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Generalized Form of Hermite Matrix Polynomials via the Hypergeometric Matrix Function

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Abstract

The object of this paper is to present a new generalization of the Hermite matrix polynomials by means of the hypergeometric matrix function. An integral representation, differential recurrence relation and some other properties of these generalized forms are established here. Moreover, some new properties of the Hermite and Chebyshev matrix polynomials are obtained. In particular, the two-variable and two-index Chebyshev matrix polynomials of two matrices are presented.

Keywords

Hermite and Chebyshev Matrix Polynomials, Three Terms Recurrence Relation, Hypergeometric Matrix Function and Gamma Matrix Function

1. Introduction

Special functions have been developed deeply in the last decades to special matrix functions due to their applications in certain areas of statistics, physics and engineering. The Laguerre and Hermite matrix polynomials are introduced in [1] as examples of right orthogonal matrix polynomial sequences for appropriate right matrix moment functionals of integral type. The Hermite matrix polynomials, $H_n(x,A)$, have been introduced and studied in [2] [3] where $H_n(x,A)$ involves a parameter $A \in \mathbb{C}^{r\times r}$ whose eigenvalues are all situated in the open right-hand half of the complex plane. The two-variable Hermite matrix polynomials, $H_n(x,y,A)$, have been presented in [4] as an extension of $H_n(x,A)$. Moreover, some properties and other generalizations of $H_n(x,A)$ are given in [5]-[11]. As one of qualitative properties of the two-variable Hermite matrix polynomials, the Chebyshev matrix polynomials of the second kind are introduced in [4], see also [12] [13].

The main aim of this paper is to consider a new generalization of the Hermite matrix polynomials and to

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derive some properties for the Hermite and Chebyshev matrix polynomials. The structure of this paper is the following. This section summarizes previous results essential in the rest of the paper and gives the development of the two-variable Hermite matrix polynomials. A matrix version of Kummer's first formula for the confluent hypergeometric matrix function is derived in Section 2. In Section 3, the addition theorem and three terms recurrence relation for the Chebyshev matrix polynomials of the second kind are obtained and further we introduce and study the two-variable and two-index Chebyshev matrix polynomials of two matrices. Finally, Section 4 deals with the study of the Generalized Hermite matrix polynomials by means of the hypergeometric matrix function.

In what follows, $\mathbb{C}^{r\times r}$ denotes the set of complex matrices of size $r\times r$ and the matrices I and θ in $\mathbb{C}^{r\times r}$ denote the matrix identity and the zero matrix of order r, respectively. For a matrix A in $\mathbb{C}^{r\times r}$, its spectrum $\sigma(A)$ denotes the set of all eigenvalues of A. We say that a matrix A in $\mathbb{C}^{r\times r}$ is a positive stable if

$$\operatorname{Re}(\mu)>0 \text{ for every eigenvalue } \mu\in\sigma(A). \tag{1}$$

If $f(z)$ and $g(z)$ are holomorphic functions of the complex variable z, which are defined in an open set Ω of the complex plane and A is a matrix in $\mathbb{C}^{r\times r}$ with $\sigma(A)\subset\Omega$, then from the properties of the matrix functional calculus ([14], p. 558), it follows that $f(A)g(A)=g(A)f(A)$.

If D_0 is the complex plane cut along the negative real axis and Log(z) denotes the principle logarithm of z, then $z^{1/2}$ represents $\exp\left(\frac{1}{2}\operatorname{Log}(z)\right)$. If A is a matrix in $\mathbb{C}^{r\times r}$ with $\sigma(A)\subset D_0$, then $A^{\frac{1}{2}}=\sqrt{A}$ denotes the image by $z^{1/2}$ of the matrix functional calculus acting on the matrix A.

Let A be a matrix in $\mathbb{C}^{r\times r}$ which satisfies the condition (1). The two-variable Hermite matrix polynomials **[2VHMPs]** are generated by [4]

$$\exp\left(xt\sqrt{2A}-yt^2I\right)=\sum_{n\ge0}\frac{1}{n!}H_n(x,y,A)t^n, \tag{2}$$

and are defined by the series

$$H_n(x,y,A)=n!\sum_{k=0}^{\lfloor n/2\rfloor}\frac{(-1)^k y^k}{k!(n-2k)!}\left(x\sqrt{2A}\right)^{n-2k}, \tag{3}$$

where $\lfloor v\rfloor$ is the standard floor function which maps a real number v to its next smallest integer.

