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From Marbles to Numbers—Estimation Influences Looking Patterns on Arithmetic Problems

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Flexibly spotting and applying shortcut options in arithmetic is often a major challenge for children as well as adults. Recent work has suggests that children benefit in terms of such flexibility from tasks requiring estimation or other operations with quantities that they cannot easily enumerate. Such tasks often require comparison of quantities by fixation and as such necessitate long-range eye movements, e.g. across the whole screen. We tested whether fixation patterns account for transfer from estimation to arithmetic tasks. Conceivably, participants who first solve estimation tasks are more flexible in spotting and applying shortcuts on later arithmetic tasks, because they stick to scanning the screen with long-range eye movements (which were necessary for solving the estimation task). To test this account, we manipulated the location of the marbles in an estimation task so that one group of participants had to make long-range eye movement, whereas another group did not need long-range eye movements to solve the task. Afterwards participants of both groups solved addition problems that contained a shortcut option based on the commutativity principle. We tested whether shortcut usage and fixation patterns in the arithmetic problems were influenced by the variant of the estimation task provided beforehand. The experiment allowed us to explore whether flexibility in spotting and using arithmetic shortcuts can be fostered by applying a prior task that induces flexible looking patterns. The results suggest that estimation tasks can indeed influence fixation patterns in a later arithmetic task. While shortcut search and application is reflected in fixation patterns, we did not obtain evidence for the reverse influence. Changed fixation patterns did not lead to higher shortcut usage. Thus, the results are in line with top-down accounts of strategy change: fixation patterns reflect rather than elicit strategy change.

Keywords: Mental Arithmetic; Estimation; Fixation Pattern; Transfer; Commutativity Principle

Introduction

For some elementary school children, arithmetic seems to be a tremendous challenge while for others it's a child's play. For all children it includes the discovery and use of general as well as abstract principles, which allow them to solve (apparently) hard problems easily and fast. Unfortunately the connection between understanding, identifying and applying shortcut options is at best moderate. Many researchers found large interindividual variability in the children's solution times and their use of shortcut strategies. For example, Dubé and Robinson (2010) found that 1/4 of the children did not use any shortcut at all. Robinson and Dubé (2012) later argued that children have different attitudes toward accepting strategies that are highly efficient but novel. For instance, from grade three onwards commutativity-based shortcuts are used spontaneously (Gaschler, Vaterrodt, Frensch, Eichler, & Haider, 2013). Children used a shortcut when subsequent addition problems contained the same addends in different order (e.g., $6 + 2 + 3 = ?$ followed by $2 + 6 + 3 = ?$). Furthermore, spontaneous application of two

different commutativity-based shortcuts correlated positively. However, while it could be documented that commutativity-based shortcuts were used spontaneously, the rate of usage was rather low. Apart from difficulties in spontaneously using shortcuts, past research has documented overgeneralization (e.g. Siegler & Stern, 1998). Once children started to use one shortcut, some tended to apply it irrespective of whether the underlying mathematical principle was applicable to the current arithmetic problem or not. This research showed that during elementary school the flexible use of shortcut strategies is not very balanced yet.

One potential way to foster adaptive flexibility in strategy usage (Verschaffel, Luwel, Torbeyns, & Dooren, 2009) might be to employ estimation tasks. By this, the mathematical principles and the corresponding shortcuts might be conveyed at early age. This might provide a head start for tackling the respective shortcuts in mental arithmetic. Several studies on commutativity have shown that children have at least some understanding of the concept of commutativity before entering school (Canobi, Reeve, & Pattison, 2003; Cowan & Renton, 1996; Resnick, 1992; Wilkins, Baroody, & Tiilikainen, 2001). Fur-

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thermore, there is evidence for that toddlers develop an informal understanding of relations between objects in the real world before entering school (Baroody & Gannon, 1984; Baroody, Ginsburg, & Waxman, 1983; Canobi, Reeve, & Pattison, 2002; Gallistel & Gelman, 1992; Gilmore, McCarthy, & Spelke, 2010; Resnick, 1992; Siegler & Jenkins, 1989; Sophian, Harley, & Martin, 1995). This idea is based on Resnick's (1992) model of mathematical thinking, in which on the first level, the level of protoquantities, mathematical thinking is strictly object-bound.

