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Modeling the Distribution of Marketable Timber Products of Private Teak (*Tectona grandis* L.f.) Plantations

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Management of marketable products of private plantations will not be sustainable without class girth being identifiable readily. Modeling marketable products is a key to obtain good fitness between observed and theoretical girth distribution. We determine the best parameter recovery method with the Weibull function for two sylvicultural regimes (coppice and high forest). Data on stand variables were collected from 1101 sample plots. The three Weibull function parameters were estimated with three parameters recovery methods: the maximum likelihood method, the method of moments and the method of percentiles. Stepwise regression and the simultaneously re-estimated parameter using the Seemingly Unrelated Regression Estimation were applied to model each parameter. The results indicated that the three methods successfully predicted girth size distributions within the sample stands. The method of moments was the best one with lowest values of Reynolds error index and Kolmogorov-Smirnov statistic however the sylvicultural regimes. The Weibull parameter distribution model developed for each of the two sylvicultural regimes was quite reliable.

Keywords: Weibull; Parameter Recovery Method; Reynolds Index; Sylvicultural Regime; Poles; Logs

Introduction

The multipurpose management of small woodlots by smallholder forestry has been gaining more importance (Harrison et al., 2002). The growing of the demand for forest products (Scheer, 2004) explained the importance of their management mainly for the smallholder farmer to generate substantial income (Aoudji et al., 2012). Teak (*Tectona grandis* L.f.) is the most important reforestation and commercial plant species in coastal West Africa due to its fast growing potential (Niskane, 1998), good-quality timber (Louppe, 2008). Reforestation with this specie has increased the above ground biomass and carbon stock at 10-year-old about 45% higher than a nearby degraded secondary forest (Odiwe et al., 2012). In Benin, the success of state-owned plantations has encouraged farmers to invest in teak sylviculture, establishing plantations on small plots ranging from 0.05 ha to 28.10 ha (Atindogbé, 2012). Various problems constrain both traders and smallholder farmers (Aoudji et al., 2012). These include the lack of market information, high transaction costs, difficulties for traders to get timber supplies (Anyonge and Roshetko, 2003; Nawir et al., 2007), and the low return to smallholder farmers (Maldonado & Louppe, 1999; Nawir et al., 2007). According to the above problems, efforts were needed to have information on the different classes of the merchants products on stand before harvesting. Forest owners and managers have no reliable tools to provide them with a comprehensive scheme of resources available and monitoring, harvesting and sales operations. Therefore, one challenge is to determine the minimum level of information required to characterize harvests (Lafond et al., 2012).

Stand tables for total or marketable volume are based on the distribution of tree diameters using traditionally probability density functions (PDFs) (Parresol et al., 2010). Many functions have been suggested for establishing tree diameters size class distribution (e.g., normal, exponential, beta, Johnson's S_B, Gamma, Weibull, logit-logistic). However, the Weibull function appears the most often used (Little, 1983; Rennolls et al., 1985; Rondeux et al., 1992; Lindsay et al., 1996; Liu et al., 2004; Newton & Amponsah, 2005; Lei, 2008) owing to its flexibility (Hafley & Schreuder, 1977; Kilkki et al., 1989) and the best description of diameter structure. This function can also model many types of failure rate behaviors when appropriate parameters are included.

Many techniques for estimating Weibull function parameters have been developed: the graphical methods and the analytical methods. The accuracy of the estimate depends on the size of the sample and the method used. Graphical methods tend to provide crude estimates, while analytical methods provide better estimates that include confidence limits (Murthy et al., 2004) and are reported to be more accurate (Razali et al., 2009). The common analytical methods are the method of moments (*MOM*),

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the maximum likelihood method (*MLM*), the method of percentiles (*MOP*) and the method of least squares (*MLS*). However, the most suitable method depends on the stands characteristics (Liu et al., 2004; Lei, 2008).

The aim of this study was to determine the best estimator method for Weibull function parameters for two different sylvicultural regimes: the coppice and the high forest. The Modeled parameters were then used to predict the distribution of marketable products of the private teak plantations as a useful management tool.

Methods

Study Site and Data

This study was carried out in the Guinea-Congo zone of Benin (West Africa) located between 6˚17' and 6˚58'N, and 1˚56' and 2˚31'E. The region has a bimodal rainfall regime, with a mean annual precipitation of 1100 mm and a daily mean temperature of 29.9˚C over the period 1971-2009 (www.Worldclim.org, 2005). Clayey-sand and vertisol are the dominant soil types. The original native vegetation, a semi-evergreen dry forest, was strongly influenced by human activities and is now reduced to a few relict forests and forest reserves.

