

Full-Filling Subcarrier Power Allocation in OFDMA-Based Cognitive Radio Systems

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ABSTRACT

When implementing an appropriate windowing, the interference from a Cognitive Radio (CR) system to licensed systems (primary users) will be significantly reduced. Consequently, power allocated to subcarriers can be increased, especially subcarriers having far spectral distance to primary user bands can be allocated full of its maximum possible power. In this paper, we propose a new class of sub-optimal subcarrier power allocation algorithm that significantly reduces complexity of OFDMA-based CR systems. Two sub-optimal proposals, called Pre-set Filling Range (PFR) and Maximum Filling Range (MFR) are studied. Investigations show that this new power allocating algorithm allows CR systems obtain high throughput while retaining low complexity.

KEYWORDS

Cognitive Radio; Windowing; Power Allocation; Full Filling

1. Introduction

Today, rapid growth of data demands requires solutions, especially higher spectrum efficiency solution, to satisfy the bandwidth hunger. Thanks to its ability of dynamically exploiting unused spectrum, OFDM-based Cognitive Radio (CR) has been considered widely as a solution to ameliorate spectrum efficiency. However, because of the use of unused spectrum of licensed users (or primary user-PU), CR and primary users often exist in side by side bands, thus interference may cause from CR to PU system and vice versa. As presented in [1-4], being based on OFDMA technology, capacity of a CR system depends not only on powers allocated to each of its subcarrier but also on interference level caused from PU users to CR user's band. It was further explained in [1], [5]. This mutual interference is due to the non-orthogonality of OFDM signals. This mutual interference leads to the reduced performance of both PU and CR networks. In principle, higher powers allocated to its subcarriers, higher capacity CR could obtain, but higher level of interference could cause PU. With the presence of mutual

interference, amounts of power to be allocated to each CR's subcarriers must be assessed in a way to ensure the total interference caused from all subcarriers to PU's bands not exceed a pre-defined threshold to protect licensed primary user. Therefore, improving transmit capacity while keeping interference to licensed user at an acceptable level is a challenge which needs to be studied in the implementation of CR system.

So far, a number of studies have been carried out to find solutions to reduce mutual interference with aims to increase subcarrier power and hence CR capacity. A side-lobe suppression mechanism has been proposed in [6] in order to minimize the adjacent channel interference. A max-min algorithm was proposed in [7] for sub-channel, bit and power allocation in a multiuser OFDM-based CR system which shows a better system performance compared with the case that uses guard-bands to protect the active PU users. In [5], the mutual interference problem was studied under the condition that CR system uses OFDM modulation. In [1], two suboptimal schemes have been also proposed in which different step sizes were used in allocating power to each sub-carrier.

In [2], several sub-optimal schemes (scheme A and B) were suggested to reduce numerical calculation steps. The effect of a subcarrier nulling and performance of classical power loading algorithm (water filling, uniform power allocation) has been investigated.

In [8], the authors proposed using windowing to reduce the interference from CR to PU. Results of this study show that, as can be seen from **Figure 1**, only a few subcarriers nearby the PU are affected and their allocated power need to be adjusted carefully, other CR subcarriers can be allocated full power as they introduce negligible interference to PU system. This does not happen in case when no windowing is introduced to CR in which almost CR subcarriers are not allocated the full power (see **Figure 2**). In terms of system capacity, employing windowing significantly improved the capacity.

Based on the results of study in [8], this paper proposes a new class of power allocation algorithm, namely

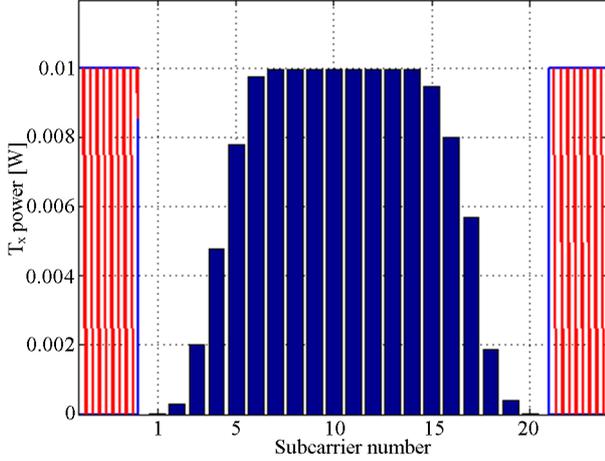


Figure 1. Optimal CR's power allocation (with Tukey window, $I_{th} = 5 \sigma^2$).

