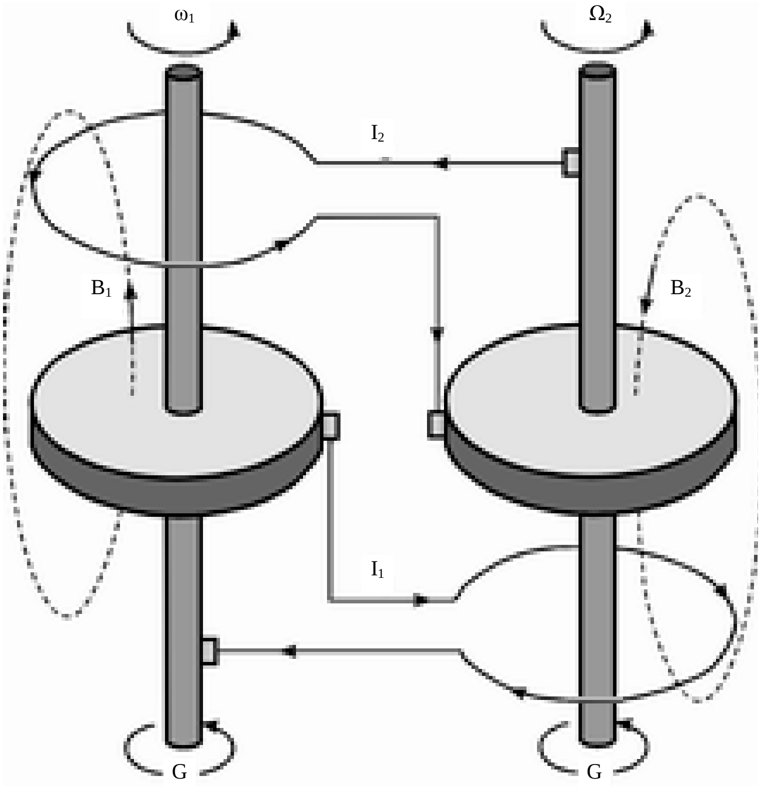

Figure 1. Rikitake two-disk dynamo [1] [2].

Disk D_2 and its axis of rotation A_2 by conducting brushes. Current loop L_1 causes a magnetic field B_2 to pierce through the rotating disk D_2, and current loop L_2 causes a magnetic field B_1 to pierce through the rotating disk D_1. According to Faraday's law and Lenz's law magnetic field B_1 crossing disk D_1 induces an emf between the center of D_1 and its rim causing an induced inward current I_1' to occur in the opposite direction to I_1 which reduces the original current I_1. A similar situation happens in disk D_2 and magnetic field B_2, but of opposite polarity to that of B_1 and D_1, where the induced emf is also between the center of D_2 and its rim causing an induced outward current I_2' to occur in the opposite direction to I_2 which reduces the original current I_2. This process will result in electric current reversals in both loops which causes a reversal in the corresponding magnetic fields. Chaos shall result under particular initial conditions.

The dynamics of Rikitake system has been discussed by many researchers. E. C. Bullard [3]-[7] has extensively discussed the behavior of earth's magnetic field and its simulation with dynamos. He first discussed the magnetic field within the earth [5], then the similar behavior between a set of homogeneous dynamos and terrestrial magnetism [6] and in 1955 [7] discussed the stability of a homopolar dynamo. Liu Xiao-Jun, *et al.* [8] analyzed the dynamics of Rikitake two-disk dynamo to explain the reversals of the Earth's magnetic field. They concluded that the chaotic behavior of the system can be used to simulate the reversals of the geomagnetic field. The Rikitake chaotic attractor was studied by several authors. T. McMillen [9] has studied the shape and dynamics of the Rikitake attractor. J. Llibre *et al.* [10] used the Poincare compactification to study the dynamics of the Rikitake system at infinity. Chien-Chih Chen *et al.* [11] have studied the stochastic resonance in the periodically forced Rikitake dynamo.