Data were collected using a snowball sampling method which yielded 1101 private teak plantations: 844 coppices and 257 high forests. The size of each plantation (area) was measured. Then five (for plantations <0.5 ha) or ten (for plantations ≥0.5 ha) replicates strips of five trees were randomly sampled. On each strip, the planting space between trees (*e*) and between lines (*l*), the survival rate (*t*), and the girth at breast height (cbh) for all trees over 10 cm (lowest girth size of the marketable products) were measured. Timber merchants use height classes of marketable products based on girth classes: small poles (10 - 19 cm), medium poles (20 - 39 cm), large poles (40 - 49 cm), small posts (50 - 64 cm), large posts (65 - 79 cm), small logs (80 - 109 cm) and large logs (≥110 cm).

Statistical Parameter Modeling

The complete three-parameter Weibull probability density function of trees girth x is given by (Bailey & Dell, 1973)

$$f(x;\theta)=\frac{\gamma}{\beta}\left(\frac{\mathrm{x}-\alpha}{\beta}\right)^{\gamma-1}\exp\left[-\left(\frac{\mathrm{x}-\alpha}{\beta}\right)^{\gamma}\right] \qquad (1)$$

$$\text{for } \mathrm{x}\geq\alpha,\ \alpha\geq 0,\ \beta>0,\ \gamma>0$$

where $\theta=(\alpha,\beta,\gamma)'$, α is a location parameter, β is a scale parameter, and γ is a shape parameter. The recovery methods based on maximum likelihood, the maximum likelihood method (*MLM*), on moments, the method of moment (*MOM*) and on percentiles, the method of percentiles (*MOP*) were compared.

In relation to the unknown parameters α, β, γ and *n* the number of trees, the logarithm of the likelihood function, $\log(L(\theta))$ of Equation (1) is given by:

$$\log L(\theta)=\sum_{i=1}^{n}\log\left[\frac{\gamma}{\beta}\left(\frac{\mathrm{x}_i-a}{\beta}\right)^{\gamma-1}\exp\left[-\left(\frac{\mathrm{x}_i-a}{\beta}\right)^{\gamma}\right]\right] \qquad (2)$$

For estimating these parameters with the *MLM*, the Equation (2) was maximized with a three-equation system as follows:

$$\begin{cases}\hat{\beta}=\left[(n-1)(\mathrm{x}_i-\hat{\alpha})^{\hat{\gamma}}\right]^{\hat{\gamma}-1}\\ \hat{\gamma}=\left[\left(\sum_{i=1}^{n}(\mathrm{x}_i-\hat{\alpha})^{\gamma}\log(x_i-\hat{\alpha})\right)\left(\sum_{i=1}^{n}(\mathrm{x}_i-\hat{\alpha})^{\hat{\gamma}}\right)^{-1}\right.\\ \qquad\left.-n^{-1}\sum_{i=1}^{n}\log(\mathrm{x}_i-\hat{\alpha})^{-1}\right]\\ (\hat{\gamma}-1)\sum_{i=1}^{n}(\mathrm{x}_i-\hat{\alpha})-\hat{\gamma}\hat{\beta}^{-\hat{\gamma}}\sum_{i=1}^{n}(\mathrm{x}_i-\hat{\alpha})^{\hat{\gamma}}=0\end{cases} \qquad (3)$$

where *n* is the number of trees in the plantation and x_i the girth of tree *i*. The SAS software (SAS 9.2) was used to solve iteratively the equation system (3).

The moment order *k* (μ_k) of the Weibull function is given by:

$$\mu_k=\beta^k\sum_{j=0}^{k}(-1)^j\binom{k}{j}\Gamma\left(\frac{k-1}{\gamma}+1\right)\left[\Gamma\left(\frac{1}{\gamma}+1\right)\right]^j \qquad (4)$$

with Γ the gamma function written for a real value *s* as: $\Gamma(s)=\int_0^{\infty}x^{s-1}e^{-x}\mathrm{d}x,\ (s>0)$. The parameters α, β and γ were estimated by *MOM* with two processes. The moments of order 1 (μ), order 2 (σ^2) and order 3 (μ_3) (Razali et al., 2009) of the equation 4 were computed as follows:

$$\begin{cases}\mu=\hat{\alpha}\hat{\beta}+\Gamma\left(1+\frac{1}{\hat{\gamma}}\right)\\ \sigma^2=\hat{\beta}^2\Gamma\left(1+\frac{2}{\hat{\gamma}}\right)-\left[\Gamma\left(1+\frac{1}{\hat{\gamma}}\right)\right]^2\\ \mu_3=\hat{\beta}^3\left\{\Gamma\left(1+\frac{3}{\hat{\gamma}}\right)-3\Gamma\left(1+\frac{1}{\hat{\gamma}}\right)\Gamma\left(1+\frac{2}{\hat{\gamma}}\right)+2\left[\Gamma\left(1+\frac{1}{\hat{\gamma}}\right)\right]^3\right\}\end{cases} \qquad (5)$$