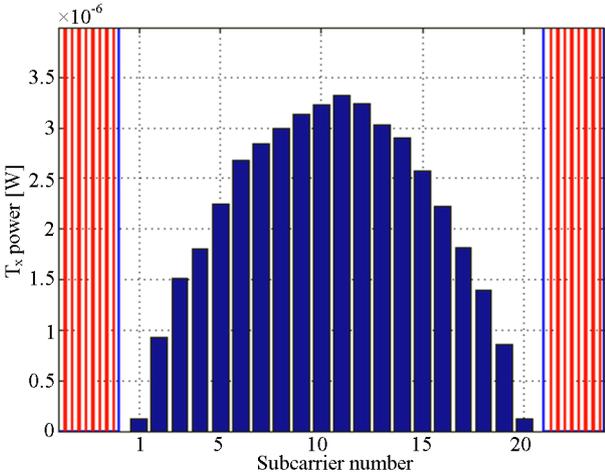


Figure 2. Optimal CR's power allocation (without Tukey window, $I_{th} = 5 \sigma^2$).

full-filling subcarrier power allocation and assesses impact of this new algorithm to system capacity and computational complexity. Two sub-optimum proposals, called Pre-set Filling Range (PFR) and Maximum Filling Range (MFR) are studied. The central idea of those proposals is allocating full power to sub-carriers that have negligible interference to PU. Investigations show that this new power allocating algorithm allows CR systems obtain high throughput while retaining low complexity.

2. System Model

In an OFDM-based CR system operating with presence of PU users, the maximum transmission capacity can be expressed mathematically as [1]:

$$C = \max \sum_{i=1}^N \Delta f \log_2 \left(1 + \frac{|h_i^{ss}|^2 P_i}{\sigma^2 + \sum_{l=1}^L J_i^{(l)}} \right) \quad (1)$$

subject to:

$$\sum_{i=1}^L \sum_{i=1}^N I_i^{(l)}(d_{il}, P_i) \leq I_{th} \quad (2)$$

and

$$P_i \geq 0 \quad \forall i = 1, 2, \dots, N \quad (3)$$

where C denotes the transmission capacity of the CR user, N denotes the total number of OFDM subcarriers, I_{th} denotes the interference threshold can be acceptable by l -th PU band and is a finite deterministic value, σ^2 is the Additive White Gaussian noise; f is the spectrum bandwidth of a subcarrier, h_i^{ss} is the fading gain, $J_i^{(h)}$ is the interference from PU to i_{th} OFDM subcarrier, I_i^l is the interference from i_{th} OFDM subcarrier to PU bands, P_i is the transmission power of i_{th} OFDM subcarrier.

2.1. Interference from CR to PU

From Equations (1) and (2), we can see that the CR transmission capacity is affected by the acceptable interference I_{th} that PU can tolerate.

Assuming signal is an ideal Nyquist pulse, then i_{th} subcarrier's power spectrum density in CR system can be expressed as [1]:

$$\phi_i(f) = P_i T_s \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2 \quad (4)$$

where P_i is the transmission power of i_{th} OFDM subcarrier, T_s is the symbol duration. Interference from subcarrier to PU's bands is integration of this subcarrier spectrum density across the l_{th} PU band. This interference can be expressed as [1]:

$$I_i^{(l)}(d_{il}, P_i) = |h_i^{sp}|^2 P_i T_s \int_{d_{il} - \frac{B_l}{2}}^{d_{il} + \frac{B_l}{2}} \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2 \quad (5)$$

where d_{il} is the spectrum distance between CR's i_{th} sub-carrier and PU band, B_l is bandwidth of l_{th} PU.

With window at the output, the PSD of CR will be shaped. We will consider a more realistic case at the next section.

2.2. Interference from PU Users to CR Users

Power spectrum density of the PU signal after M-fast Fourier transform (FFT) can be expressed as [2]:

$$E\{I_N(\omega)\} = \frac{1}{2\pi M} \int_{-\pi}^{\pi} \phi_{PU}(e^{j\omega}) \left(\frac{\sin(\omega - \psi)M/2}{\sin(\omega - \psi)/2} \right)^2 d\omega \quad (6)$$

where ω is the frequency normalized to the sampling frequency, Φ_{PU} is power spectrum density of PU signal.