Moreover, Sherman & Bisanz (2009) showed that working with non-symbolic material can encourage subsequent strategy use in symbolic equivalence problems. In this context, research hints at that estimation might positively influence exact calculation. Gilmore, McCarthy and Spelke (2007) found that even preschoolers were able to solve symbolic problems as long as they were instructed to just estimate the results, rather than to calculate the exact result. Along these lines, Hansen and colleagues (submitted) tested whether children would profit from an estimation task involving commutative arithmetic problems on later arithmetic problems. They confirmed the assumption that symbolic estimation increased the spontaneous spotting and applying of commutativity-based shortcuts in a later arithmetic task. Surprisingly, the positive effect seemed to be confined to actual usage of commutativity-based shortcuts (procedural knowledge). In a task measuring conceptual understanding of the commutativity principle, children showed liberalization in the response criterion rather than improved or maintained performance when before confronted with a commutativity-based estimation task.

Apparently, symbolic as well as non-symbolic estimation tasks support the use of commutativity-based shortcuts—but not by activating conceptual knowledge of the mathematical principle. This suggests to explore potential ways of transfer between the task formats that side-track conceptual knowledge. One potential account for the incoherent transfer results reported by Hansen et al. (submitted) and Sherman and Bisanz (2009) might be a transfer of eye movement patterns. Participants might profit from the estimation task in spotting and applying shortcuts in later arithmetic problems, because (a) long-range eye movements are helpful in both contexts and are (b) transferred from the estimation task to the arithmetic task. Specifically, short cut strategies that entail comparing addends across subsequent addition problems should necessitate unusually long eye movements. Potentially, such looking patterns—triggered by an estimation task—would still be present when later faced with an addition task and raise the chance that a child spots and applies the shortcut.

Our hypothesis was that a variant of an estimation task that requires long-range eye movements, would lead to a larger amount of shortcut usage in a later arithmetic task (as compared to an estimation task not requiring long-range eye movements). To test this hypothesis, we actively manipulated the eye movements in order to investigate the influence of long-range eye movements on the detection and application of possible shortcuts. Eye movement patterns can influence spatial reasoning by means of an implicit eye-movement-to-cognition link. For instance, Thomas and Lleras (2007) showed an implicit compatibility between spatial cognition and the eye movement. However, until now the influence of eye movement patterns (i.e., those triggered by an estimation task) on arithmetic problems presented later on has been neglected. Therefore, we contrasted (a) one experimental condition starting with an estimation task necessitating long-range eye movements (scattered group) with (b) another group of primary school children starting with an estimation task without such demands (centered group). As dependent measure we tested the extent to which children saved calculation effort by a commutativity-based shortcut in a mental arithmetic task presented afterwards. Comparing addends in subsequent arithmetic problems, one could avoid calculation on problems that presented the same addends as the predecessor problem (e.g., 6 + 2 + 3 = ? followed by 2 + 6 + 3 = ?), this shortcut we called *addends compare strategy*.

Method

Participants

Thirty-four elementary school children (16 females and 18 males) with an age range between 6 and 11 years took part in the experiment. The mean age of the children in the *scattered group* was 8.78 years (*SD* = .92), and in the *centered group* 8.47 years (*SD* = 1.02). The children attended second to fifth grade of various Berlin elementary schools, most of them in the fourth grade (50%). The children were randomly assigned to either the scattered group (18 children) or the centered group (16 children).

Materials

The experiment comprised two parts. In the first part, the children were presented with an estimation task that differed between the groups (scattered/centered). In the second part, both groups were presented with an arithmetic task. All material was computerized.

For the *estimation task* four sets of eight estimation problems were designed, depicting different quantities of marbles. The two sets for the centered group consisted of one quantity of marbles, which belonged to one fictional character (either "Tim" or "Lisa"). In order to provoke a small spatial range and low number of saccades in the *centered group*, the quantity of marbles was presented centrally. Children were asked to estimate if the character owned few or many marbles (see **Figure 1**). The two sets for the *scattered group* included two different quantities of marbles of which one belonged either to "Tim" or "Lisa". To trigger a larger spatial range and higher number of saccades (compared to the centered group), the quantities of marbles were presented at right and left edge of the presentation frame. The children had to indicate which of the characters owned more marbles or if both own the same amount of marbles.