The system (5) was solved using the *R* package rootSolve (R2.14.1).

The parameter recovery method based on percentiles (*MOP*) requires computation of the 0^{th} (minimum girth), 25^{th}, 50^{th}, and 95^{th} percentiles of the distribution of the girth as x_0, x_{25}, x_{50}, and x_{95}, respectively. The three parameters were estimated by solving the following three equations simultaneously (6) (Borders et al., 1987):

$$\begin{cases}\hat{\alpha}=\dfrac{n^{1/3}\mathrm{x}_0-\mathrm{x}_{50}}{n^{1/3}-1}\\ \hat{\gamma}=\dfrac{\ln\left[\dfrac{\ln(1-0.95)}{\ln(1-0.25)}\right]}{\ln(x_{95}-\hat{\alpha})-\ln(x_{25}-\hat{\alpha})}\\ \hat{\beta}=-\dfrac{\hat{\alpha}\Gamma\left(1+\frac{1}{\hat{\gamma}}\right)}{\Gamma\left(1+\frac{2}{\hat{\gamma}}\right)}\\ \qquad+\sqrt{\dfrac{\alpha^2}{\Gamma^2\left(1+\frac{2}{\hat{\gamma}}\right)}\left[\Gamma^2\left(1+\frac{1}{\hat{\gamma}}\right)-\Gamma\left(1+\frac{2}{\hat{\gamma}}\right)\right]+\dfrac{\overline{\mathrm{x}}_q^2}{\Gamma\left(1+\frac{2}{\hat{\gamma}}\right)}}\end{cases} \qquad (6)$$

where *n* is the number of trees in the plantation, Γ is the gamma function, $\overline{\mathrm{x}}_q$ is the quadratic mean girth of the plantation, and ln is the natural logarithm.

Comparison Criteria

Two statistics were used to assess the goodness of fit of the three methods: the Kolmogorov–Smirnov statistic (KS) and the prediction index error of Reynolds (*e*) (Reynolds et al., 1988). The optimal recovery parameter method is the one with low value for the two criteria. The prediction index error of Reynolds (Equations (7) and (8)) was computed (Pauwels, 2003) as:

$$e(\%) = \frac{\sum_{j=1}^{m} \left| N_j - \hat{N}_j \right|}{N} \times 100 \quad (7)$$

where N_j is the observed and $\hat{N}_j$ is the estimated frequency of trees in girth size class *j*, N is the total number of trees, and *m* is the number of classes. $\hat{N}_j$ is estimated as follows:

$$\hat{N}_j = N \int_{l_j}^{u_j} f(x)\,\mathrm{d}x \quad (8)$$

where u_j and l_j are the upper and lower limits of class *j*.

Modeling Weibull Parameters

Multiple regression method was used to establish the relationship between the estimated parameters α, β, and γ (dependent variables) and dendrometric characteristics of stands. The predictor variables were density ($N \cdot ha^{-1}$), surface, and basal area of stand ($G \cdot m^2 \cdot ha^{-1}$); the girth of the tree of mean basal area (x_g cm); the mean, maximum, and minimum of girth; and the 25th (P_{25}), 50th (P_{50}), and 75th (P_{75}) percentiles of girth distribution. Models were established and tested for each set of dendrometric characteristics. A stepwise regression was then used to select the best subset of two variables. The estimated parameters were simultaneously re-estimated using the seemingly unrelated regression estimation (SURE). This can account for correlation errors between equations and is asymptotically efficient in the absence of specific errors (Liu et al., 2004).

Results

Data Summary

Overall, values for girth, density and basal area of trees averaged 21.3 cm, 5170 stems/ha and 18.4 $m^2 \cdot ha^{-1}$, respectively (**Table 1**). While the mean girth was larger in high forests than in coppices, the reverse trend was obtained for the density and basal area of trees.