Interference caused by l_{th} PU signal to the i_{th} CR's sub-carrier is expressed in [1]:

$$J_i^{(l)}(d_{il}, P_l) = |h_i^{sp}|^2 P_l T_s \int_{d_{il}-B_l/2}^{d_{il}+B_l/2} E\{I_N(\omega)\} d\omega \quad (7)$$

2.3. Impact of Windowing on Interference

The system capacity could be improved by decreasing interference from CR to PU. It could be achieved by windowing transmitting signal of OFDM symbols that sharpens out-of-band subcarrier spectrum which goes down smoothly and quickly. In WLAN system, raised-cosine window is commonly used [9]. This window is defined in time domain as in Equation (10), and associated Fourier transform can be shown as in Equation (11) as in [10].

In case of windowing, the received signal $x(k)$ is windowed in the time domain by window function $w(k)$ result in:

$$\hat{X}(k) = x(k)w(k) \quad (8)$$

and the Fourier transform $\hat{X}(e^{j\omega})$ of $\hat{x}(k)$ is a convolution of the Fourier transform $X(e^{j\omega})$ and $W(e^{j\omega})$ of these respective time signals $x(k)$ and $w(k)$

$$\hat{X}(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) W(e^{j(\omega-\psi)}) d\psi \quad (9)$$

$$G(f) = \frac{\pi^2 \left(\sin\left(\frac{T\omega}{2}(1-a)\right) + \sin\left(\frac{T\omega}{2}\right) \right)}{\omega \left(\pi^2 - \omega^2 a^2 \left(\frac{T}{2}\right)^2 \right)} \quad (11)$$

2.4. PU's Signal Model

In theory, cognitive radio can work in any frequency bands, but in practical broadcasting UHF band is focused as a high potential to host the first secondary wireless networks. TV UHF band has been chosen thank to its suitable propagation characteristics for wireless systems like CRS and to a fact that in this band, close-by terrestrial transmitting TV stations normally use different frequencies to avoid interference between them and this opens "white space" chance for secondary systems to use.

Based on this fact, in this study we assume PU's signal is a DVB-T signal for interference assessment between CR and PU.

3. Full-Filling Algorithm

As mentioned in Section I, when implementing windowing, only subcarriers nearby PU users create significant amount of interference to CR, for subcarriers having far spectral distance, interference are negligible. Therefore, those subcarriers could be loaded with maximum power level. Based on this observation, we have developed a new sub-optimum algorithm class, namely full-filling, as follow:

The first proposed algorithm, called Max Filling Range (MFR), is:

FOR each frame transmission

- Collect channel information;

$$\max_{\substack{k \in \left[1, \left\lceil \frac{N}{2} \right\rceil\right]^+ \\ l \in \left[\left\lfloor \frac{N}{2} \right\rfloor, N \right] \\ l \geq k}} (l-k)$$

subject to

$$\text{for each } l: \sum_{i=k}^l I_i^l \leq I_{th}$$

$$f(n) = \begin{cases} \frac{1}{2} \left(1 + \cos\left(\frac{2\pi}{aT} \left[t - \frac{T}{2}(1-a) \right] \right) \right), & \frac{T}{2}(1-a) \leq t \leq \frac{T}{2}, 0 \leq a \leq 1 \\ 1, & -\frac{T}{2}(1-a) \leq t \leq \frac{T}{2}(1-a) \\ \frac{1}{2} \left(1 - \cos\left(\frac{2\pi}{aT} \left[t + \frac{T}{2} \right] \right) \right), & -\frac{T}{2} \leq t \leq -\frac{T}{2}(1-a) \end{cases} \quad (10)$$

Set $K \rightarrow k$ & $L \rightarrow l$

$$C_{MFR} = \sum_{i=K}^L \Delta f \log_2 \left(1 + \frac{|h_i^{ss}|^2 P_{\max}}{\sigma^2 + \sum_{l=1}^L J_l^l} \right)$$

IF no MFR solution: solve (1).

The merit of MFR algorithm is its simplicity. The algorithm is very fast to converge, there for it is suitable for power saving and low computation capacity CR's applications.

