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Thermal Effect on Vibration of Parallelogram Plate of Bi-Direction Linearly Varying Thickness

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Abstract

In this paper, the effect of thermal gradient on the vibration of parallelogram plate with linearly varying thickness in both direction having clamped boundary conditions on all the four edges is analyzed. Thermal effect on vibration of such plate has been taken as one-dimensional distribution in linear form only. An approximate but quiet convenient frequency equation is derived using Rayleigh-Ritz technique with a two-term deflection function. The frequencies corresponding to the first two modes of vibration of a clamped parallelogram plate have been computed for different values of aspect ratio, thermal gradient, taper constants and skew angle. The results have been presented in tabular forms. The results obtained in this study are reduced to that of unheated parallelogram plates of uniform thickness and have generally been compared with the published one.

Keywords: Parallelogram Plate, Vibration, Thermal Gradient, Linearly Thickness, Both Directions

1. Introduction

Parallelogram plates have quite a good number of applications in modern structures. This type of plates can be found frequently in modern constructions in the form of reinforced slabs or stiffened plates. Such structures are widely used as floor in bridges, ship hulls, buildings etc. these plates are also used in the construction of wings, tails and fins of rockets and missiles.

In the modern time, people have started taking lot of interest in effect of temperature on solids, as it has a lot of role in space technology, high-speed atmospheric flights and in nuclear energy applications. In the modern technology, mechanical parts of different machines have to operate high temperature, which effect efficiency. The reason for it is that during heating up period of structures exposed to high intensity heat fluxes, the material properties under go significant vibrations.

Notable contributions [1-10] are available on vibration of skew plate but none of them considered the thermal effect. It is well known [11] that in the presence of a constant thermal gradient, the elastic coefficients of homogeneous material become functions of the space variable. Tomar and Gupta [12-13] have considered the effect of thermal gradient of the frequency of an orthotropic plate of variable thickness. Bhatnagar and Gupta [14-15] have studied the effect of thermal gradient on vibration of visco-elastic plate of variable thickness. Singh and Saxena [16] have studied the transverse vibration of skew plates with variable thickness. Gupta and Khanna [17] have studied the vibration of visco-elastic rectangular plate with linearly thickness variation in both directions. Gupta, Kumar and Gupta [18] have analyzed on vibration of visco-elastic parallelogram plate with parabolic thickness variation. Free vibration of super elliptical plates with constant and variable thickness by Ritz method has been discussed by Bambill, Maize and Rossi [19]. Effect of thermal gradient on vibration of non-homogeneous visco-elastic elliptical plate of variable thickness has been analyzed by Gupta and Kumar [20]. Li [21] discussed the vibration analysis of rectangular plate with general elastic boundary supports. Sakiyama, Haung, Matuda and Morita [22] solved the problem of free vibration of orthotropic square plate with square hole.

The aim of the present study is to find the effect of linear thermal gradient on vibration of a clamped parallelogram plate with linearly varying thickness in both directions. All the edges are taken as clamped. The Rayleigh-Ritz technique has been used to determine the frequency equation of the plate. The frequency to the first two modes of vibration is obtained for a clamped parallelogram plate for various values of aspect ratio (a/b), thermal gradient (α), taper constants (β_1, β_2) and skew angle (θ).

2. Analysis and Equation of Motion

A parallelogram plate R (axb) with skew angle θ is shown in **Figure 1**. The skew plate is assumed to be non-uniform, thin and isotropic. The skew co-ordinate are related as

$$\xi = x - y\tan\theta \text{ and } \eta = y\sec\theta \quad (1)$$

The boundaries of the plate in oblique co-ordinate are

$$\xi = 0,\ \xi = a \text{ and } \eta = 0,\ \eta = b \quad (2)$$

For free vibration of the parallelogram plate, the displacement is assumed to be of the form

$$w(\xi,\eta,t) = W(\xi,\eta)\sin\omega t \quad (3)$$

where $W(\xi,\eta)$ is the maximum displacement at time t and ω is the angular frequency. The maximum kinetic energy, T, and the strain energy, V in the plate when it is executing transverse vibration mode shape $W(\xi,\eta)$ are [1]