It is therefore evident, for $y=1$, that

$$H_n(x,1,A)=H_n(x,A), \tag{4}$$

where $H_n(x,A)$ is the Hermite matrix polynomials as given in [2]. Furthermore,

$$H_n(x,y,A)=y^{n/2}H_n\left(x/\sqrt{y},A\right).$$

According to [4], we have

$$\frac{\partial^k}{\partial x^k}H_n(x,y,A)=\left(\sqrt{2A}\right)^k\frac{n!}{(n-k)!}H_{n-k}(x,y,A);\ 0\le k\le n, \tag{5}$$

$$\frac{\partial^k}{\partial y^k}H_n(x,y,A)=\frac{(-1)^k n!}{(n-2k)!}H_{n-2k}(x,y,A);\ 0\le k\le\left[\frac{n}{2}\right]. \tag{6}$$

Also, the **2VHMPs** appear as a solution of the second order matrix differential equation in the form

$$\left[y\frac{\partial^2}{\partial x^2}-xA\frac{\partial}{\partial x}+nA\right]H_n(x,y,A)=0. \tag{7}$$

and satisfy the three terms recurrence relationship

$$H_n(x,y,A)=x\sqrt{2A}H_{n-1}(x,y,A)-2(n-1)yH_{n-2}(x,y,A);\ n\ge2 \tag{8}$$

with $H_0(x,y,A)=I$ and $H_1(x,y,A)=x\sqrt{2A}$.

From (5), the relation (8) gives

$$H_n(x,y,A)=\left(x\sqrt{2A}-2y\left(\sqrt{2A}\right)^{-1}\frac{\partial}{\partial x}\right)H_{n-1}(x,y,A). \tag{9}$$

Iteration (9) yields a another representation of the **2VHMPs** in the form

$$H_n(x,y,A)=\left(x\sqrt{2A}-2y\left(\sqrt{2A}\right)^{-1}\frac{\partial}{\partial x}\right)^n(I). \tag{10}$$

Another remarkable representation of the **2VHMPs**, which is due essentially to ([4], Theorem 7), has the elegant form:

$$H_n(x,y,A)=\exp\left(-y(2A)^{-1}\frac{\partial^2}{\partial x^2}\right)\left(x\sqrt{2A}\right)^n. \tag{11}$$

Applying (11) provides the formula

$$H_n(x,y+z,A)=\exp\left(-y(2A)^{-1}\frac{\partial^2}{\partial x^2}\right)H_n(x,z,A). \tag{12}$$

In fact, the addition and multiplication theorems are

$$H_n(ax+bz,y,A)=\sum_{k=0}^{n}\binom{n}{k}H_{n-k}\left(ax,\frac{y}{2},A\right)H_k\left(bz,\frac{y}{2},A\right), \tag{13}$$

and

$$a^nH_n(x,y,A)=H_n\left(ax,a^2y,A\right). \tag{14}$$

If $A(k,n)$ and $B(k,n)$ are matrices in $\mathbb{C}^{r\times r}$ for $n\geq 0$ and $k\geq 0$, then it follows that [10] [15] [16]:

$$\sum_{n\geq 0}\sum_{k\geq 0}A(k,n)=\sum_{n\geq 0}\sum_{k=0}^{n}A(k,n-k), \tag{15}$$

and

$$\sum_{n\geq 0}\sum_{k=0}^{n}B(k,n)=\sum_{n\geq 0}\sum_{k=0}^{\lfloor n/2\rfloor}B(k,n-k). \tag{16}$$

2. The Confluent Hypergeometric Matrix Function

In this section, the confluent hypergeometric matrix function is given. For the sake of clarity in the presentation, we recall some concepts and results related to the generalized hypergeometric matrix functions, that may be found in [15] [18] [19].

