

One Dimensional Modeling of the Shape Memory Effect

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ABSTRACT

This paper aims to build a constitutive model intended to describe the thermomechanical behavior of shape memory alloys. This behavior presents many facets, among them we have considered the simple way of shape memory, which is one of most important properties of shape memory alloys. Because of numerous stages of this effect, the subject was divided into three independent parts. For each part, we built the corresponding thermodynamic potential and we deduced the constitutive equations. To make this model workable, we have developed an algorithm. The simulation was performed using the NiTi as shape memory alloy.

Keywords: Shape; Strain; Detwinned Martensite; Region

1. Introduction

The known behavior of conventional materials has allowed their use in many applications but the discovery of new properties, coming from a singular behavior of materials known as shape memory alloys, opened a way for other applications from medical to aerospace.

This unusual behavior has attracted a significant attention of scientists and researchers. Therefore, various models were proposed.

These models are based on thermodynamics laws and frameworks theories as generalized standard materials. Halphen and Nguyen [1] used by Lexcellent and Licht [2], Edelen's formalism [3] used by Tanaka and Nagaki [4].

These models can be classified as follows:

1) Macroscopic models: Built of the thermomechanical behavior, they are generally simpler in formulation and geared more towards engineering applications. Thus, the detailed physics of phase transformation are usually not rigorously addressed.

2) Micromechanical models are based on the micro-mechanics of a single crystal. Chu and James [5]; James *et al.*, [6]; Lexcellent *et al.*, [7]; Govindjee and Hall [8]; Berveiller *et al.* [9]; Patoor *et al.* [10]. Generally, this class of model was derived from more fundamental thermodynamic principles and the shape strains of different martensite variants are included.

In this paper, we focus on the effect of simple way

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shape memory which is the first property discovered. The adopted approach is based on the sharing of the study on three following steps:

- 1) Orientation of twinned Martensite;
- 2) Heating for austenite;
- 3) Cooling for detwinned Martensite.

2. Methods

2.1. Presentation of the Subject (Figure 1)

The thermomechanical cycle to the memory effect of a simple shape is defined the following:

1) Applying a mechanical load under Temperature T_1 lower than M_f^0 (Temperature of transformation start of Martensite): The material is deformed first elastically, followed by an important deformation due to twinned Martensite orientation. When the load is cancelled the deformation is not fully recovered, only elastic part is recovered.

2) Heating to a temperature above A_f^0 (Temperature of transformation finish of Austenite): When the temperature reaches A_s^0 (Temperature of transformation start of Austenite) the deformation begins to recover.

3) Cooling to T_1 : Austenite begins to transform to Martensite and finally the material is at the origin of the cycle.

2.2. Definition of Regions (Figure 2)

The study will concern three regions which are planes (ϵ ,

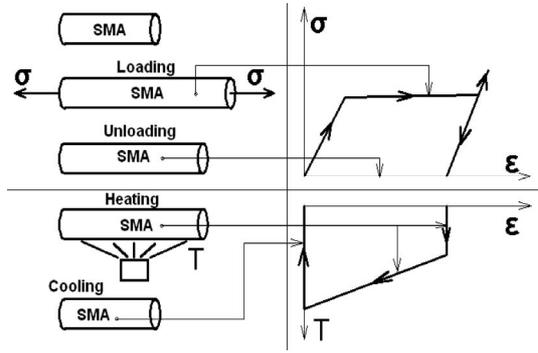


Figure 1. Thermomechanical cycle of way memory effect.

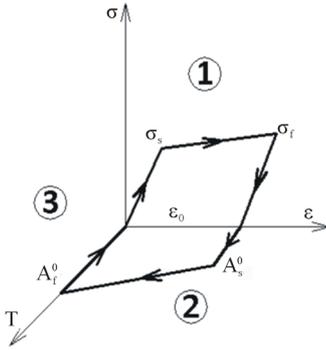


Figure 2. Regions of study.

σ); (ε, T) and axe T :

- 1) Plane (ε, σ) : Region of mechanical loading;
 - 2) Plane (ε, T) : Region of thermal loading (heating);
 - 3) Axe T : Region of cooling.
- σ_s : Stress of orientation start of twinned Martensite;
 σ_f : Stress of orientation finish of twinned Martensite;
 ε_0 : Maximum deformation of orientation.

2.3. Constitutive Equations (Figure 3)

In each case, we consider an elementary volume

$$V = V_p + V_f,$$

where V_p is volume of parent phase and V_f is volume of incipient phase.

$$f = \frac{V_f}{V}$$

is fraction of incipient phase.