$$T = \frac{1}{2}\rho\omega^2\cos\theta\iint hW^2 d\xi d\eta \quad (4)$$

and

$$\begin{aligned} V = \frac{1}{2\cos^3\theta}\iint D\Big[& W_{,\xi\xi^2} - 4\sin\theta W_{,\xi\xi}W_{,\xi\eta} \\ & + 2\left(\sin^2\theta + v\cos^2\theta\right)W_{,\xi\xi}W_{,\eta\eta} \\ & + 2\left(1+\sin^2\theta - v\cos^2\theta\right)W_{,\xi\eta^2} \\ & - 4\sin\theta W_{,\xi\eta}W_{,\eta\eta} + W_{,\eta\eta^2}\Big]d\xi d\eta \end{aligned} \quad (5)$$

A comma followed by a suffix denotes partial differential with respect to that variable. Here D is the flexural rigidity.

Assume that plate is subjected to a study one dimensional temperature along the length *i.e.* in ξ-direction as

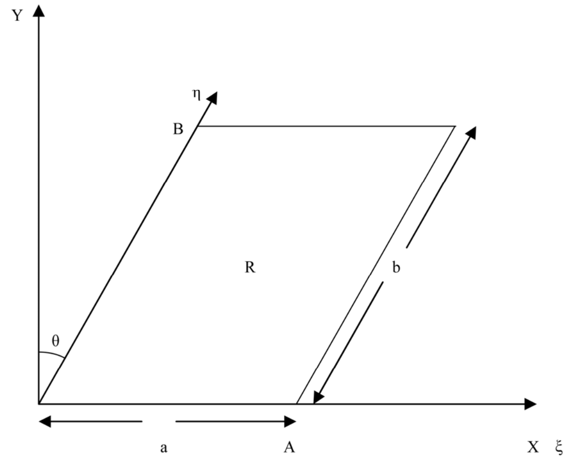

Figure 1. Plate in x-y plane.

$$\tau = \tau_0\left(1 - \frac{\xi}{a}\right) \quad (6)$$

where τ is the temperature excess above the reference temperature at any point at a distance $\frac{\xi}{a}$ and τ_0 at $\xi = $ a.

The temperature dependence of the modules of elasticity is given by

$$E(\tau) = E_0(1 - \gamma\tau) \quad (7)$$

where E_0 is the Young Modules at $\tau = 0$.

Using (6) and (7) one obtains

$$E(\xi) = E_0\left(1 - \alpha\left(1 - \frac{\xi}{a}\right)\right) \quad (8)$$

where $\alpha = \gamma\tau_0$ ($0 \le \alpha < 1$), is a parameter known as temperature gradient.

The thickness variation of the parallelogram plate is assumed to be linear in both directions

$$h = h_0\left(1 + \beta_1\left(\frac{\xi}{a}\right)\right)\left(1 + \beta_2\left(\frac{\eta}{b}\right)\right) \quad (9)$$

where β_1 and β_2 are taper constants in ξ-direction and η-direction respectively. And $h_0 = h$ when $\xi, \eta, = 0$.

Using(8) and (9) in Equations (4) and (5) one gets

$$T = \frac{1}{2}h_0\rho\omega^2\int_0^b\int_0^a\left(1 + \beta_1\left(\frac{\xi}{a}\right)\right)\left(1 + \beta_2\left(\frac{\eta}{b}\right)\right)W^2 d\xi d\eta \quad (10)$$