The reciprocal gamma function denoted by $\Gamma^{-1}(z)=1/\Gamma(z)$ is an entire function of the complex variable z. Then, for any matrix A in $\mathbb{C}^{r\times r}$, the image of $\Gamma^{-1}(z)$ acting on A, denoted by $\Gamma^{-1}(A)$ is a well-defined matrix. Furthermore, if

$$A+nI \text{ is invertible for every integer } n\geq 0, \tag{17}$$

then $\Gamma(A)$ is invertible, its inverse coincides with $\Gamma^{-1}(A)$ and it follows that ([17], p. 253)

$$(A)_n=A(A+I)\cdots(A+(n-1)I);\ n\geq 1, \tag{18}$$

with $(A)_0=I$.

If A is a positive stable matrix in $\mathbb{C}^{r\times r}$, then the gamma matrix function, $\Gamma(A)$, is well defined as [18]

$$\Gamma(A)=\int_0^{\infty}\mathrm{e}^{-t}t^{A-I}\mathrm{d}t. \tag{19}$$

From ([19], p. 206), we have

$$(A)_n = \Gamma(A+nI)\Gamma^{-1}(A); \quad n \ge 0. \tag{20}$$

Definition 2.1 [15] *Let* p *and* q *be two non-negative integers. The generalized hypergeometric matrix function is defined in the form*:

$${}_pF_q(A_1,\cdots,A_p;B_1\ldots,B_q;z) = \sum_{n\ge0}(A_1)_n\cdots(A_p)_n\left[(B_1)_n\right]^{-1}\cdots\left[(B_q)_n\right]^{-1}\frac{z^n}{n!}, \tag{21}$$

where A_i *and* B_j $(1\le i\le p, 1\le j\le q)$ *are matrices in* $\mathbb{C}^{r\times r}$ *such that the matrices* B_j *satisfy the condition* (17).

According to [15], it follows that:

- If $p \le q$, then the power series (21) converges for all finite z.
- If $p = q+1$, then the power series (21) is absolutely convergent for $|z|<1$ and diverges for $|z|>1$.
- If $p \le q$ then the power series (21) diverges for $z \ne 0$.

With $p=1$ and $q=0$ in (21), one gets the following relation due to ([3], p. 213)

$$(1-z)^{-A} = \sum_{n\ge0}\frac{1}{n!}(A)_n z^n, \quad |z|<1, \tag{22}$$

which can be written by (19) and (20) in the form

$$(1-z)^{-A} = \Gamma^{-1}(A)\int_0^\infty e^{(z-1)t}t^{A-I}\,dt. \tag{23}$$

For $p=2$ and $q=1$ in (21), we obtain the hypergeometric matrix function as given in [3] in the form

$$F(A,B;C;z) = \sum_{n\ge0}(A)_n(B)_n\left[(C)_n\right]^{-1}\frac{z^n}{n!}; \quad |z|<1. \tag{24}$$

Moreover, the confluent hypergeometric matrix function is well defined for all finite z, when $p=q=1$, in the form

$$\Phi(A;B;z) = \sum_{n\ge0}(A)_n\left[(B)_n\right]^{-1}\frac{z^n}{n!}. \tag{25}$$

One can easily get the following result.

Proposition 2.2

$$\frac{d}{dz}\Phi(A;B;z) = A\Phi(A+I;B+I;z)B^{-1} \tag{26}$$

and

$$\frac{d^k}{dz^k}\Phi(A;B;z) = (A)_k\,\Phi(A+kI;B+kI;z)\left[(B)_k\right]^{-1}. \tag{27}$$

In [20], the following theorem was proved:

Theorem 2.3 *Let A and B be two matrices in* $\mathbb{C}^{r\times r}$ *such that*

1. A *and* $B-A$ *are positive stable,*
2. $AB=BA$,
3. $B+jI$ *is invertible for all* $j\ge0$.

Then for a positive integer $n\ge0$ *the following holds*

$$F(-nI,A;B;1) = \Gamma(B-A+nI)\Gamma^{-1}(B+nI)\Gamma^{-1}(B-A)\Gamma(B). \tag{28}$$

Indeed, by (20) we can rewrite the formula (28) in the form

$$F(-nI,A;B;1) = (B-A)_n\left[(B)_n\right]^{-1}. \tag{29}$$

A matrix version of Kummer's first formula for the confluent hypergeometric matrix function is presented in the following theorem:

Theorem 2.4 *Let A and B be two matrices in* $\mathbb{C}^{r\times r}$ *satisfy the conditions of Theorem* 2.3. *Then*

$$\Phi(B-A;B;-z)=\mathrm{e}^{-z}\Phi(A;B;z). \tag{30}$$

Proof. From (15) and (25) we have

$$\mathrm{e}^{-z}\Phi(A;B;z)=\sum_{n\geq 0}\sum_{k=0}^{n}\frac{(-nI)_k}{k!}(A)_n\left[(B)_n\right]^{-1}\frac{(-z)^n}{n!}. \tag{31}$$

By (29) and taking into account the conditions of Theorem 2.3 we find

$$\mathrm{e}^{-z}\Phi(A;B;z)=\sum_{n\geq 0}(B-A)_n\left[(B)_n\right]^{-1}\frac{(-z)^n}{n!}, \tag{32}$$

and so (30) follows. □

3. Generalized Chebyshev Matrix Polynomials

In [20], the Chebyshev matrix polynomials of the first kind $T_n(x,A)$ was defined by

$$T_n(x,A)=\sum_{k=0}^{n}\frac{(-1)^k n(n+k-1)!}{2^k k!(n-k)!}\Gamma(A)\Gamma^{-1}(A+kI)(1-x)^k. \tag{33}$$

From (20) and (25) with the use of

$$n!=\int_0^\infty \mathrm{e}^{-t}t^n\mathrm{d}t, \tag{34}$$

we give an integral representation of $T_n(x,A)$ in the form

$$T_n(x,A)=\frac{1}{(n-1)!}\int_0^\infty \mathrm{e}^{-t}t^{n-1}\Phi(-nI;A;t(1-x)/2)\mathrm{d}t. \tag{35}$$

The generalized Chebyshev matrix polynomials of the second kind **[GCMPs]** are defined by the series [4]

$$U_n(x,y,A)=\sum_{k=0}^{[n/2]}\frac{(-1)^k(n-k)!\left(\sqrt{2A}\right)^{n-2k}}{k!(n-2k)!}x^{n-2k}y^k \tag{36}$$

and specified by the integral representation

$$U_n(x,y,A)=\frac{1}{n!}\int_0^\infty \mathrm{e}^{-t}t^nH_n\left(x,\frac{y}{t},A\right)\mathrm{d}t. \tag{37}$$

According to (14), the integral representation (37) becomes

$$U_n(x,y,A)=\frac{1}{n!}\int_0^\infty \mathrm{e}^{-t}H_n(xt,yt,A)\mathrm{d}t. \tag{38}$$

The use of the relations (5) and (8) in (37) yields the differential recurrence relation

$$nU_n(x,y,A)=x\frac{\partial}{\partial x}U_n(x,y,A)-2y\left(\sqrt{2A}\right)^{-1}\frac{\partial}{\partial x}U_{n-1}(x,y,A).$$

According to [4], we have $U_n(x,1,A)=U_n(x,A)$ and $U_n(x,y,A)=y^{n/2}U_n\left(x/\sqrt{y},A\right)$, where $U_n\left(x/\sqrt{y},A\right)$ is the Chebyshev matrix polynomials of the second kind **[CMPs]**.

As a direct consequent of ([4], Lemma 5), we state the following result.

Proposition 3.1 *For a real number* $K>2$, *it follows that*

$$\left\|U_n(x,y,A)\right\|\leq K^n\sqrt{\frac{(2y)^n}{n!}}\left(1-\frac{x^2}{2}\right)^{-(n+2)/2}\Gamma\left(\frac{n+2}{2}\right), \tag{39}$$

where $|x|<K/\left\|\sqrt{2A}\right\|$ *and* $y>0$.