2.3.1. Region 1

Parent phase is twinned Martensite and incipient phase is detwinned Martensite

Free energy of Gibbs:

$$G_1(\sigma, T, f) = -\frac{\sigma^2}{2E_M} - \sigma \cdot f \cdot \varepsilon_0 + f \cdot B(T - M_s^0) + C \cdot f(1-f) \quad (1)$$

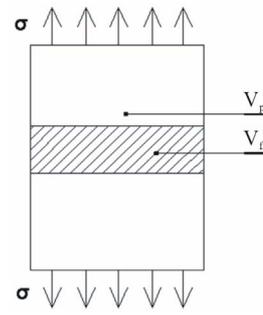


Figure 3. Representative elementary volume.

E_M : Young modulus of Martensite,
 B, C : Coefficients to be determined by tests,

$\frac{\sigma^2}{2E_M}$: Elastic energy,

$\sigma \cdot f \cdot \varepsilon_0$: Energy of deformation due to transformation,

$f \cdot B(T - M_s^0)$: Free energy of phase change,

$C \cdot f(1-f)$: Energy of interaction between two phases.

Assuming that the dissipation occurs during processing Clausius inequality can be written as following:

$$-\frac{\partial G_1}{\partial f} \frac{df}{dt} \geq 0 \quad (2)$$

Let us write the term:

$$-\frac{\partial G_1}{\partial f} = F^{th}$$

which is the driving force.

The dissipative force can be written:

$$F^{di} = a_1 f + a_1 \quad (3)$$

The transformation occurs when this condition is satisfied:

$$F^{th} = F^{di} \quad (4)$$

It will result:

$$\sigma \cdot \varepsilon_0 - B \cdot (T - M_s^0) - C \cdot (2f - 1) - a_1 f - a_1 = 0 \quad (5)$$

Let us use the function:

$$\varphi_1(\sigma, T, f) = \sigma \cdot \varepsilon_0 - B \cdot (T - M_s^0) - C \cdot (2f - 1) - a_1 f - a_1 \quad (6)$$

To write the evolution of the fraction we use the following expression:

$$\frac{d\varphi_1(\sigma, T, f)}{dt} = \frac{\partial \varphi_1}{\partial \sigma} \frac{d\sigma}{dt} + \frac{\partial \varphi_1}{\partial T} \frac{dT}{dt} + \frac{\partial \varphi_1}{\partial f} \frac{df}{dt} = 0 \quad (7)$$

$$\varepsilon_0 \dot{\sigma} - (2C + a_1) \dot{f} = 0 \quad (8)$$

$$\dot{f} = \frac{\dot{\sigma} \cdot \varepsilon_0}{2C + a_1} \quad (\dot{f} > 0) \quad (\dot{\sigma} > 0) \quad (9)$$

The coefficients a_1 and C can be determined using limit values of σ, T, f when:

$$\varphi_1(\sigma, T, f) = 0; \quad \sigma = \sigma_s; \quad T = T_1; \quad f = 0 \quad (10)$$

$$\varphi_1(\sigma, T, f) = 0; \quad \sigma = \sigma_f; \quad T = T_1; \quad f = 1 \quad (11)$$

It is denoted that parameter B is identified using pseudoelasticity test.

2.3.2. Region 2

Parent phase is detwinned Martensite and incipient phase is Austenite

Following the same approach:

$$G_2(\sigma, T, f) = -\frac{\sigma^2}{2} \left(\frac{1-f}{E_A} + \frac{f}{E_M} \right) - \sigma \cdot f \cdot \varepsilon_0 + f \cdot B \cdot (T - M_s^0) + H \cdot f \cdot (f-1) \quad (12)$$

$$\varphi_2(\sigma, T, f) = \frac{\sigma^2}{2} \left(\frac{1}{E_M} - \frac{1}{E_A} \right) + \sigma \cdot \varepsilon_0 - B \cdot (T - M_s^0) - H \cdot (2f-1) + a_2 f + b_2 \quad (13)$$

Parameter H can also be determined by pseudoelasticity test; the coefficients a_2 and b_2 are defined using limit values of σ, T, f when:

$$\varphi_2(\sigma, T, f) = 0; \quad \sigma = 0; \quad T = A_s^0; \quad f = 1 \quad (14)$$

$$\varphi_2(\sigma, T, f) = 0; \quad \sigma = 0; \quad T = A_f^0; \quad f = 0 \quad (15)$$