Many researchers start working on controlling the chaotic behaviors. Harb and Ayoub [12] have designed a nonlinear controller based on backstepping nonlinear theory to control the chaotic behaviour in Rikitake system. Harb and Harb [13] have designed a nonlinear controller to control the chaotic behavior in the phase-locked loop by means of nonlinear control. Harb and Abdel-Jabber [14] used both linear and nonlinear control to mitigate the chaotic oscillations in electrical power system. Nayfeh *et al.* [15], Chiang *et al.* [16] and Abed *et al.* [17] have controlled the chaotic oscillations in power systems using nonlinear control. Chang and Chen [18] investigated bifurcation characteristics of nonlinear systems under a PID controller. The main objective of control is to stabilize and delay a chaotic oscillation and reduce the amplitude of bifurcation solution. In [19], they used a control law to transform an unstable subcritical bifurcation point into a stable supercritical bifurcation point.

Lately, the control aspect of the Rikitake chaotic attractor was studied through self coupling of single state variable and nonlinear feedback controller with partial systems states [20].

In this paper, we used the nonlinear control based on slide mode control to control the chaotic oscillations of the Rikitake two-disk dynamo.

2. Mathematical Model

As shown in **Figure 1**, the Rikitake two-disk system consists of two conducting rotating disks. These disks are connected into two coils. The current in each coil feeds the magnetic field of the other. The self inductance (L) and resistance (R) are the same in each circuit. An external constant mechanical torque (G) for each circuit is applied on the axis to rotate with an angular velocity. The RL circuit equations as shown in **Figure 1** are given as:

$$RI_1 + L\frac{\mathrm{d}I_1}{\mathrm{d}t} = MI_2\omega_1 \tag{1}$$

$$RI_2 + L\frac{\mathrm{d}I_2}{\mathrm{d}t} = MI_1\omega_2 \tag{2}$$

where $MI_2\omega_1$ and $MI_1\omega_2$ are the rotation voltages V_1 and V_2. M is the mutual inductance.

If the moment of inertia of each disk, C is considered, the system mathematical model can be written after [3] as follows:

$$\dot{x}_1 = x_2x_3 - ax_1 \tag{3}$$

$$\dot{x}_2 = (x_3 - p)x_1 - ax_2 \tag{4}$$

$$\dot{x}_3 = 1 - x_1x_2 \tag{5}$$

where $a = R\sqrt{\frac{LC}{GM}}$ and $p = (\omega_1 - \omega_2)\sqrt{\frac{CM}{GL}}$.

3. Uncontrolled System

Simulation Results and Discussion

The equilibrium solution or constant solution of the system can be found if we let the set of Equations (3)-(5) to be zero. One will end up with a set of nonlinear algebraic equations. One of the solutions of this set of equations willbe function of one of the control parameters. The stability of this constant solution, can be studied by using eigenvalue analysis which is based on the Jacobian matrix derived based on linearization. The eigenvalues of the Jacobian matrix of the set of Equations (3)-(5) evaluated at the equilibrium point. In this paper, we used our own program for calculating and analyzing the stability of the fixed points and their bifurcations rather than using software packages in the market.

On the other hand, the stability of the periodic solution is studied by means of the Floquet theory. At the value of the control parameter $p = 3.4641$, we found that the system is experiencing chaotic oscillations as shown in **Figure 2** and **Figure 3**. In this paper, the main objective is to find a way to eliminate the chaotic oscillations. So, a nonlinear controller based on backstepping nonlinear theory control is designed. In the next section, the nonlinear controller design will be discussed.

4. Controlled System

4.1. Slide Mode Nonlinear Controller

Sliding mode control (SMC) is a variable structure control utilizing a high-speed switching control law to drive a system state trajectory onto a specified and user chosen surface, so called sliding surface, and to maintain the