Optimal Method

Descriptive statistics of the estimated parameters for coppices are presented (**Table 2**). For *MLM*, the parameters $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ averaged 8.14, 12.78, and 2.34, respectively. For *MOM*, mean values were 8.97, 11.79, and 2.20, respectively. For *MOP*, the three parameters averaged 9.20, 12.17, and 2.41, respectively. Statistics on the estimated parameters for high forests were presented (**Table 2**). For *MLM*, mean values for the parameters $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ varied 9.89, 15.97, and 3.07, respectively. For *MOM*, these values were 11.28, 13.94, 2.52 respectively. For *MOP*, the three parameters means were 10.14, 18.50, 3.72, respectively.

For coppices, the percentages of plantations that fitted the Weibull distribution were 0.95% for *MLM*, 0.97% for *MOM*, and 0.91% for *MOP*. For high forests, these percentages were 0.94% for *MLM*, 0.99% for *MOM*, and 0.96% for *MOP*. The *MOM* method showed the lowest values for the error index of Reynolds (e%) and the Kolmogorov-Smirnov statistic (KS) for both coppices and high forests (**Table 3**).

Table 1.
Density, N (/ha), quadratic mean of girth, x_g (cm) and basal area (G $m^2 \cdot ha^{-1}$) of the study plantations.

Sylvicultural regimes	Variables	Mean	Min.	Max.	SE
Coppices (n = 844)	x_g (cm)	20.2	10.7	51.6	0.2
	$N \cdot ha^{-1}$	5952	289	22,458	104
	$G \cdot m^2/ha^{-1}$	20.1	1.0	133.6	0.50
High forests (n = 257)	x_g (cm)	24.4	11.3	58.4	0.43
	$N \cdot ha^{-1}$	2798.9	632.5	8590.1	69.1
	$G \cdot m^2 \cdot ha^{-1}$	13.2	1.8	58.1	0.45

Min. and Max. are minimal and maximal values of the dendrometric parameters; SE is the standard error of the mean.

Table 2.
Estimated parameters of Weibull for the two sylvicultural regimes; α, β, and γ are the Weibull position, scale, and shape parameters, respectively. *MLM* is the maximum likelihood method; *MOM* the method of moments and *MOP* the method of percentiles.

	$\hat{\alpha}$				$\hat{\beta}$				$\hat{\gamma}$			
Method	Mean	Min.	Max.	SE	Mean	Min.	Max.	SE	Mean	Min.	Max.	SE
a) Coppices regime												
MLM	8.14	0.00	17.05	0.08	12.78	0.01	55.58	0.23	2.34	0.06	8.43	0.03
MOM	8.97	0.00	19.00	0.10	11.79	1.18	46.70	0.22	2.20	0.56	3.50	0.02
MOP	9.20	0.00	20.00	0.12	12.17	0.66	129.38	0.35	2.41	1.00	32.00	0.08
b) High forests regime												
MLM	9.89	0.00	19.88	0.22	15.47	1.91	65.11	0.53	3.07	0.92	6.30	0.06
MOM	11.28	0.00	19.96	0.25	13.94	2.29	67.08	0.49	2.52	0.68	3.52	0.03
MOP	10.14	0.00	19.62	0.32	18.50	1.32	180.15	1.31	3.72	1.00	39.98	0.27

MLM is the maximum likelihood method, *MOM* the method of moments and *MOP* the method of percentiles; Min and max are the minimum and the maximum respectively; SE is the standard error.

Table 3.
Comparison of the efficiency of the three parameters estimation methods: values of the error index of Reynolds (e%) and the Kolmogorov-Smirnov statistic (KS) for the two sylvicultural regimes.

	Coppices				High forests			
	e%		KS		e%		KS	
Methods	Mean	SE	Mean	SE	Mean	SE	Moy	SE
MLM	13.03	0.40	0.29	0.01	12.60	0.77	0.27	0.02
MOM	11.35	0.31	0.26	0.01	10.12	0.56	0.21	0.01
MOP	19.26	1.15	0.43	0.03	13.12	1.28	0.28	0.03

Consequently, *MOM* was chosen as the most appropriate method for modeling the distribution parameters of the Weibull function for private teak plantations. The models revealed significant differences between coppices and high forests for all three parameters estimated using *MOM* (**Table 4**).