For a better performance while still keeping low complexity, we propose the second algorithm, called Pre-set Filling Range (PFR), as following:

FOR each frame transmission

STEP 1)

- Collect channel information;
- Set the filling range: K, L ;
- Pre-calculate:

$$C_{FF} = \sum_{i=K}^L \Delta f \log_2 \left(1 + \frac{|h_i^{ss}|^2 P_i}{\sigma^2 + \sum_{l=1}^L J_l^l} \right)$$

STEP 2)

- Construct the sub-optimization problem and solve:

$$C = \sum_{i=1}^N \Delta f \log_2 \left(1 + \frac{|h_i^{ss}|^2 P_i}{\sigma^2 + \sum_{l=1}^L J_l^l} \right) + C_{FF}$$

Subject to

$$\bigvee_{i=1}^L \left(\sum_{i=1}^N I_i^l(d_{il}, P_i) + \sum_{i=K}^M I_i^l(d_{il}, P_{\max}) \right) \leq I_{th} \quad 0 \leq P_i \leq P_{\max} \quad \forall i = \{1, \dots, N\} \setminus \{K, \dots, M\}$$

RETURN Step 1) for next frame transmission

In comparison with optimal solution, PFR takes a different approach. It allocates lower power to nearby sub-carriers in exchange for higher power for far away sub-carriers.

The MFR and PFR algorithms are expressed as in **Figures 3** and **4**:

Normally, in practical frequency assignment, two TV stations are not assigned frequency channels adjacent to each other. The void frequency will serve as gap-band to avoid interference. When the two TV channels are with the same bandwidth, $B_1 = B_2$, then the power allocated to CR's sub-carriers will have the symmetrical form.

4. Numerical Results

4.1. Simulation Parameters

For comparison purpose, a numerical simulator is devel-

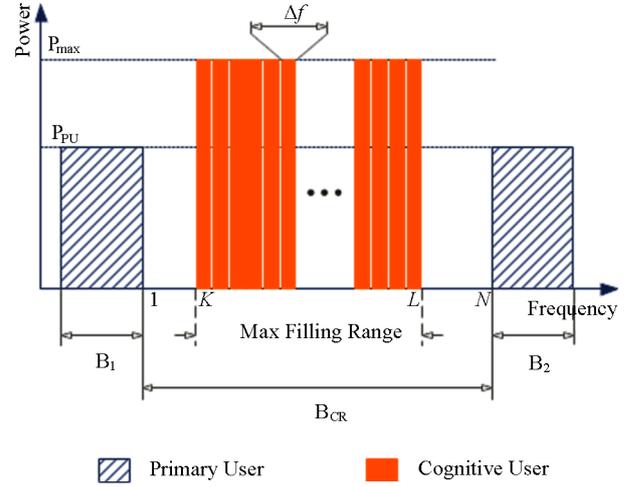


Figure 3. MFR algorithm.

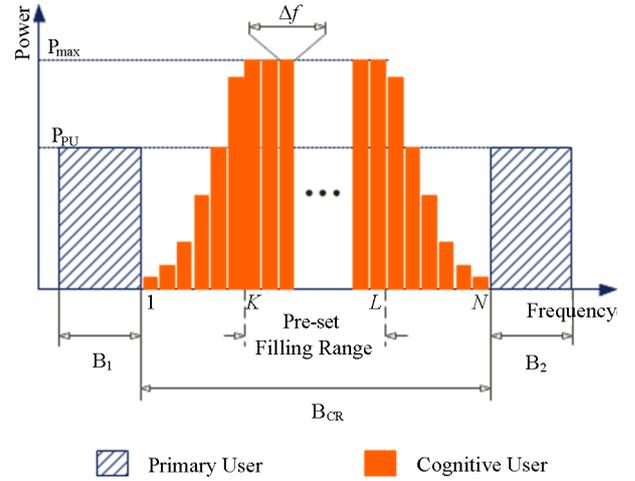


Figure 4. PFR algorithm.

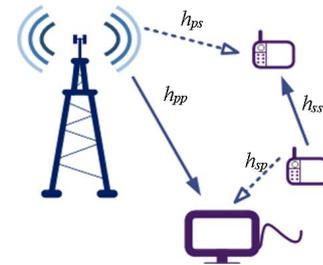


Figure 5. CR's working model.

oped in Matlab. The simulation results are presented in this section; we assume that CR system has 20 sub-carriers to operate in 2 PU systems. We assume the value of T_s to be 4 μ seconds and Δf has been assigned the value of 0.3125 MHz. B_1, B_2 varied as $k\Delta f$. The value of σ^2 is assumed to be 10^{-3} . The value of P_{PU} is assumed to be 10 mW. P_{CR} is calculated at each transmission and is assumed to be no larger than P_{PU} .