Let us now introduce the two-variable and two-index Chebyshev matrix polynomials of two matrices **[2V2ICMP]** through the integral representation

$$U_{m,n}\left(x,y,A,B\right)=\frac{1}{m!n!}\int_0^{\infty}\mathrm{e}^{-t}H_m\left(xt,t,A\right)H_n\left(yt,t,B\right)\mathrm{d}t, \tag{40}$$

where A and B are two matrices in $\mathbb{C}^{r\times r}$ satisfy the condition (1). From (3) and (34) we obtain that

$$U_{m,n}\left(x,y,A,B\right)=\sum_{i=0}^{\lfloor m/2\rfloor}\sum_{j=0}^{\lfloor n/2\rfloor}\frac{(-1)^{i+j}(m+n-i-j)!}{i!j!(m-2i)!(n-2j)!}\left(x\sqrt{2A}\right)^{m-2i}\left(x\sqrt{2B}\right)^{n-2j}. \tag{41}$$

Indeed, by (14), the integral representation (40) becomes

$$U_{m,n}\left(x,y,A,B\right)=\frac{1}{m!n!}\int_0^{\infty}\mathrm{e}^{-t}t^{m+n}H_m\left(x,\frac{1}{t},A\right)H_n\left(y,\frac{1}{t},B\right)\mathrm{d}t. \tag{42}$$

It is worthy to mention that, on taking $n=0$ or $m=0$, the Equations (40), (41) and (42) of the **2V2ICMP** reduce to the Equations (38), (36) and (37) of the **[CMPs]**, respectively.

It is evident that the formula (37) provides

$$U_n\left(x+z,A\right)=\frac{1}{n!}\int_0^{\infty}\mathrm{e}^{-t}t^nH_n\left(x+z,\frac{1}{t},A\right)\mathrm{d}t. \tag{43}$$

Thus, by applying (13) in (43), we obtain

$$U_n\left(x+z,A\right)=\sum_{k=0}^{n}\frac{1}{k!(n-k)!}\int_0^{\infty}\mathrm{e}^{-t}t^nH_{n-k}\left(x,\frac{1}{2t},A\right)H_k\left(z,\frac{1}{2t},A\right)\mathrm{d}t,$$

which, in view of (14), one gets

$$U_n\left(x+z,A\right)=2^{-n/2}\sum_{k=0}^{n}\frac{1}{k!(n-k)!}\int_0^{\infty}\mathrm{e}^{-t}t^{(n-k)+k}H_{n-k}\left(\sqrt{2}x,\frac{1}{t},A\right)H_k\left(\sqrt{2}z,\frac{1}{t},A\right)\mathrm{d}t.$$

This, by the formula (42), leads to the addition theorem for the Chebyshev matrix polynomials of the second kind in the form

$$U_n\left(x+z,A\right)=2^{-n/2}\sum_{k=0}^{n}U_{n-k,k}\left(\sqrt{2}x,\sqrt{2}z,A,A\right). \tag{44}$$

4. Generalized Hermite Matrix Polynomials

By using the hypergeometric matrix function it is convenient to consider a new generalized form of the Hermite matrix polynomials. The generalized Hermite matrix polynomials **[GHMPs]** of two matrices and two variables are presented here. Let A and B be two matrices in $\mathbb{C}^{r\times r}$ such that A satisfies the condition (1) and B satisfies the condition (17). We can define the **GHMPs** in the form:

$$H_n^B\left(x,y,A\right)=n!\sum_{k=0}^{\lfloor n/2\rfloor}\frac{(-1)^k}{k!(n-2k)!}B_{n,k}\left(x\sqrt{2A}\right)^{n-2k}, \tag{45}$$

where

$$B_{n,k}=y^kF\left(-kI,B;-nI;y\right). \tag{46}$$

Note that, by (24), the expression (46) can be written in the form

$$B_{n,k}=\sum_{j=0}^{k}\frac{k!(n-j)!}{j!(k-j)!}(B)_j\,y^{k+j}. \tag{47}$$

when B is the zero matrix, then the **GHMPs** reduce to the two-variable Hermite matrix polynomials, *i.e.*, $H_n^{\theta}\left(x,y,A\right)=H_n\left(x,y,A\right)$.