and

$$\begin{aligned} V = \frac{E_0h_0^3}{24\left(1-v^2\right)\cos^4\theta}\int_0^b\int_0^a & \left(1 - \alpha\left(1 - \frac{\xi}{a}\right)\right)\left(1 + \beta_1\left(\frac{\xi}{a}\right)\right)^3\left(1 + \beta_2\left(\frac{\eta}{b}\right)\right)^3 \\ & \times\Big[W_{,\xi\xi^2} - 4\left(\frac{a}{b}\right)\sin\theta W_{,\xi\xi}W_{,\xi\eta} + 2\left(\frac{a}{b}\right)^2\left(\sin^2\theta + \cos^2\theta\right)W_{,\xi\xi}W_{,\eta\eta} \\ & + 2\left(\frac{a}{b}\right)^2\left(1 + \sin^2\theta - v\cos^2\theta\right)W_{,\xi\eta^2} - 4\left(\frac{a}{b}\right)^3\sin\theta W_{,\xi\eta}W_{,\eta\eta} + \left(\frac{a}{b}\right)^4 W_{,\eta\eta^2}\Big]d\xi d\eta \end{aligned} \quad (11)$$

3. Solution and Frequency Equation

In using the Rayleigh-Ritz technique, one requires that the maximum strain energy must be equal to the maximum kinetic energy. It is, therefore, necessary for the problem under consideration that

$$\delta\left(V-T\right)=0 \tag{12}$$

for arbitrary variations of W satisfying relevant geometric boundary conditions.

For a parallelogram plate clamped along all the four edges the boundary conditions are $W = W_{,\xi} = 0$ at $\xi = 0$, a and

$$W=W_{,\eta}=0 \text{ at } \eta=0,b \tag{13}$$

and corresponding two term deflection function is taken as [1].

$$W\left(\xi,\eta\right)=\left(\frac{\xi^2}{a^2}\right)\left(\frac{\eta^2}{b^2}\right)\left(1-\frac{\xi}{a}\right)^2\left(1-\frac{\eta}{b}\right)^2 \times\left[A_1+A_2\left(\frac{\xi}{a}\right)\left(\frac{\eta}{b}\right)\left(1-\frac{\xi}{a}\right)\left(1-\frac{\eta}{b}\right)\right]. \tag{14}$$

Now Equation (12) becomes after using Equation (14)

$$\delta\left(V_1-\lambda^2T_1\right)=0 \tag{15}$$

where

$$V_1=\frac{1}{\cos^4\theta}\int_0^b\int_0^a\left(1-\alpha\left(1-\frac{\xi}{a}\right)\right)\left(1+\beta_1\frac{\xi}{a}\right)^3\left(1+\beta_2\frac{\eta}{b}\right)^3 \times\Bigg[W_{,\xi\xi^2}-4\left(\frac{a}{b}\right)\sin\theta W_{,\xi\xi}W_{,\xi\eta} +2\left(\frac{a}{b}\right)^2\left(\sin^2\theta+\cos^2\theta\right)W_{,\xi\xi}W_{,\eta\eta} +2\left(\frac{a}{b}\right)^2\left(1+\sin^2\theta-\nu\cos^2\theta\right)W_{,\xi\eta^2} -4\left(\frac{a}{b}\right)^3\sin\theta W_{,\xi\eta}W_{,\eta\eta}+\left(\frac{a}{b}\right)^4W_{,\eta\eta^2}\Bigg]d\xi d\eta$$

$$T_1=\int_0^b\int_0^a\left(1+\beta_1\left(\frac{\xi}{a}\right)\right)\left(1+\beta_2\left(\frac{\eta}{b}\right)\right)W^2d\xi d\eta$$

and

$$\lambda^2=\frac{12a^4\omega^2\rho\left(1-\nu^2\right)}{E_0h_0^2} \text{ is a frequency parameter.}$$

Equation (15) involves the unknown A_1 and A_2 arising due to the substitution of W (ξ, η) from Equation (14). These unknowns are to be determined from Equation (15) for which

$$\frac{\partial}{\partial A_n}\left(V_1-\lambda^2T_1\right)=0,\ n=1,2 \tag{16}$$

The above equation simplifies to

$$b_{n1}A_1+b_{n2}A_2=0,\ n=1,2 \tag{17}$$

where b_{n1}, b_{n2} (n = 1,2) involve parametric constants and the frequency parameter.