The working model of CR is demonstrated in **Figure 5**

with difference channel gain, communication and interference paths.

Regarding to channel, the channel gains h_i^{sp} , h_i^{ps} , and h_i^{ps} are assumed to be Rayleigh fading. Complete channel information is assumed to be known at CR. We consider worst case scenario: the simulator assumes that CR and PU systems are always transmit data. This approach is reflected in the channel setting as following: $h^{ss} = 5$ dB, $h^{ps} = h^{pp} = 10$ dB; $h^{sp} = 15$ dB.

Regarding to PU's signal, in this paper, we assume that the transmitting spectrum mask of PU signal is conformable with emission mask defined in the ITU-R Recommendation SM.1541-4 [11]. The **Figure 6** is spectrum limit mask for 8MHz DVB-T system.

Regarding to CR's signal, based on the previous study in [8] we apply the Tukey window to CR system. The normalized PSD of Tukey window is demonstrated in **Figure 7**. With the introduction of window, the interference from CR to PU depends significantly on the roll-off factor of Tukey window.

The values of break points of this mask are in the **Table 1**:

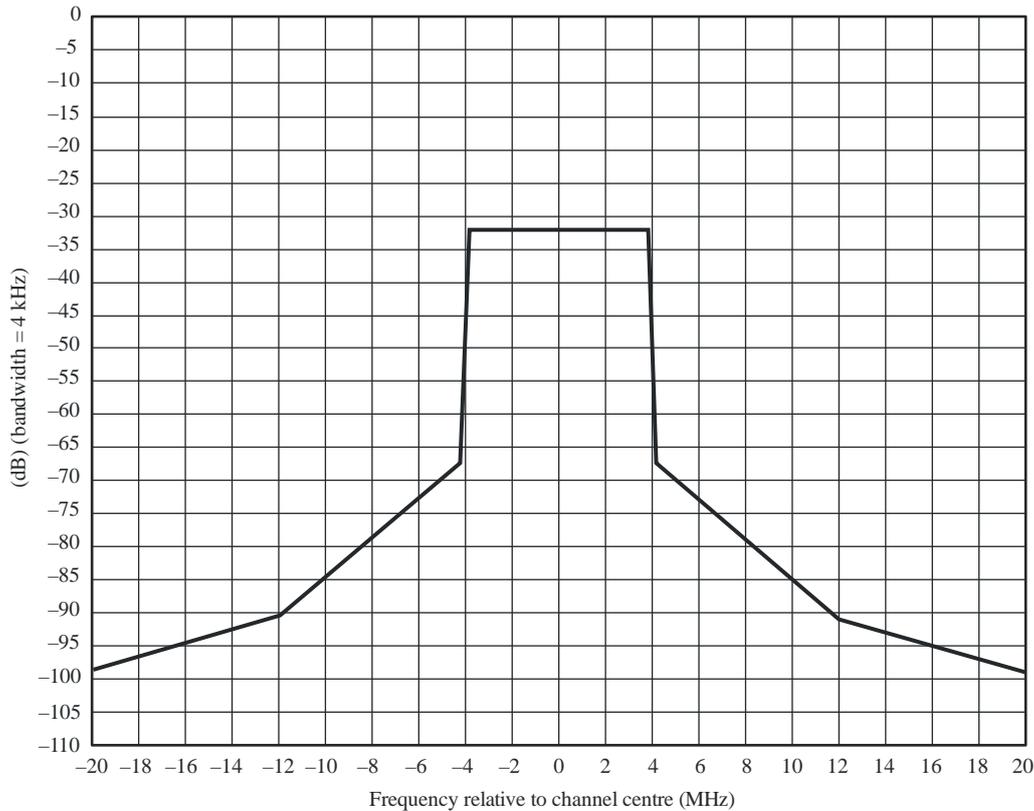
The CR transmission speed will much depend on the tolerable level of PU against interference that is from CR. As in [8], it is reasonable that the achievable throughput

of CR increases in accordance with that tolerable level. The roll-off factor plays a critical role in the maximum achievable throughput of CR system. Throughput increases as the roll-off factor becomes more stringent. Simulation in this paper is conducted with roll-off factor of 0.3.

