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Solving a Traveling Salesman Problem with a Flower Structure

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Abstract

This works aims to give an answer to the problem P = NP? The result is positive with the criteria that solve the Traveling Salesman Problem in polynomial cost of the input size and a proof is given. This problem gets a solution because a polyhedron, with a cut flower looking, is introduced instead of graph (e.g. tree).

Keywords

Traveling Salesman Problem, Polyhedron, Flower, NP-Complete

1. Introduction

The traveling salesman problem (TSP) has origin in 1832 in a hand book of Hamilton. It is a NP-complete problem and NP-hard problem and till now only some special cases are found to be polynomially executable with a reducible Turing machine. I propose a method which solves the general case in polynomial cost of the input size. To today I read [1], an annealing algorithm about TSP [2], and about the bottleneck TSP but I have also the opportunity to read [3] about another NP-complete problem, the job shop scheduling, with which I measure the size of NP problem. This problem and consequent solution are really important for industry.

Problem Definition and Notations

The TSP can be definined like the problem for salesman to go throught each cities a, b, c, ⋯, v, z passing once on them and returning the shortest travel distance $S = w_1 + \cdots + w_n$ as the sum of the weight of the arcs joining cities that are visited.

2. Method

Consider a polyhedron. Cities are disposed on the axis, so a, b, c, ⋯, z are positions on the axis of the cities from

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a common origin in the space (a center) and therefore cities are on vertex of polyedron (the limit of 26 letter doesn't mean limit of 26 cities, that's valid to all functions involved). The distance beetween two cities is the weight of the arc joining them. A cycle on the polyhedron that pass once on each city is a flower with the corolla as origin (center) and the coordinate axis of cities petals. To find a, b, ⋯, x, z we must solve the linear system for each variable that define the position on axis:

$$\begin{aligned}
&w_{ab} = \sqrt{a^2 + b^2} \\
&w_{ac} = \sqrt{a^2 + c^2} \\
&\cdots \\
&w_{az} = \sqrt{a^2 + z^2} \\
&\cdots \\
&w_{xz} = \sqrt{x^2 + z^2}.
\end{aligned}$$

The equation for node a is $W_a = w_{ab}^2 + w_{ac}^2 + \cdots + w_{az}^2 = (N-1)\cdot a^2 + b^2 + c^2 + \cdots + z^2$. We compute W_k; $a \le k \le z$ having N variable in N equations and obtain a^2 will give easy access to a.

2.1. Theorem

Be O the set of petals. Be x the minimum and $O_1 = O \setminus \{x\}$ then $m_k = \min O_k$; $M_k = \min O_k \setminus \{m_k\}$; $O_{k+1} = O_k \setminus \{m_k, M_k\}$. The optimal sequence for petals is given by

$$\cdots, m_3, m_2, m_1, x, M_1, M_2, M_3, \cdots. \tag{1}$$

2.2. Proof

In this section we will consider the petals a, b, c, ⋯ as variables $x_1, x_2, \cdots, x_n$.

$$\begin{aligned}
\min S &= \min\left\{\sqrt{x_1^2 + x_2^2} + \sqrt{x_2^2 + x_3^2} + \sqrt{x_3^2 + x_4^2} + \cdots + \sqrt{x_1^2 + x_n^2}\right\} \\
&= \min\left\{f(x_1, x_2) + f(x_2, x_3) + f(x_3, x_4) + \cdots + f(x_1, x_n)\right\} = \min \sum f
\end{aligned} \tag{A}$$

Consider an inequality on functions for the swap of two elements

$$f(x_a, x_b) + f(x_c, x_d) + f(x_e, x_f) + \cdots < f(x_a, x_c) + f(x_b, x_d) + f(x_e, x_f) + \cdots \tag{B}$$

$f(x_a, x_b) - f(x_a, x_c) + f(x_c, x_d) - f(x_b, x_d) < 0$ with $\dfrac{\partial f(x, y)}{\partial x} = \dfrac{2x}{f(x, y)}$ so with equality of Δx the df is higher at lower y, therefore if $x_a < x_d$ with $x_b < x_c$. Further consider that $f(x_a, x_b) + f(x_c, x_d)$ must be soddisfied from one value of x_a, x_b to one value of x_c, x_d and therefore $x_a < x_c$ and $x_b < x_d$ is a stronger relation togheter with $x_a < x_d$ and $x_b < x_c$. So x_a, x_b minimum with x_c, x_d maximum with a complete satisfy condition is the lowest: $\{x_i, x_j\}_{\in f_1} < \{x_h, x_k\}_{\in f_2}$. This condition can be extended to more than two elements so for example $f(x_a, x_b) + f(x_c, x_d) + f(x_e, x_f)$ minimum if $\{x_i, x_j\}_{\in f_1} < \{x_h, x_k\}_{\in f_2} < \{x_m, x_n\}_{\in f_3}$ (Optimality Condition OC 1).