The evolution of the fraction of Martensite can be expressed by:

$$\dot{f} = -\frac{B \cdot \dot{T}}{2H - a_2} \quad (\dot{f} < 0) \quad (\dot{T} > 0) \quad (16)$$

2.3.3. Region 3

Parent phase is Austenite and incipient phase is twinned Martensite

$$G_3(\sigma, T, f) = f \cdot B \cdot (T - M_s^0) + H \cdot f \cdot (f-1) \quad (17)$$

$$\varphi_3(\sigma, T, f) = -B \cdot (T - M_s^0) - H \cdot (2f-1) - a_3 f - b_3 \quad (18)$$

a_3, b_3 : Parameters to be defined by test using following conditions:

$$\varphi_3(\sigma, T, f) = 0; \quad T = M_s^0; \quad f = 0 \quad (19)$$

$$\varphi_3(\sigma, T, f) = 0; \quad T = M_f^0; \quad f = 1 \quad (20)$$

$$\dot{f} = -\frac{B \cdot \dot{T}}{2H + a_3} \quad (\dot{f} > 0) \quad (\dot{T} < 0) \quad (21)$$

2.4. Application of the Model

Using K.L. Ng. and al work [11], where a tensile test was performed on NiTi and the following parameters were determined:

$$B = 0.248 \text{ MPa} \cdot \text{K}^{-1}; \quad C = 4.3232 \text{ MPa};$$

$$a_1 = 10.678 \text{ MPa}; \quad a_2 = -8.130 \text{ MPa};$$

$$b_2 = 12.5 \text{ MPa}; \quad H = 0.152 \text{ MPa};$$

$$a_3 = 9.37 \text{ MPa}; \quad b_3 = -0.152 \text{ MPa};$$

$$E_M = 18442 \text{ MPa}; \quad E_A = 25775 \text{ MPa};$$

$$M_s^0 = 279 \text{ K}; \quad M_f^0 = 240 \text{ K};$$

$$\sigma_s = 70 \text{ MPa}; \quad \sigma_f = 120 \text{ MPa};$$

$$A_s^0 = 296 \text{ K}; \quad A_f^0 = 330 \text{ K}; \quad E_0 = 0.04064$$

Temperature of orientation loading $T_1 = 230 \text{ K}$; mechanical load $\sigma_1 = 300 \text{ MPa}$;

3. Results (Figures 4-6)

Discussions

The results obtained coincide well with the experimental

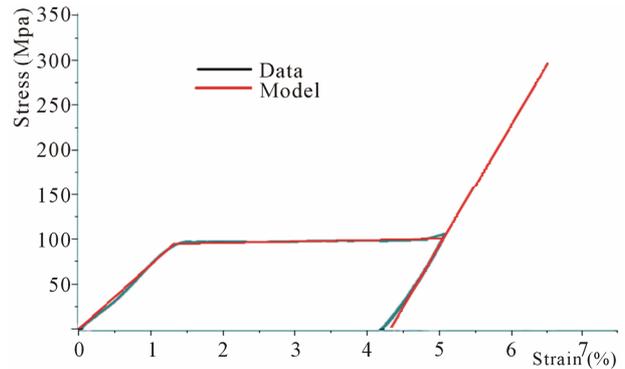


Figure 4. Results of tensile test at $T = 23^\circ\text{C}$ (Test of orientation) performed to validate the model.

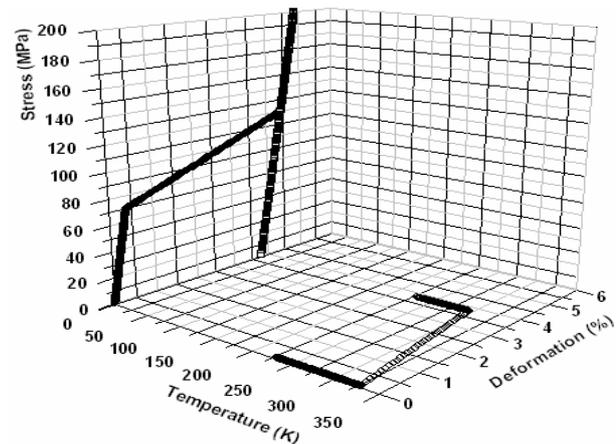


Figure 5. Representation of the shape memory effect in 3D space.