The introduction on Tukey window has significantly reduced the interference from CR to PU as seen in **Figures 1** and **2**. Do notice at the difference in y-axis scale of transmission power. Without Tukey window, CR can only transmit with much lower power than that of the Tukey window case. Beside its main objective to reduce interference, Tukey window also opens up possibilities for new suboptimal solutions.

4.2. Simulation Analysis

Although PFR algorithm is a sub-optimal solution, it can still yield a comparable result with the optimal solution as in **Figure 8**, except for the very stringent requirement ($I_{th} = 1/2 \sigma^2$). The introduction of windowing, Tukey window in this case, has significantly improved the system performance. As Tukey is very effective in reducing out-of-band interference, CR's sub-carriers far from PU's band will have negligible effect. Therefore, optimal algo-



SM.1541-23

Figure 6. Emission mask of 8 MHz DVB-T transmitter.

Table 1. Breakpoints for 8 MHz DVB-T systems.

Frequency relative to centre of the 8 MHz channel	Relative level in 4 kHz measurement bandwidth (dB)
-20	-99
-12	-91
-4.2	-67.8
-3.81	-32.8
3.81	-32.8
4.2	-67.8
12	-91
20	-99

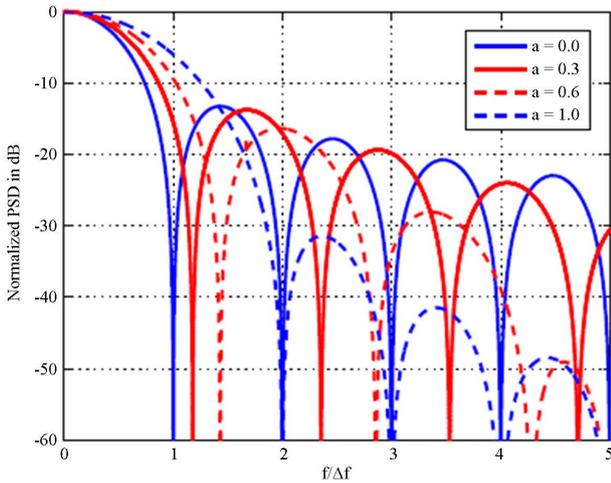


Figure 7. PSD of Tukey window with different roll-off factors.

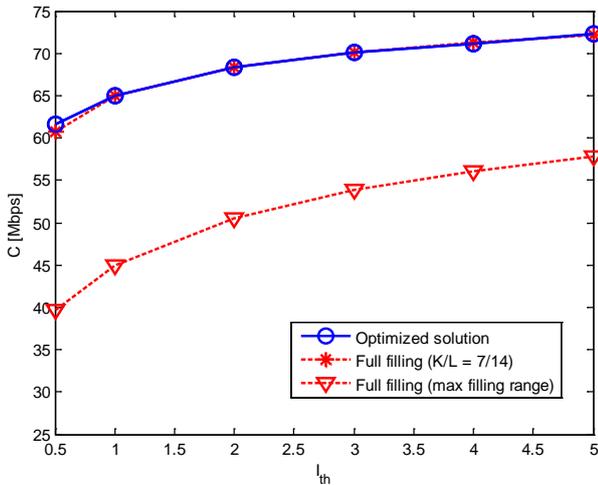


Figure 8. Channel capacity

gorithm only plays its role to sub-carriers closed to PU's band. As a result it is not much different in terms of channel capacity when comparing optimal and sub-optimal PFR algorithms.

For comparison purpose, we provide the optimized power allocation solution in **Figures 9, 10**. With the si-

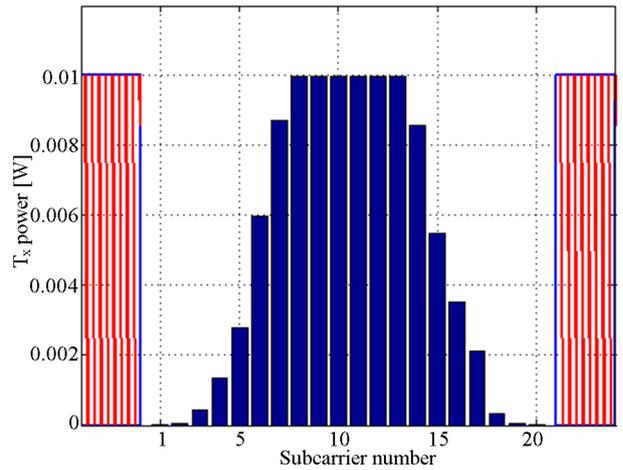


Figure 9. Optimized power allocation ($I_{th} = 1 \sigma^2$).