The 12 *arithmetic problems* were presented in two groups of six simultaneously depicted problems on two consecutive screens (see **Table A1**). We presented six problems in black on grey background simultaneously on the screen. Digits were approximately .5 cm wide and 1 cm tall. The distance both between the lines and columns of digits was 5 cm. Each addition problem consisted of three addends between 2 and 9 (e.g., 6 + 2 + 3 = ?). Each number occurred only once in a problem. Small and large numbers were balanced across the different problems. Each screen contained two commutative problem-pairs, in which the addends compare strategy could be used—one problem and its repetition with a different order of the same addends. All other problems were filler problems, with no shortcut option.
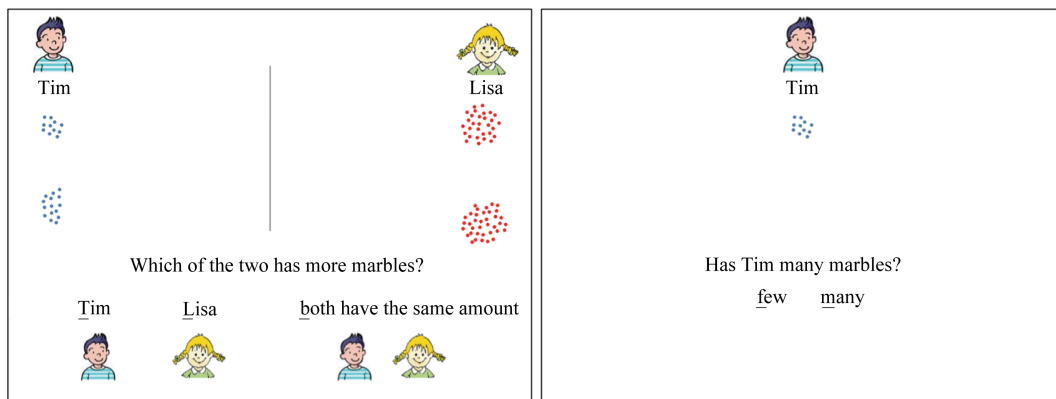
Figure 1.
Example of the estimation task for the scattered group (left) and the centered group (right).

Procedure

Each child was tested individually in the laboratory of the Department of General Psychology at Humboldt-University, Berlin. The children were seated in front of a 22 inch TFT computer monitor, which was equipped with the SMI RED stationary eye tracking system recording at 250 Hz. Children were asked to find a comfortable position (about 60 cm from the screen) and reminded to sit as still as possible. After five-point calibration, they received the instruction of the estimation task, which introduced them to fictional characters "Tim" and "Lisa" as owners of different quantities of marbles. They were then presented with an example problem. The children were reminded that the task did not require counting but only estimation and that they should answer as fast and as correct as possible.

In the estimation task, eight different estimation problems were presented consecutively. In a between subject design we wanted to manipulate the range and the number of the saccades during the estimation task to investigate the influence on discovering and using the commutativity shortcut strategy on later addition problems. For the centered group the marble quantities were presented centrally and the child had to decide if the pictured character had few or many marbles. For the scattered group the marble quantities were presented on the right and left edges of the screen and the child had to decide if one of the pictured characters had more marbles or if both had the same amount. After presenting each estimation problem for two seconds, the screen went blank. The time limit ensured that children had to rely on estimation. The experimenter entered the verbal answer of the child and started the next trial of the estimation task.

After completing the estimation task, the children calculated simple addition problems (arithmetic task). The children were instructed to work through the problems (six per screen) in strict order from top to bottom and not to leave out any. The experimenter entered the given answers so it was directly visible and remained visible when working on subsequent problems. After completing the first six problems, the experimenter started the second and last screen presenting the next six problems. Overall the procedure took about 10 minutes.

Results

The aim was to evaluate whether different eye movement patterns induced by different estimation tasks (scattered and centered) lead to different eye movement patterns and/or increased use of the addends-compare strategy in the arithmetic task.