Parameters' Model Development

The results of the SURE analysis for coppices are presented in **Table 5**. Stepwise regression revealed that the best subset of two stand characteristics is P_{25} and P_{75} for $\hat{\gamma}$, P_{75} and G^2 for $\hat{\beta}$, and LP_{50} and P_{75} for $\hat{\gamma}$ with $\left(LP_{50} = \ln\left(P_{50}/\mathrm{x}_g\right)\right)$. The final regression equations are:

$$\hat{\alpha} = 6.736 + 0.405P_{25} - 0.167P_{75}$$

$$\hat{\beta} = 5.5895 + 0.243P_{75} + 0.001G^2$$

$$\hat{\gamma} = 1.989 + 0.024P_{75} + 8.116\ln\left(P_{50}/\mathrm{x}_g\right)$$

The associated P_{value} are less than 0.001. The model to predict marketable products from coppices is:

$$F(\mathrm{x}) = 1 - \exp\left[-\left(\frac{\mathrm{x} - 6.736 - 0.405P_{25} + 0.167P_{75}}{5.895 + 0.243P_{75} + 0.001G^2}\right)^{\nu}\right]$$

where $\nu = 1.989 + 0.024P_{75} + 8.116LP_{50}$, $F(\mathrm{x})$ is the Weibull function, x is the girth of the tree, G is the basal area, P_{25}, P_{50}, and P_{75} are the 25th, 50th, and 75th are the *th* percentiles, and LP_{50} the weighted percentile. Some dependent variables such as P_{25} and P_{75} were computed with the best adjustment as follows:

$$P_{25} = -9.79 + 5.69\sqrt{\mathrm{x}_g} \quad \text{and} \quad P_{75} = -26.6 + 11.1\sqrt{\mathrm{x}_g}$$

with $P_{\text{value}} = 0.000$. The results of the SURE analysis for high forest are presented in **Table 6** and the best models equations with P_{value} less than 0.023 are:

$$\hat{\alpha} = 5.249 + 0.476\mathrm{x}_{\min} - 0.111G$$

$$\hat{\beta} = 10.567 + 0.004G^2 - 23.519\ln\left(P_{50}/\mathrm{x}_g\right)$$

$$\hat{\gamma} = 1.867 + 8.283\ln\left(P_{50}/\mathrm{x}_g\right) + 0.014N^{0.5}$$

The equation to predict the marketable products for high forests is then:

$$F(\mathrm{x}) = 1 - \exp\left[\left(\frac{\mathrm{x} - 5.249 - 0.476\mathrm{x}_{\min} + 0.111G}{10.567 + 0.004G^2 - 23.519LP_{50}}\right)^{w}\right]$$

where $w = 1.867 + 8.283LP_{50} + 0.014\sqrt{N}$, $F(\mathrm{x})$ is the Weibull distribution function, x is the girth of the tree, $\mathrm{x}_{\min}$, is the minimum girth, G is the basal area, N is the density, P_{50} is the 50th percentile or median, and LP_{50} the weighted 50th percentile.

Table 4.
Comparison of the sylvicultural regimes according to estimated parameters of Weibull function by the best estimator method *MOM*.

	Coppices		High forests			
Parameters	Mean	SE	Mean	SE	*t*	*P*
$\hat{\alpha}$	8.97	0.10	11.28	0.25	8.58	0.000
$\hat{\beta}$	11.79	0.22	13.94	0.49	4.00	0.000
$\hat{\gamma}$	2.20	0.02	2.52	0.03	8.88	0.000

$\hat{\alpha}$, $\hat{\beta}$, and $\hat{\gamma}$ are the Weibull position, scale, and shape parameters, respectively. *MOM* is the method of moments; *t* is the statistic of student, and *P* the associated probability value.

Discussion

Efficiency of the Parameter Recovery Methods

Regardless of the parameter recovery method used, the observed and theoretical distributions of the plantations were much closed according to the Kolmogorov-Smirnov test. Moreover, for both sylvicultural practices, coppices and high forests, the average value of the shape parameter γ of the Weibull distribution was lower than 3.6, suggesting that the distribution of trees is left-skewed. Using the optimal method (*MOM*), the parameter α, whose value is associated with the minimum girth, was 8.97 for coppices and 11.28 for high forest, both were close to the minimum girth of this study (10 cm).