For a non-trivial solution the determinant of the coefficients of Equation (17) must be zero.

Therefore one gets the frequency equation as

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22}\end{vmatrix}=0. \tag{18}$$

From Equation (18), one can obtained the quadratic equation in λ^2 from which two values of λ^2 can found.

4. Result and Discussion

The frequency Equation (18) is quadratic in λ^2 from which two roots can be determined. The frequency parameter λ corresponding to the first two modes of vibration of clamped parallelogram plate have been computed for various values of temperature gradient (α), aspect ratio (a/b), taper constants (β_1), (β_2) and skew angle (θ). These results are summarized in **Tables 1-5**.

For numerical computation the value of Poisson's ratio ν is taken 0.3.

Table 1 contains the value of frequency parameter of a clamped parallelogram plate for different values of thermal constant (α) and for fixed aspect ratio (a/b) =1.0 for the first two modes of vibration for values of taper constants (β_1 = 0.0 and β_2 = 0.0, β_1 = 0.0 and β_2 = 0.6, β_1 = 0.4 and β_2 = 0.6) and two values of skew angle (θ). It can be seen from the table that as thermal constant increase, frequency parameter decreases in all the cases. Also effect of thermal constant is more for $\theta = 45^\circ$ in comparison to $\theta = 30^\circ$. **Table 2** comprises the value of frequency parameter of a clamped parallelogram plate for different values of aspect ratio (a/b) for the first two modes of vibration for fixed value of taper constant (β_1 = 0.4and β_2 = 0.6) and fixed thermal constant (α = 0.2) for two values of skew angle (θ). It is noted from the table that as aspect ratio is increasing, frequency parameter increasing in all the cases and effect of aspect ratio is more for $\theta=45^\circ$ in comparison to $\theta = 30^\circ$.

Table 3 gives the values of frequency parameter of a clamped parallelogram plate for various values of taper constant (β_1) for the first two modes of vibration for fixed value of taper constant (β_2 = 0.0, 0.6), thermal gradient (α = 0.4), aspect ratio (a/b = 1.0) and for two values of skew angle (θ). It is observed from the table that as taper constant is increasing, frequency parameter is also increasing.

Table 4 shows the values of frequency parameter of a clamped parallelogram plate for various values of taper

Table 1. Frequency parameter λ of a clamped parallelogram plate for a/b = 1.0.

α	$\theta = 30°$					
	$\beta_1 = \beta_2 = 0.0$		$\beta_1 = 0.0, \beta_2 = 0.6$		$\beta_1 = 0.4, \beta_2 = 0.6$	
	I mode	II mode	I mode	II mode	I mode	II mode
0.0	61.4283	236.5564	67.2606	258.9453	81.1022	318.4257
0.2	58.7339	226.1332	63.7993	245.2736	77.4094	304.5002
0.4	55.9060	215.2066	59.9905	230.7934	73.5239	289.9084
0.6	52.9215	203.6964	56.0027	215.3417	69.4106	274.5449
0.8	49.7499	191.4977	51.7080	198.6919	65.0218	258.2730

$\theta = 45°$					
$\beta_1 = \beta_2 = 0.0$		$\beta_1 = 0.0, \beta_2 = 0.6$		$\beta_1 = 0.4, \beta_2 = 0.6$	
I mode	II mode	I mode	II mode	I mode	II mode
79.5005	302.5225	105.7948	401.7948	127.3479	499.3421
75.4208	286.9980	100.1417	380.3892	121.5072	477.7926
71.1074	270.5844	94.2215	357.7051	115.3581	455.2276
66.5149	253.1086	87.9034	333.4815	108.8434	431.4886
61.5808	234.3329	81.0943	307.3547	101.8854	406.3723

Table 2. Frequency parameter λ of a clamped parallelogram plate for $\beta_1 = 0.4$, $\beta_2 = 0.6$, $\alpha = 0.2$.

a/b	$\theta = 30°$		$\theta = 45°$	
	I mode	II mode	I mode	II mode
0.5	51.8014	206.8539	121.7636	316.5569
1.0	77.4094	304.5002	121.5072	477.7926
1.5	129.6335	514.0130	201.7746	804.2516
2.0	207.3843	827.5726	319.2430	1283.8269
2.5	309.2726	1237.4373	472.4142	1906.5696

Table 3. Frequency parameter λ of a clamped parallelogram plate for a/b = 1.

