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Effect of Interplanetary Matter on the Spin Evolutions of Venus and Mercury

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Abstract

Differs from other planets in the Solar System, the Venus has a retrograde and long-period rotation. To explain the special spin of the Venus, mechanisms such as core mantle friction inside planet [1], atmospheric tide [2-7], or twain effects together [8-11], and impact with a giant object [12,13] have been suggested. These mechanisms, however, need specific initial conditions with a remote probability [3,5]. The slow spin of Mercury cannot be explained very well. One viewpoint is that the unusual spins of Venus and Mercury might be naturally evolved from similar initial states by interaction with interplanetary matter during long-time evolution. Based on the theory of planet formation and the orderliness of planetary distance, we discuss the possibility that the radial density distribution of interplanetary matter is undulated, and the wave function satisfies the formal Schrödinger equation. We calculate the evolution of planet spins under the effect of interplanetary matter during planets revolution and rotation. The results show that planets can naturally evolve to the current state (particularly the negative spin of the Venus) given the similar initial quick and positive spins.

Keywords: Schrödinger Equation; Planets Rotation; Planets Revolution

1. Introduction

The regular distribution of planets in the Solar System implies that the density distribution of the original nebular disk undulated, which played important role on planets growth and spin. During the formation of the Solar System, nebular particles behave as random thermal motion and concurrent revolution around the Sun. It is impossible to trace each particle in microcosmic scope. However, nebular particles follow a probability orbit, the so-called Keplerian orbit. We attempt to describe the probability orbits of nebular particles in the Solar System by using the wave function ψ as we do to describe the probability orbits of electrons in the atom system, which satisfies the formal Schrödinger equation,

$$-\frac{K^2}{2\mu}\nabla^2\psi = \left[E + \frac{GM\mu}{r}\right]\psi \tag{1}$$

where G is gravitational constant, μ and M are masses of the nebular particle and the Sun, respectively, r is the distance between the particle and the Sun, K is the large-scale constant. If total energy $E < 0$, the normalized radial function is

$$R_{nl}(r) = -\left\{\left(\frac{2}{na_0}\right)^3 \frac{(n-l-1)!}{2n\left[(n+1)!\right]^3}\right\}^{\frac{1}{2}} e^{-\frac{\rho}{2}} \rho^l L_{n+1}^{2l+1}(\rho) \tag{2}$$

where

$$L_{n+1}^{2l+1}(\rho) = \sum_{k=0}^{n-l-1} (-1)^{k+1} \frac{\left[(n+l)!\right]^2 \rho^k}{(n-l-1-k)!(2l+1+k)!k!},$$

$a_0 = K^2/GM\mu^2$, $\rho = 2r/na_0$, $n = 1,2,3\cdots$, and $l = 0,1,.$ $2,\cdots,n-1$ Based on the equiprobability principle, the radial probability density of the nebular particles is expressed as

$$m(r) = \sum_{nl} R_{nl}^2(r) r^2 \tag{3}$$

It has been found that the protosolar cloud is composed of ~0.71 hydrogen, ~0.27 helium, and ~0.02 heavy elements [14]. The interplanetary space is filled with atomic hydrogen [15], since H_2 molecules are very vulnerable to photons with energies below the Lyman limit

[16]. The distributions of H and He play crucial role on the formation and evolution of planets. From $a_0 = K^2/GM\mu^2$, we take $a_0 = 1$ AU for H, and $a_0 = 1/16$ AU for He. The calculated $m(r)$ for H is shown in **Figure 1(a)**. Significant facts were found by Nie[17,18] that the giant planets and the center of the Kuiper Belt are all at the wave crests, and their masses order is almost consistent with that of the amplitude of $m(r)$, the terrestrial planets including the Earth are all in the first wave, and their masses order is consistent with that of the intensity of $m(r)$. The calculated $m(r)$ for He is shown in the bottom wave line of **Figure 1(b)**. It is found that each terrestrial planet is located in a separate wave of He from second to the fifth, and the Asteroid Belt is located in the sixth wave. The intensity of H is too small to form planet in the first wave of He.