Eye Movement Data

The eyetracking data of one child were not recorded correctly so we present the data of 33 children. In the analysis of the eye movements we focused on saccade distances, which were computed as Euclidean distance in angular degree. The saccade distances from commutative problems were compared to all other (non-commutative) addition problems. **Figure 2** depicts a bimodal distribution of saccade distances for the centered and scattered group. The bimodal character seemed more pronounced for commutative problems and especially so for the centered group.

For a more detailed analysis, we differentiated between horizontal and vertical saccades. To test if the two groups (scattered and centered group) differ in the saccade distances in the arithmetic task, we conducted ANOVAs for horizontal and vertical saccades separately. The results showed a significant difference between both groups for the horizontal saccade distances in the commutative problems, $F(1, 31) = 13.57$, $p = .001$. Surprisingly, the horizontal saccade distance for commutative problems was *lower* for the scattered as compared to the centered group (see **Table 1**). There were neither differences for the horizontal saccade distances for the non-commutative problems, $F(1, 31) = 1.33$, $p = .26$, nor between-group differences for the vertical saccade distances for non-commutative problems, $F(1, 31) = .65$, $p = .43$, and the commutative problems, $F(1, 31) = .780$, $p = .38$.

Additionally, we conducted a 2 (horizontal versus vertical saccades) $\times$ 2 (commutative versus non-commutative problems) $\times$ 2 (condition: scattered versus centered) ANOVA. As suggested by **Table 1**, the horizontal saccade distances were longer than the vertical saccade distances (main effect for horizontal versus vertical saccade distances, $F(1, 31) = 12605.84$, $p < .001$; $\eta p^2 = .998$). The ANOVA did not show a significant overall difference in the saccade distance for the commutative versus non-commutative problems, $F(1, 31) = 1.35$, $p = .25$; $\eta p^2 = .042$. However, we found a significant interaction of commutative versus non-commutative problems and condition, $F(1, 31) = 10.64$, $p = .01$; $\eta p^2 = .255$. This indicates that condition had an impact on the difference of saccade distances for commutative
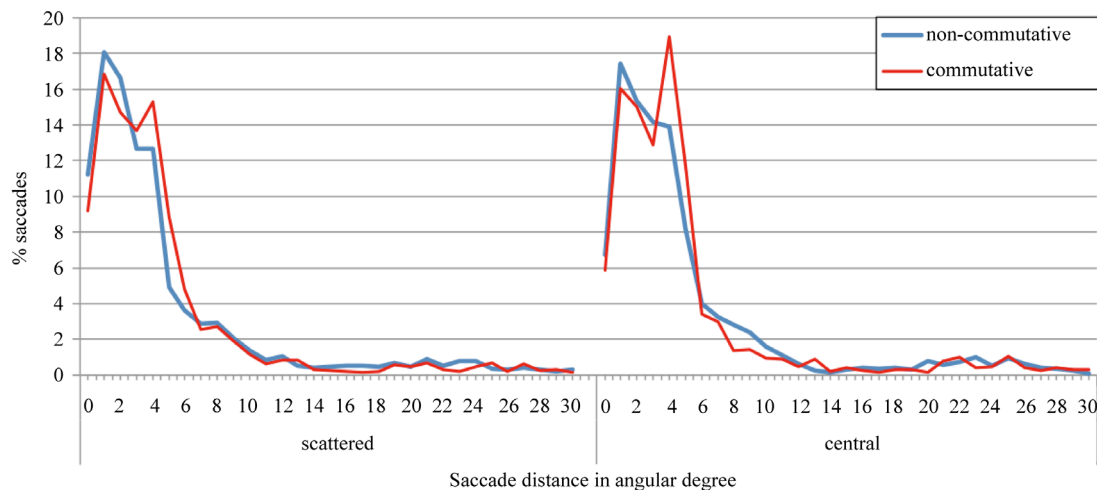
Figure 2.
Distribution in percent of saccade distances in angular degree for commutative and non-commutative problems for scattered group (left) and centered group (right).

Table 1.
Mean saccade distance horizontal and vertical for commutative and non-commutative problems analysed for condition.