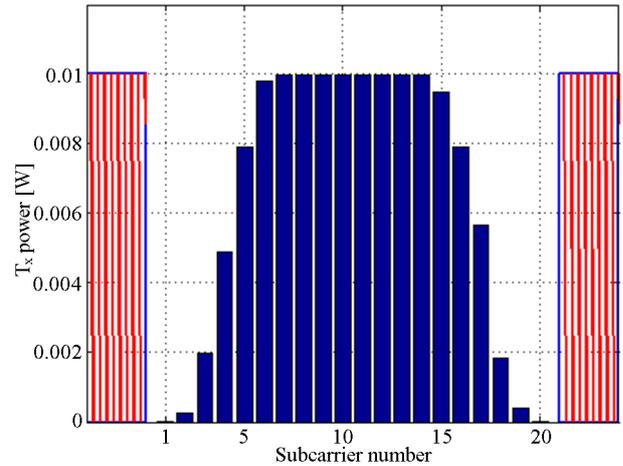


Figure 10. Optimized power allocation ($I_{th} = 5 \sigma^2$).

mulation setting for PFR in this paper, filling range is set to sub-carriers from 7 to 14, the simulation results are given in **Figures 11, 12**. When the requirements are quite stiff, the difference between PFR and optimum solution in power allocation pattern is quite obvious as in **Figures 9, 11**. When the requirement is less stringent, as in **Figures 10, 12**, the power allocation patterns of PFR and the optimum one are quite similar.

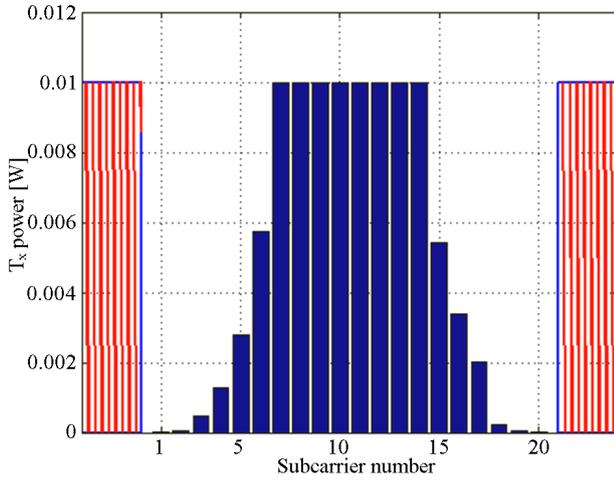


Figure 11. PFR's power allocation ($I_{th} = 1 \sigma^2$).

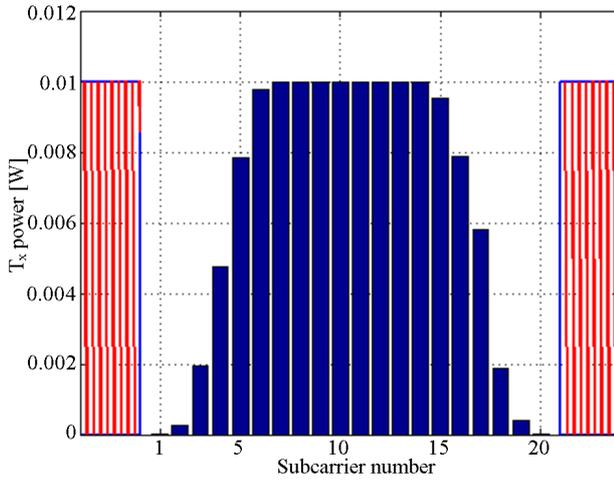


Figure 12. PFR's power allocation ($I_{th} = 5 \sigma^2$).

We now turn to the power allocation of MFR algorithm.

As the sub-carriers adjacent to PU's bands cause the most interference to PU, even in the optimal solution, the power allocated to those sub-carriers are quite low indeed. Therefore, we can shut down those sub-carriers to reduce the computation complexity with small trade-off in the system throughput.

The MFR presents a light weight solution to the problem. It tries to allocate as much power as possible for suitable subcarrier, depending on the channel condition. Figures 13, 14 show different power allocation for a transmission in different conditions.