2.2.1. Case of Four Dimensions

So if we have $\{x_1, x_2\}, \{x_2, x_3\}, \{x_3, x_4\}, \{x_4, x_1\}$ we start from the maximum $x_4 > x_{j=1,2,3}$ so that for condition OC 1 $x_{i=3} > x_{j=1,2}$ and $x_{i=1} > x_{j=2,3}$. The swap of x_1 with x_3 is indifferent because we have not prevalent constrains so solutions are $x_2 < x_1 < x_3 < x_4$ and $x_2 < x_3 < x_1 < x_4$. Those are symmetric respect to the minimum.

2.2.2. Case of More Dimensions

So if we have $A_1 := \{x_1, x_2\}, \{x_2, x_3\}, \{x_3, x_4\}, \{x_4, x_5\}, \{x_5, x_6\}, \{x_6, x_1\}$. In such sets we start from the maximum $x_6 > x_{j=1,2,\cdots,5}$ so that, for condition OC 1, elements $x_5 > x_{j=1,\cdots,4}$ and $x_1 > x_{i=2,\cdots,4,5}$ are maximum. Here we will find only a symmetric solution considering $x_5 > x_1$. Further $x_5 > x_{j=1,2,3,4}$ therefore, for condition OC 1, $x_4 > x_{i=1,2,3}$ from the confront of others elements not sequenced; so we have to choose the order for x_4, x_1, but $x_4 < x_1$ because notice that we have an even number of elements, two at high adjacent values $\{x_4, x_5\}, \{x_6, x_1\}$ and two at lower adjacent values $\{x_3, x_4\}, \{x_1, x_2\}$; the behaviour of down level $\{x_1, x_2\}, \{x_2, x_3\}, \{x_3, x_4\}$ is symmetric and specular of the choice of up level; the down level can have a good move symmetrically equivalent from uplevel choice whereas the uplevel depends on the choice, and so is convinient to have a good choice at up level (Optimality Condition OC 2). Further $x_1 > x_{j=2,3,4}$ and therefore, for condition OC 1, $x_2 > x_{3,4}$ but, for condition OC 2, $x_4 > x_2$. So the sequence became $x_3 < x_2 < x_4 < x_1 < x_5 < x_6$, with $\{x_2, x_3\}$ minimum.

So the order is the best solution. Therefore the solution with elements at right and left of the minimum in n-dimensions case is $x < m_1 < M_1 < m_2 < M_2 < m_3 < M_3 \cdots$ that for simmetry is the same than $x < M_1 < m_1 < M_2 < m_2 < M_3 < m_3 < \cdots$. Then (1) is true.

2.3. Algorithm

Node: a list of nodes
$w : NXN \to R$
Solve the linear system for a, b, ⋯, z obtain node
While $\text{result.size}(\ ) < N$ **do**
  minimum $= \infty$;
   for $i = 0 \to i < N$
    if $\text{node}[i] < \text{minimum} \wedge i \notin \text{result}$ **then**
      minimum $= \text{node}[i]$;
      next $= i$;
    end if
   end for
   if $\text{result.size}(\ )_{\text{mod}\,2} \equiv 0$ **then**
    result.add (position(last),next);
   else if $\text{result.size}(\ )_{\text{mod}\,2} \equiv 1$ **then**
    result.add (position(init),next);
   else if
  end while

Test

The algorithm has been tested on several instances 9 × 9 and in all of them the best solution is found in few seconds.

3. Complexity

The linear system can be found in $O(N^3)$ with Gauss method [4] wheras the cycle is $O(N^2)$ so the method is of $O(N^3)$. Then P = NP.

4. Discussion

The complexity can be reduced also changing the criteria to achieve an objective.

References

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Appendix

Method to Solve the Linear System, Jacobi

$Ax = b$ with x vector of solution and b vector of costant terms. Choose $A = P - N$ so that P easy to invert. With $x^{(k+1)} = P^{-1}Nx^{(k)} + P^{-1}b$ and stop condition dependent from τ that must be tunned depending for example from the minimum x: $\left|x^{(k)} - x^{(k-1)}\right| < \tau\left|x^{(k)}\right|$ [5].