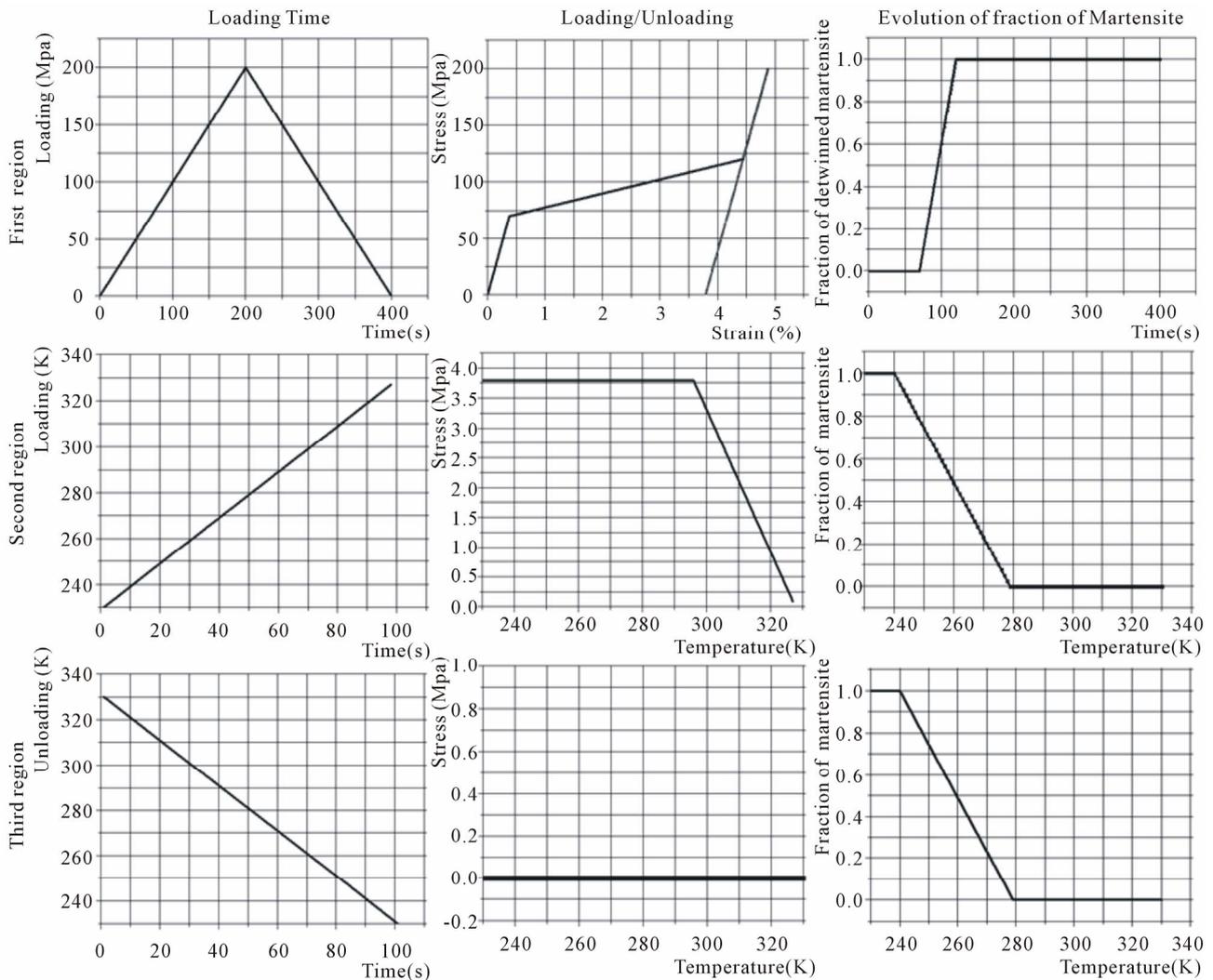


Figure 6. Results of simulation.

data in the case of orientation test, in case of heating to make martensite changing into austenite which occurs with recovering previous deformation and finally changing of martensite into austenite during the cooling. It seems that our results are consistent with practical data of the selected material (NiTi).

In each case we see clearly that deformation is function of transformation of martensite. The cycle beginning from lower temperature reaches the final temperature with recovering the deformation caused by applied mechanical load.

4. Conclusion

This constitutive model presented in this paper was built using a simple formalism. We have divided the work into three parts, and each one has its proper particularities after simulating the model. The results appear in good agreement with experimental data. This model can be used in

engineering fields.

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