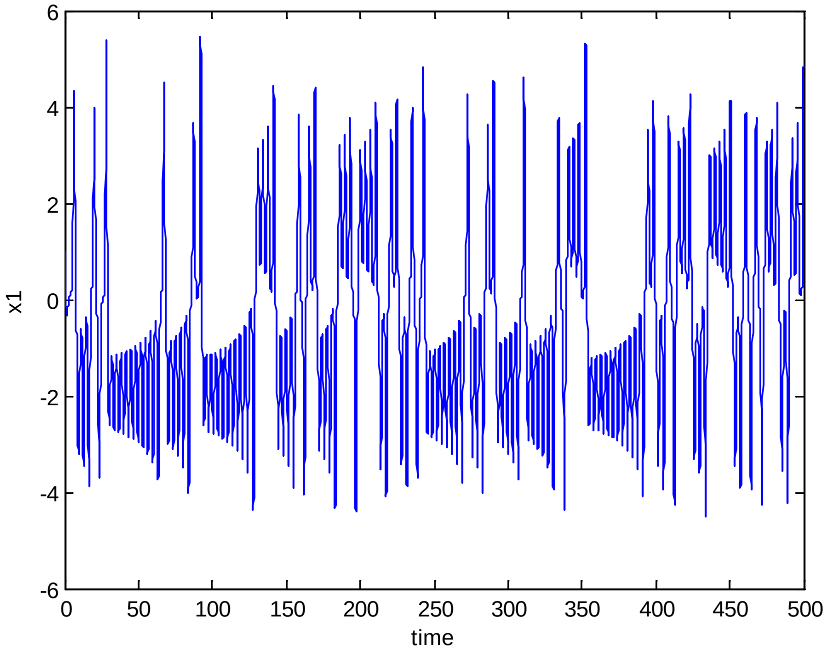

Figure 2. Time history (chaotic behavior, without controller).

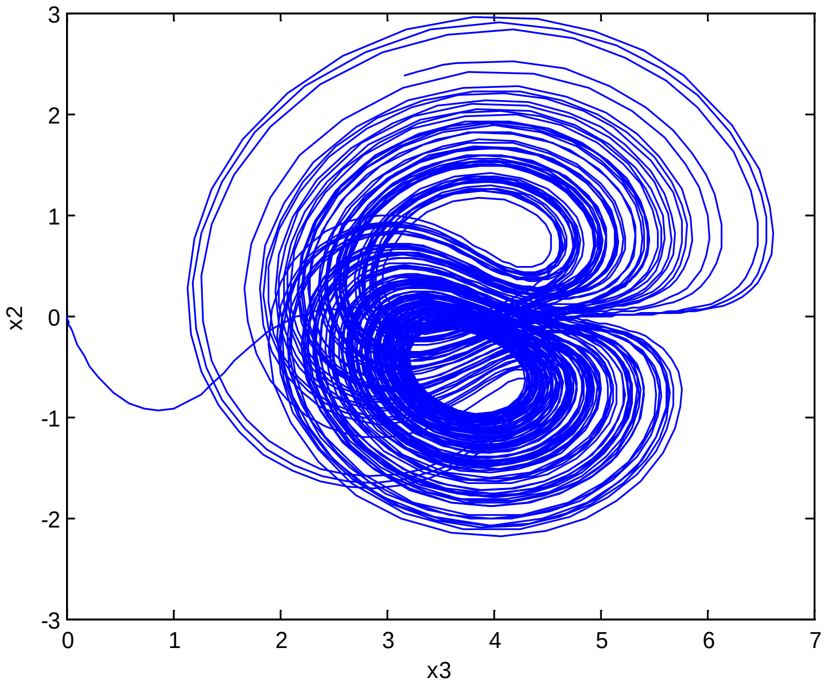

Figure 3. State space trajectory projection (chaotic behavior, without controller).

system state trajectory on the sliding surface at subsequent times. Many researchers have been used such a controller, for example; Korondi *et al.* [21], Hung *et al*. [22], Slotine and Sastry [23], and Slotine and Li [24].

To this end, let us rewrite the Equations (3)-(5) as follows:

$$\dot{x}_1 = x_2 x_3 - a x_1 \tag{6}$$

$$\dot{x}_2 = (x_3 - p) x_1 - a x_2 + u \tag{7}$$

$$\dot{x}_3 = 1 - x_1 x_2 \tag{8}$$

where u, is the control signal need to be designed.

The two separate phases of the sliding mode design method are:

Phase 1:

Sliding-surface design: an appropriate sliding surface must be selected to yield desirable performance.

$$S = a_1 (x_1 - x_{1\text{ref}}) + (x_2 - x_{2\text{ref}}) + (x_3 - x_{3\text{ref}}) \tag{9}$$

where a_1 is Tuning parameter.

Phase 2:

Sliding-reachability condition design

$$a_1 \dot{s} = -K_c s - K_d sign(s) \tag{10}$$

where K_c and K_d are positive constant design parameters.