In view of (19), (20) and (34), the expression (46) can be also written in the following integral representation

$$B_{n,k}=y^k\Gamma^{-1}(B)\int_0^\infty\int_0^\infty e^{-(t+u)}t^n u^B\left(1+\frac{yu}{t}\right)^k \mathrm{d}t\mathrm{d}u. \tag{48}$$

It is clear that

$$H_n^B(-x,y,A)=(-1)^n H_n^B(x,y,A).$$

and

$$H_n^B(x,0,A)=\left(x\sqrt{2A}\right)^n.$$

By using (19), (20) and (34), the formula (45) leads to

$$H_n^B(x,y,A)=\Gamma^{-1}(B)\int_0^\infty\int_0^\infty t^n e^{-(t+u)}u^{B-I}\sum_{k=0}^{\lfloor n/2\rfloor}\frac{(-1)^k\left(x\sqrt{2A}\right)^{n-2k}}{k!(n-2k)!}y^k\left(1+\frac{yu}{t}\right)^k \mathrm{d}t\mathrm{d}u.$$

Hence, by (3), we obtain the integral representation of the **GHMPs** in the form

$$H_n^B(x,y,A)=\frac{\Gamma^{-1}(B)}{n!}\int_0^\infty\int_0^\infty t^n e^{-(t+u)}u^{B-I}H_n\left(x,y\left(1+\frac{yu}{t}\right),A\right)\mathrm{d}t\mathrm{d}u. \tag{49}$$

In view of (12), the integral representation (49) becomes

$$H_n^B(x,y,A)=\frac{\Gamma^{-1}(B)}{n!}\int_0^\infty e^{-u}u^{B-I}\int_0^\infty t^n e^{-t}H_n\left(x,\frac{y^2u}{t},A\right)\mathrm{d}t\mathrm{d}u\exp\left(-y(2A)^{-1}\frac{\partial^2}{\partial x^2}\right),$$

which, by (37), provides the following form by means of the generalized Chebyshev matrix polynomials

$$H_n^B(x,y,A)\exp\left(y(2A)^{-1}\frac{\partial^2}{\partial x^2}\right)=\Gamma^{-1}(B)\int_0^\infty e^{-u}u^{B-I}U_n\left(x,y^2u,A\right)\mathrm{d}u. \tag{50}$$

Thus, by exploiting (19), (20) and (36) in (50), one gets

$$H_n^B(x,y,A)\exp\left(y(2A)^{-1}\frac{\partial^2}{\partial x^2}\right)=\sum_{k=0}^{\lfloor n/2\rfloor}\frac{(-1)^k(n-k)!y^{2k}}{k!(n-2k)!}(B)_k\left(x\sqrt{2A}\right)^{n-2k}.$$

By (11) and (6), it follows that

$$\begin{aligned}H_n^B(x,y,A)&=\sum_{k=0}^{\lfloor n/2\rfloor}(B)_k\frac{(n-k)!y^{2k}}{k!n!}\frac{(-1)^k n!}{(n-2k)!}H_{n-2k}(x,y,A)\\&=\sum_{k=0}^{\lfloor n/2\rfloor}(B)_k\left[(-nI)_k\right]^{-1}\frac{(-1)^k y^{2k}}{k!}\frac{\partial^k}{\partial y^k}H_n(x,y,A).\end{aligned}$$

Hence from (25) we arrive at the following representation of the **GHMPs**

$$H_n^B(x,y,A)=\Phi\left(B;-nI;-y^2\frac{\partial}{\partial y}\right)H_n(x,y,A). \tag{51}$$

The use of the second order matrix differential Equation (7) in the integral representation (49) gives

$$\frac{\Gamma^{-1}(B)}{n!}\int_0^\infty\int_0^\infty e^{-(t+u)}t^n u^{B-I}\left[y\left(1+\frac{yu}{t}\right)\frac{\partial^2}{\partial x^2}-xA\frac{\partial}{\partial x}+nA\right]H_n\left(x,y\left(1+\frac{yu}{t}\right),A\right)\mathrm{d}t\mathrm{d}u=\theta,$$

which, with the help of (5), obtaining the differential recurrence relation

$$\left[y\frac{\partial^2}{\partial x^2}-xA\frac{\partial}{\partial x}+nA\right]H_n^B\left(x,y,A\right)=B\frac{\partial}{\partial x}H_{n-1}^{B+I}\left(x,y,A\right)\sqrt{2A}.$$

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