Depending on the channel at the moment of transmission, the power allocation may not have symmetry form, as clearly shown in Figure 14. However, when we view the average power allocation to all sub-carriers in the long term, we will have a perfect symmetrical form, as in Figure 15.

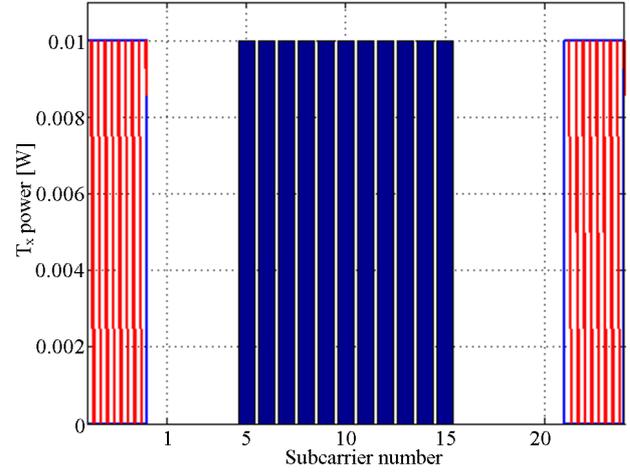


Figure 13. MFR power allocation (one snapshot, $I_{th} = 2 \sigma^2$).

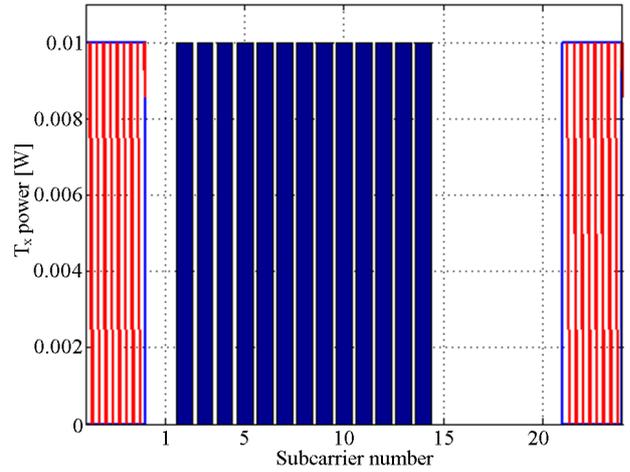


Figure 14. MFR power allocation (one snapshot, $I_{th} = 5 \sigma^2$).

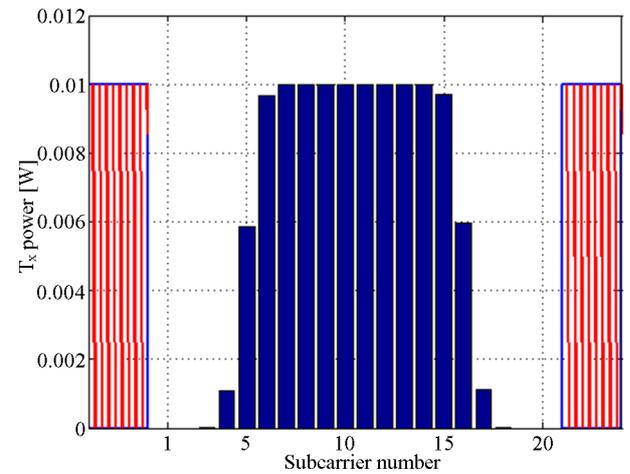


Figure 15. MFR's average power allocation ($I_{th} = 2 \sigma^2$).

5. Conclusions

In this paper, we have introduced a new class of sub-optimal power loading algorithm, called full-filling algo-

rithm, to CR systems. Two sub-optimal solutions are proposed, MFR and PFR. Through extensive numerical analysis, we have shown the effectiveness of the proposed full-filling algorithm.

One of the striking advantages of full-filling algorithm is its low computational complexity. The proposed algorithm has significantly reduced the number of variables in the optimization problem. This directly leads to: simpler implementation, faster convergence rate, saving CR's computation power while achieving practically the same transmission rate.

The MFR algorithm proves to be a very promising proposal in term of low complexity. Beside simplicity, one of its shortcomings is that it does not take into account 2 adjacent sub-carriers to the filling range. In MFR, those two sub-carriers are set to 0. We believe that allocating small power to those 2 sub-carriers can improve the CR's throughput with non-significant additional computation.

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