4.2. Construction of the Control Law

By differentiating the sliding surface and equating with the reachability condition and solving out for u, the control signal will be:

$$u = -K_c S - K_d sign(S) + a_1 \dot{x}_1 - (x_3 - p) + a x_2 + \dot{x}_2 \tag{11}$$

4.3. Simulation Results and Discussion

As mentioned previously, the Rikitake system is experiencing a chaotic behavior as shown in **Figure 2**, **Figure 3**, and this was when no control signal added to the system. But once we add the designed control signal (u) in Equation (11) to the mathematical model of the Rikitake system Equations (6)-(8), one can see that all the chaotic oscillations, shown in **Figure 2** and **Figure 3**, have been eliminated as shown in **Figures 4-7**. **Figure 6** and **Figure 7** show the time history of the system when the designed controller u, is applied after 50 sec. One can see how effective the controller in eliminating the chaotic oscillations.

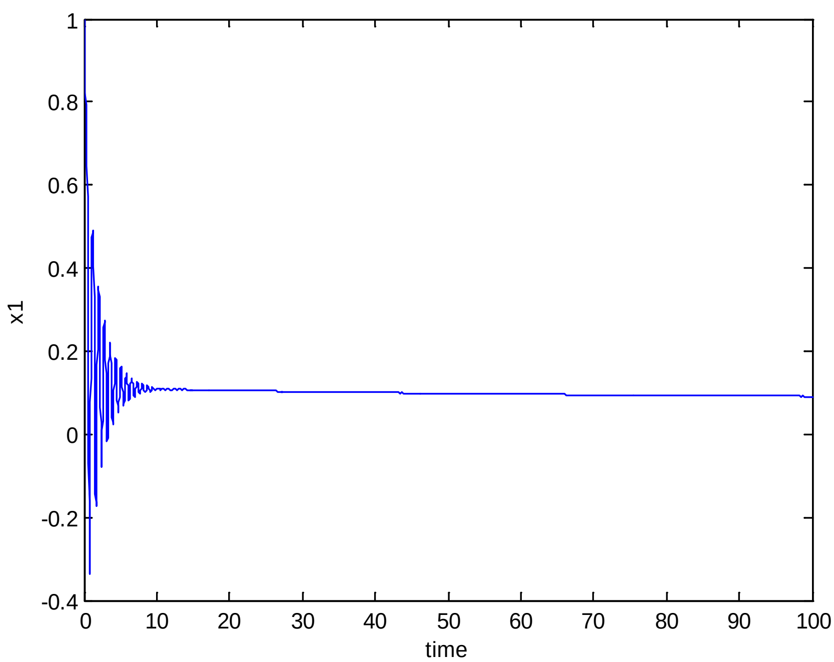

Figure 4. Time history (with controller).

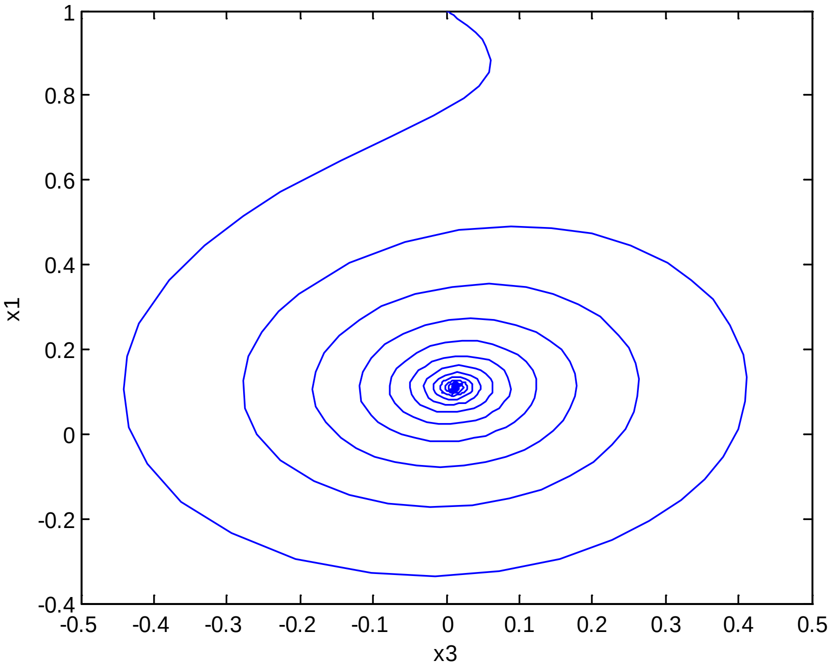

Figure 5. State space trajectory projection (with controller).