β_1	$\theta = 30°$				$\theta = 45°$			
	$\alpha = 0.4, \beta_2 = 0.0$		$\alpha = 0.4, \beta_2 = 0.6$		$\alpha = 0.4, \beta_2 = 0.0$		$\alpha = 0.4, \beta_2 = 0.6$	
	I mode	II mode	I mode	II mode	I mode	II mode	I mode	II mode
0.0	45.2395	174.2904	59.9905	230.7934	71.1074	270.5843	94.2216	357.7051
0.2	50.4341	194.2100	66.6139	260.3359	79.2771	301.4143	104.5958	407.2275
0.4	55.9060	215.2067	73.5239	289.9084	87.8845	333.8754	115.3581	455.2276
0.6	61.5759	237.0058	80.6690	319.7503	96.8045	367.5542	126.4684	502.6380
0.8	67.3896	259.413	88.0023	349.9568	105.9515	402.1589	137.8679	549.9366

Table 4. Frequency parameter λ of a clamped parallelogram plate for a/b = 1.

β_2	$\theta = 30°$				$\theta = 45°$			
	$\alpha = 0.4, \beta_1 = 0.0$		$\alpha = 0.4, \beta_1 = 0.6$		$\alpha = 0.4, \beta_1 = 0.0$		$\alpha = 0.4, \beta_1 = 0.6$	
	I mode	II mode	I mode	II mode	I mode	II mode	I mode	II mode
0.0	45.2395	174.5793	61.5759	237.0058	71.1074	270.5005	96.8045	367.5542
0.2	49.8864	192.1109	67.6357	266.7037	78.3851	298.0087	106.2491	416.8518
0.4	54.8293	211.0362	74.0234	293.7256	86.1307	327.2656	116.1637	460.8392
0.6	59.9405	230.7934	80.6690	319.7503	94.2216	357.7051	126.4684	502.6380
0.8	65.3156	251.1870	87.5153	345.6646	102.5688	389.1129	137.0843	543.8896

constant (β_2) for the first two modes of vibration for fixed value of taper constant (β_1 = 0.0, 0.6), thermal gradient (α = 0.4) ,aspect ratio (a/b = 1.0) and for two values of skew angle (θ). It is observed from the table that as taper constant is increasing, frequency parameter is also increasing.

Table 5 contains the values of frequency parameter of a clamped parallelogram plate for various values of skew angle (θ) for the first two modes of vibration for fixed value of taper constant (β_1 = 0.0, β_2 = 0.0 and β_1 = 0.4 β_2 = 0.6), thermal gradient (α = 0.2) and aspect ratio (a/b = 1.0). It is observed from the table that as skew angle is increasing, frequency parameter is also increasing.

5. Conclusions

The results for a uniform isotropic clamped rectangular plate are compared with the results published by the authors [1] and found to be in close agreement.

After comparing, the authors conclude that as the skew angle increases, the frequency parameter increases. Further, it is interesting to note that effect of taper in ξ -direction is small in comparisons of η–direction. Therefore, engineers are provided with a method to develop plates in a manner so that they can fulfill the requirements.

Table 5. Frequency parameter λ of a clamped parallelogram plate for a/b = 1.

θ	$\alpha = 0.2, \beta_1 = \beta_2 = 0.0$		$\alpha = 0.2, \beta_1 = 0.4\ \beta_2 = 0.6$	
	I mode	II mode	I mode	II mode
0°	34.1524	133.6542	55.4962	212.4850
30°	47.9838	184.8624	77.4094	304.5002
45°	75.4208	286.9980	121.5072	477.7926
60°	157.4119	593.0757	253.5286	993.6003

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