Here $m(r)$ is the non-normalized probability density of interplanetary matter. To calculate planets rotation, the mass density $\rho(r)$ should be given by $m(r)$ and the near-Earth mass density $\rho_\oplus$. Based on HST [19-22] observations of the nebular disks around the young stars, it is known that most of the interplanetary matter is distributed in the nebular disk. The thickness of the nebular disk is expressed as $h(r) \propto r^\beta$. For instance, β = 1.10-1.32 for HK Tau/c [20], 9/8 for Orion 114-426 [21], 1.29 for IRAS 04302+2247 [22], and 1.25 for HH 30 IRS [23]. Here we take $\beta = 1$ when $r < 6$ AU in the solar system. Taking $r_\oplus$ to be the distance of Earth from the Sun, $m(r_\oplus)$ probability density in the near Earth space, and $h_\oplus$ the scale height of the nebular disk at $r_\oplus$, due to $h_\oplus/h(r) = r_\oplus/r$ and $m(r)/m(r_\oplus) = [\rho(r)\cdot 2\pi r \cdot h(r)]/[\rho_\oplus \cdot 2\pi r_\oplus \cdot h_\oplus]$, $\rho(r)$ can be expressed as

$$\rho(r) = \frac{\rho_\oplus r_\oplus^2}{m(r_\oplus)} \cdot \frac{m(r)}{r^2} \tag{4}$$

2. Spin Evolution

To calculate the evolution of planets spin, two models for planets revolution and rotation should be established. Assuming the planet spin axis is perpendicular to the orbital plane and the spin is anticlockwise similar to its revolution. We take the radius of the planet Hill sphere as *R*. Drawing a thin ring with the width $\mathrm{d}r$ on the surface of the Hill sphere which is perpendicular to the line connecting the Sun with the planet, and letting *r* and r_0 are the heliocentric distances of the ring center and the planet center, the spin angular momentum obtained during the period of revolution by the ring in collision with

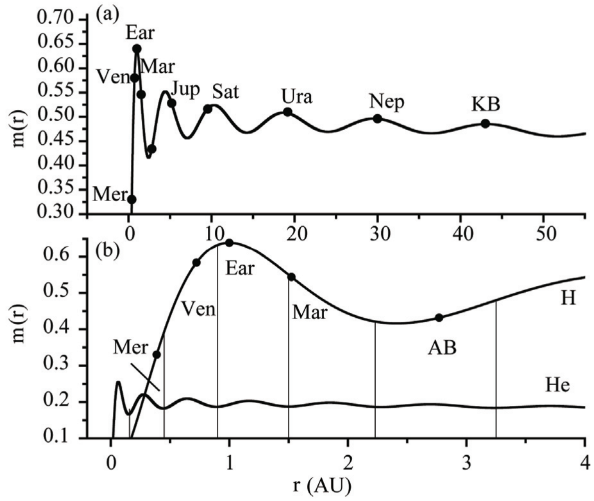

Figure 1. Radial probability density distribution of interplanetary matter. Upper panel a) is the distribution of H with 3/4 of the nebular mass. Giant planets and the center of Kuiper Belt are all at the wave crests. Terrestrial planets are all at the first wave. Bottom panel b) is the distribution of He with 1/4 of the nebular mass and the first wave of H. Each terrestrial planet lies at a separate wave of He. It is interesting that the Kuiper Belt and Asteroid Belt lie in the sixth wave of H and He, and the widths of belts are consistent with wavelengths, respectively.

interplanetary matter is

$$\mathrm{d}L = -\rho(r)\left[2\pi r \cdot 2\sqrt{R^2-(r-r_0)^2}\,\mathrm{d}r\right]\eta v(r-r_0) \tag{5}$$

where v is the revolution velocity of a planet, ηv is the relative velocity of planets to interplanetary gas. Theory of the origin of the solar system suggests that solids and gas do not have the same angular velocities in solar nebula, and angular velocity of the gas is smaller than that of a large body [24], so $0 < \eta < 1$. Assuming $m(r) = m(r_0) + kx$ in the scale of the planet, where $x = r - r_0$, and taking the angular velocity of revolution to be Ω, the integral of Equation (5) is

$$\begin{aligned}\Delta L_1 &= -4\pi\eta\Omega\frac{\rho_\oplus r_\oplus^2}{m(r_\oplus)}\int_{-R}^{R}\left[m(r_0)+kx\right]\sqrt{R^2-x^2}\,x\mathrm{d}x \\ &= -\frac{\rho_\oplus r_\oplus^2\pi^2\eta}{2m(r_\oplus)}\Omega R^4 k\end{aligned} \tag{6}$$

This is the angular momentum obtained by the planet during the period of revolution. If the curvilinear slope or derivative of $m(r)$, $k > 0$, the spin angular momentum is decreasing, this is the case of the Venus and Mercury.