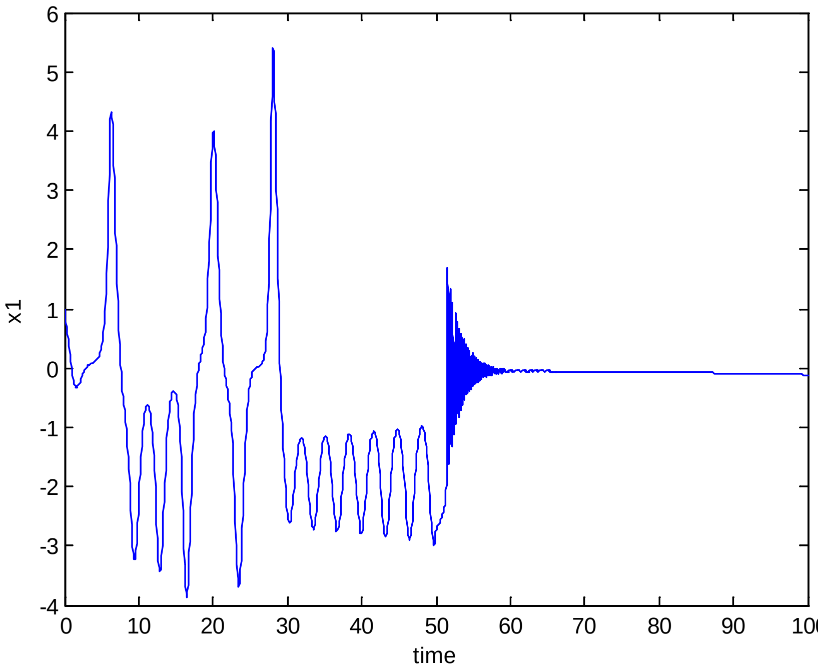

Figure 6. State space trajectory projection (with controller).

5. Conclusion

Bifurcation and chaos theory was used to investigate dynamics of the Rikitake two-disk dynamo. Rikitake system mathematical model consists of three nonlinear differential equations, which found to be the same as the mathematical model of the well known Lorenz system. This system is experiencing a chaotic behavior at certain value of the control parameter. The experienced chaotic oscillation can simulate the reversal of the Earth magnetic field. To control chaotic behavior, a nonlinear controller based on the slide mode control theory was

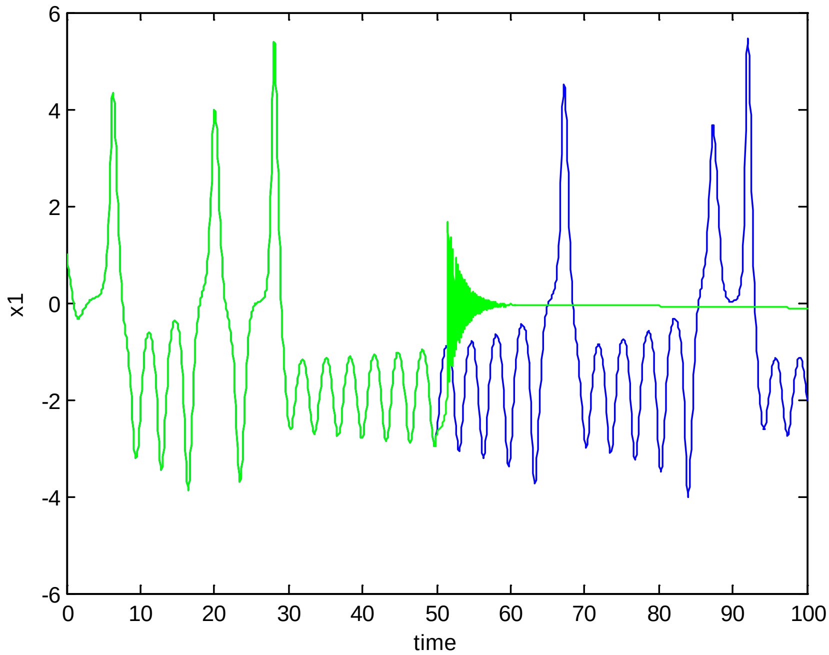

Figure 7. Comparison of state space trajectory projection between two cases (with and without controller, and the controller applied after 50 sec).

designed. The study showed that the designed controller was so effective to eliminate the unstable chaotic oscillations of the Rikitake two-disk dynamo.

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