Condition	Saccade distance horizontal		Saccade distance vertical	
	Non-commutative problems	Commutative problems	Non-commutative problems	Commutative problems
Scattered	4.38	4.11	2.28	2.24
Centered	4.24	4.41	2.32	2.3
Total	4.31	4.24	2.30	2.27

and non-commutative problems. Furthermore, this impact was different for horizontal and vertical saccade distances, because the 2-fold interaction was significant, too, $F(1, 31) = 15.28$, $p < .001$; $\eta p^2 = .330$. We did not obtain a main effect for condition, $F < 1$. In conclusion we manipulated the eye-movement in the estimation task (scattered and centered) and found different eyemovement patterns in the arithmetic task presented afterwards. For conclusions concerning the strategy use we present now the results of the solving times in the arithmetic task.

Solving Times

The solving time comprises the time between responding to one arithmetic problem and the time of the key press of the experimenter entering the verbalized answer to the subsequent problem (first key in entering the child's answer). **Figure 3** suggests that children in both experimental conditions benefitted from the addends-compare strategy. In line with exploiting the commutativity principle, children were faster when faced with the same addends in altered order for a second time on subsequent addition problems. The prior problem needed to be calculated conventionally, whereas the second problem of the commutativity pair consisted of the same addends in a different order. Supposed a child first calculated $6 + 3 + 2$ and then $6 + 2 + 3$ it would not need to calculate when confronted with the second problem—if it used the addends compare strategy. Across experimental conditions, a pairwise comparison of these two problems showed a significant benefit in solving times for commutative compared to their preceding noncommutative problems $t(32) = 3.37$, $p = .01$. Note that mean

solving times of the filler problems were higher, 9.82 sec for the scattered group and 10.04 sec for the centered group. The comparison we focused on can thus be regarded as a conservative estimate of the usage of the addends compare strategy.

To test the difference between the scattered group and the centered group in solving times, we conducted a 2 (commutativity: commutative vs. non-commutative problems) × 2 (condition: scattered vs. centered group) ANOVA. As suggested by **Figure 3**, we obtained a significant main effect of commutativity, $F(1, 31) = 10.74$, $p = .01$, $\eta p^2 = .26$, but no effect of condition and no interaction effect of commutativity and condition ($Fs < 1$). The error rate for the twelve arithmetic problems was 10.2% and individual number of errors ranged from 0 to 6 (mean = 1.18; SD = 1.29). Note that children of different age seemed to profit to a similar extent from the addends compare strategy. We obtained no correlation between age and the benefit in solving times on commutative as compared to non-commutative problems.

Discussion

Prior work (Obersteiner et al., 2013; Hansen et al., submitted) indicates that estimation can positively influence subsequent calculation tasks. Our assumption was that this influence might in part be based on transfer of eye movement patterns. In an eyetracking experiment with primary school children, we contrasted a variant of an estimation task that did necessitate long-range eye movements with one that required central fixations. In particular, we were interested in how these two variants of an estimation task would influence later spontaneous usage of
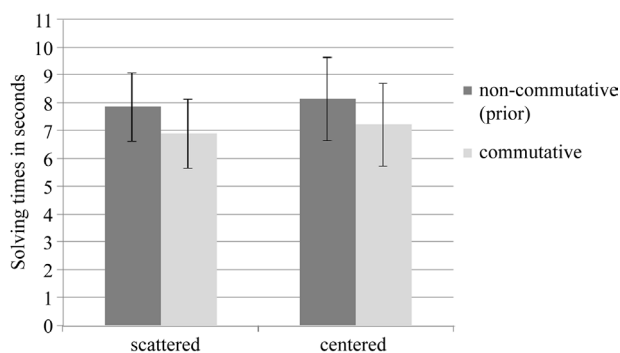
Figure 3.
The mean solving times in seconds per arithmetic problem for non-commutative (prior) problems (dark grey) in comparison to the commutative problems (light grey) for the scattered and the centered group. The error bar displays the 95% confidence interval of the comparison with each other.

commutativity-based shortcuts in mental arithmetic. While we did find an effect of estimation task variant on eye movement patterns in the arithmetic task, we did not obtain any effect on shortcut usage. Children of both experimental groups profited from the commutative addition problems to the same extent.