Assuming the planet as a sphere with uniform density, the moment of inertia is $J = 2mR_0^2/5$, where *m* is the planet mass, R_0 planetary object radius. Using $\Delta L_1 =$, $J\Delta\omega_1$ the increment of angular velocity is

$$\Delta\omega_1 = -\frac{5\pi^2 \rho_\oplus r_\oplus^2 \eta}{m(r_\oplus)} \cdot \frac{\Omega R^4 k}{m R_0^2} \tag{7}$$

The resistance of interplanetary matter during the planet rotation could also slow down its spin velocity. During a period of revolution, the mass of interplanetary matter interacting with the planet Hill sphere is $\Delta m = \rho(r_0)\cdot \pi R^2 \cdot 2\pi r_0$. Based on the conservation of angular momentum $(2mR_0^2/5)\cdot\omega = [2(m+\Delta m)R_0^2/5]\cdot(\omega+\Delta\omega_2)$, ignore the second order, then $\Delta\omega_2 = -\Delta m\omega/m$, where ω is spin angular velocity. Using $\rho(r_0) = [\rho_\oplus r_\oplus^2/m(r_\oplus)]\cdot[m(r_0)/r_0^2]$, the increment of angular velocity is

$$\Delta\omega_2 = -\frac{2\pi^2 \rho_\oplus r_\oplus^2}{m(r_\oplus)} \cdot \frac{m(r_0) R^2}{m r_0}\omega \tag{8}$$

During a period of revolution, the increment of the spin angular velocity is $\Delta\omega = \Delta\omega_1 + \Delta\omega_2$, and the planet spin angular acceleration can be expressed as

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = -\frac{\pi\rho_\oplus r_\oplus^2}{m(r_\oplus)} \cdot \frac{R^2}{m}\Omega\left[\frac{5}{8}\left(\frac{R}{R_0}\right)^2 \eta k\Omega + \frac{m(r_0)}{r_0}\omega\right] \tag{9}$$

Assuming the revolution angular velocity Ω, the planet mass m, and the Hill sphere radius R are constants, so the mass density $\rho_\oplus$ is the only variable in Equation (9). Referring to the method for studying the nebular disks around sun-like stars [25], the mass of the disk decreases as $\frac{\mathrm{d}m_{disk}}{\mathrm{d}t} \propto -m_{disk}$. Due to the effect of solar wind, the evolution of $\rho_\oplus$ can be worked out by solving equation $\frac{\mathrm{d}\rho_\oplus}{\mathrm{d}t} = -a\rho_\oplus$ to give $\rho_\oplus = \rho_0 \mathrm{e}^{-at} = \rho_0 \mathrm{b}^{-t}$, where $b > 1$. The planet spin angular velocity is derived by integral of equation (9) as

$$\omega = (Q+\omega_0)\exp\left[P\rho_0\left(b^{-t}-1\right)/\ln b\right] - Q \tag{10}$$

where $P = \frac{\pi r_\oplus^2}{m(r_\oplus)} \cdot \frac{m(r_0) R^2}{m r_0}\Omega$, and

$$Q = \frac{5\eta}{8m(r_0)}\left(\frac{R}{R_0}\right)^2 k r_0 \Omega .$$

Taking current time $t = 0$, then ω_0 and ρ_0 are the current spin angular velocity and mass density in near-Earth space, respectively. Here ρ_0 and b should be clarified. The mass density of interplanetary dust have been estimated to be 10^{-19} kg/m^3 [26], the ratio of dust to nebula is about 2%, then the density of interplanetary matter is about 5×10^{-18} kg/m^3. The hydrogen number density is ~10^4 cm^{-3} (the mass density is ~10^{-17} kg/m^3) [27]. Here we take ρ_0 = 5×10^{-18} kg/m^3 $=1.7\times10^{16}$ kg/AU^3. Taking ρ_B as the mass density at 4.5 Gyr (the Earth age) ago, then $\rho_B/\rho_0 = b^{45}$. We estimate b by Equation (10). It is found that the effects of mass density change (ρ_B/ρ_0) on the planetary rotation periods are different for different planet, it is obvious for Venus and Mercury, but unobvious for other planets. For $\rho_B/\rho_0 \sim 10^9$, according to b = 1.585, the periods of the terrestrial planets are all close to one day, which are typical and expected values. Taking Hill sphere radius $R = (m/3M)^{1/3} r_0$, $\eta = 0.5$ and $r_\oplus$ = 1 AU, $m(r_\oplus)$ = 0.64, and $M = 1.989\times10^{30}$ kg, the other parameters are derived and shown in **Table 1**.