Surprisingly, the effect of condition on eye movements in the arithmetic task was opposite to our expectations. At present we can only state that we did see an effect of the estimation task on eye movements in a later calculation task and speculate about the reasons for its unexpected direction. The estimation tasks in the scattered and centered group were designed to elicit long-range eye movements to a different extent. To minimize these saccades in the centered group we presented only one character (Tim/Lisa) centrally and slightly changed the question to, whether Tim/Lisa has many or few marbles. In comparison, the scattered group was asked in the estimation task whether Tim or Lisa has more marbles. Both tasks required some form of estimation, but the children in the centered group had to compare the centrally depicted marbles with their own concept of few or many. To the scattered group, however, the comparison array was presented simultaneously on the other side of the computer screen. Presumably, in the centred group one additional step of processing was needed (i.e., "What do I consider as few ore many marbles?"). Accordingly, one limitation of the current research is that the two variants of the estimation task not only differed considerably in the type of saccades demanded, but also in other cognitive processes involved in generating an answer. The requirements in the centered condition might have supported flexibility in thinking in this group, because the criterion to decide between *few* and *many* can adaptively change between the problems (e.g. in one problem a child might consider 8 marbles as *many*, but after seeing 20 marbles, 8 seem to be only a *few*). Presumably, such demanding comparisons might lead to more comparisons between numbers on the screen once the addition problems are being presented. This in turn, might be the reason for the centred group executing longer saccades. Yet, these differences in fixation patterns did not lead to differences in spotting and applying shortcut options. This is coherent with models emphasizing the role of top-down decisions on strategy change in skill acquisition. For instance, Haider and Frensch (1999) suggested that changes in fixation patterns in a skill acquisition task involving a shift towards a more efficient strategy reflect the voluntary decision to change

the strategy. Changes in fixation patterns might often reflect rather than cause changes in processing strategy. This is in line with the concept of adaptive expertise (Verschaffel et al., 2009), according to which learners need to autonomously regulate whether (a) to solve an arithmetic problem in a standard way or to (b) search for/apply a shortcut.

Past work (e.g. Gaschler et al., 2013; Godau et al., submitted) showed that flexible strategy use is mirrored in fixation patterns and that different commutativity-based shortcuts are mirrored in different fixation patterns. For the use of the addends-compare strategy, the eye movement pattern showed that the children looked back to the preceding problem featuring the same addends in different order. While using a shortcut strategy is reflected in the corresponding fixation patterns, influencing fixation patterns does not necessarily lead to changes in arithmetic strategies. Future studies might test more direct ways to trigger flexibility in calculation strategies by inducing flexibility in fixation patterns. Visual cues can help the learner attend to and notice relevant information in the problem, which they previously may have ignored (Madsen, Rouinfar, Larson, Loschky, & Rebello, 2013). In their study, Madsen and colleagues (2013) found that inducing participants to look at helpful areas in physics problems and to ignore distracting areas when no visual cues are present is possibly a first step to support them to reason correctly about the problem. They also found transfer effects in that students could successfully answer and reason about related but different problems without cues (Madsen et al., 2013). Guiding attention to areas with regard to contents is one point; we, on the contrary, wanted to focus more on the eye movement as such, without special attention on content. For further investigations concerning a direct intervention to support flexible strategy use, the eye movement pattern could also be manipulated in a calculation task. During solving the arithmetic problems, children who have more long distance saccades (e.g. by presenting distractors in changing corners of the screen) maybe recognize the shortcut strategy more often.

In summary, the current results suggest that estimation problems can indeed influence fixation patterns in a later arithmetic task. While shortcut search and application is reflected in fixation patterns, we did not obtain evidence for the reverse influence. Changed fixation patterns did not lead to higher shortcut usage. Thus, the results are in line with top-down accounts of strategy change: fixation patterns reflect rather than elicit strategy change (cf. Haider & Frensch, 1999).

Author Note

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Appendix

Table A1.

The 12 arithmetic problems (commutative and non-commutative problems). The results printed in italics had to be calculated by the participants.

Screen 1		
3 + 5 + 4	=	12
4 + 9 + 8	=	21
4 + 8 + 9	=	21
6 + 2 + 5	=	13
9 + 7 + 2	=	18
2 + 9 + 7	=	18
Screen 2		
6 + 3 + 2	=	11
6 + 2 + 3	=	11
8 + 9 + 6	=	23
7 + 2 + 6	=	15
6 + 7 + 2	=	15
7 + 4 + 8	=	19