Furthermore, the evolution of spin periods of the Venus and Mercury are shown in Figure 2. It is found that the spin of the Venus changes naturally from positive to retrograde around 3.5 Gyr ago, the period is from the initial about 3 day to the current – 243 day; the spin of the Mercury changes from quick to slow, the period is from the initial about 1 day to the current 58.6 day. Another fact should be noticed that the spins of the two planets remain almost steady from about 3 Gyr ago until the foreseeable future.

Appling Equation (10) to other planets, their spins change slightly. In detail, the period of the Earth decreases 0.01 days, the Mars increases 1.3 days, the Jupiter decreases 0.02 days, the changes of Saturn and Neptune are much smaller. However, Equation (10) cannot be applied to the Uranus due to the particularity of its spin axis.

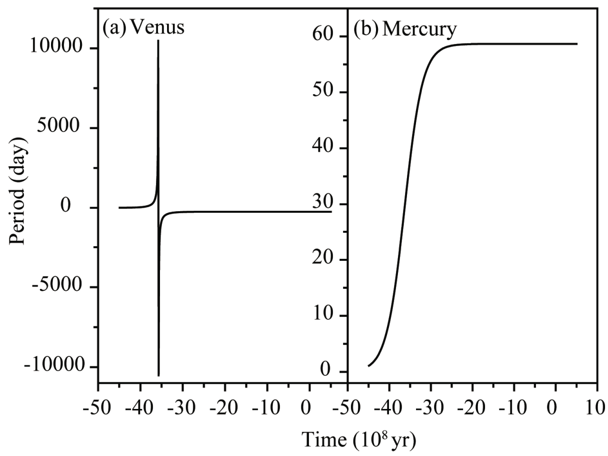

Figure 2. Evolution of the rotation periods of the Venus and Mercury in 4.5 billion years. From the initial spin period of 3.5 day to the current -243 day, the spin of the Venus experienced a reverse around 3.5 billion years ago and remained almost stable since then. From the initial spin period of 1 day to the current 58.7 day, the spin of the Mercury experienced a rapid slowing down around 3 billion years ago and then remained stable to present.

Table 1. The data for computing.

planets	m 10^{24}kg	R_0 10^{-5}AU	r_0 AU	R AU	ω_0 yr^{-1}	T yr	m (r_0)	K AU^{-1}	τ_B day	τ_0 Day
Mer	0.33	1.63	0.387	0.0015	39.11	0.241	0.33	1.03	1.054	58.6
Ven	4.87	4.04	0.723	0.0068	–9.44	0.615	0.59	0.43	3.465	–243
Ear	5.97	4.25	1.000	0.0100	2210	1.000	0.64	0	0.985	0.997
Mar	0.64	2.26	1.524	0.0073	2236	1.881	0.54	–0.24	2.314	1.026
Jup	1900	47.6	5.203	0.3551	5594	11.86	0.53	–0.05	0.412	0.41

Here m and R_0 are the planet mass and radius, respectively. R is the Hill radius, r_0 is the distance from the Sun to the planet center, ω_0 is the current spin angular velocity, and T is the revolution period, τ_B and τ_0 are the 4.5 Gyr ago and current spin periods.

3. Conclusions

In summary, the probability orbits of nebular particles in the Solar System can be described by the wave function in large scale, which satisfies the formal Schrödinger equation. The radial density distribution of interplanetary matter is undulated as a wave line, and the curvilinear slopes of the wave line at the positions of the Venus and Mercury slow down their spins. The results show that the planets spins can evolve from the initial positive spins to the current state if the original interplanetary matter density is 10^9 times to the current one and has an exponential decrease with time. Especially, the retrograde of the Venus is a natural result of the asymmetric effect of interplanetary matter.

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