The parameter β, which gives an idea of the central value of samples, had a maximum value of 46.70 for coppices and 67.08 for high forests, while the values measured, with the completed inventory of 18 plantations, were 51.6 cm for coppices and 58.4 cm for high forests. These results are similar to previous findings, which illustrated that α is a good predictor of the minimum diameter (Frazier, 1981; Knoebel et al., 1986; Leduc et al., 2001). They also support those of Lei (2008), who demonstrated that the two-parameter Weibull distribution and *MOM* provided the best estimation of the diameter distribution of Chinese pine (*Pinus tabulaeformis*). These results are also consistent with those found by Liu et al. (2004) in their study of the diameter distribution of unthinned plantations of black spruce (*Picea mariana*) in central Canada, although in their study *MOP* was the preferred method. Zhang et al. (2003) previously dem-

Table 5.
Regression coefficients of the predictors and statistics resulting from the SURE (Seemingly Unrelated Regression Estimation) analysis for coppices for the response variables i.e. $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\gamma}$ respectively the Weibull position, scale and shape parameters.

	$\hat{\alpha}$			$\hat{\beta}$			$\hat{\gamma}$		
	Constant	P_{25}	P_{75}	Constant	P_{75}	G^2	Constant	P_{75}	LP_{50}
Coef.	6.74	0.405	−0.167	5.895	0.243	0.001	1.989	0.024	8.116
P	<0.00	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
t	16.46	8.69	−6.100	7.720	7.130	4.650	25.410	6.810	10.060
SE	0.41	0.05	0.027	0.763	0.034	0.000	0.078	0.004	0.807

Coef. is the regression coefficients, G is the basal area, P_{25} and P_{75} are the 25th and 75th percentiles, and $LP_{50} = \ln\left(P_{50}/x_g\right)$ where x_g is the girth of the tree of mean basal area and P_{50} is the 50th percentile, t is the statistic of Student and P the associated probability value.

Table 6.
Regression coefficients values and their significant appreciation statistics resulting from the SURE (Seemingly Unrelated Regression Estimation) analysis for high forests with the response variables $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\gamma}$ respectively the Weibull position, scale and shape parameters.

	$\hat{\alpha}$			$\hat{\beta}$			$\hat{\gamma}$		
	Constant	x_{min}	G	Constant	G^2	LP_{50}	Constant	LP_{50}	$N^{1/2}$
Coef	5.249	0.476	−0.111	10.567	0.004	−23.519	1.867	8.283	0.014
P	<0.001	<0.001	< 0.001	<0.001	0.003	0.023	<0.001	0.003	<0.001
t	6.040	7.440	−4.060	15.000	3.640	−2.290	9.800	3.700	4.320
SE	0.870	0.064	0.027	6.705	0.001	23.519	0.191	2.241	0.003

Coef. is the regression coefficients, x_{min} is the minimum girth, G is the basal area, N is the density and $LP_{50} = \ln\left(P_{50}/x_g\right)$ where x_g is the girth of tree of mean basal area and P_{50} is the 50th percentile, t is the statistic of Student and P the associated probability value, SE is the standard error.

onstrated the effectiveness of the Weibull distribution for describing the diameter distribution of natural stands of red spruce (*Picea rubens*) and balsam fir (*Abies balsamea*) in the north-east of North America. Meanwhile, Bailey & Dell (1973) have shown that *MOM* is more efficient than *MOP* for estimating parameters of the Weibull distribution, but it requires very complex calculations. Nanang (1998) in a study on the diameter distribution of *Azadirachta indica* plantations in Ghana asserted the same thing. It was also argued that *MOM* assures compatibility between the characteristics of the observed population used in parameter recovery and those obtained through simulation (Mateus & Tomé, 2011). The differences observed between coppices and high forests for all the three parameters estimated by *MOM* confirm the need to build separate models for different sylvicultural regimes.

Predicting the Weibull Parameters

Parameters of the Weibull distribution were functions of most of the dendrometric characteristics. In all cases the regression coefficients were statistical significant with P value less than 0.001 for coppices and 0.023 for high forests. These findings are in agreement with Liu et al. (2004), who modeled the three parameters of a Weibull distribution using four characteristics of black spruce stands (age, basal area, average height, and site index). In most cases, the probability values associated to the regressions were less than 0.0001. Since the distribution model of marketable products from coppices differed from high forests, a global model combining data from these two sylvicultural regimes would not be suitable. This is supported by the observed differences between the estimated parameters. The parameter of the distribution shape, γ, is more influenced by LP_{50} for both sylvicultural practices. The location parameter α depends on the positional parameters P_{25} and P_{75} in coppices, and on the minimum girth and basal area in high forests. β is more influenced by the square of the basal area for both sylvicultural regimes. These results differ from those of Torres-Rojo et al. (2000), who found that the shape of the distribution is strongly influenced by the diameter, mean basal area, density, and dominant height of the trees; and that β is influenced by diameter and mean basal area of trees. Moreover, several studies have previously found that the minimum girth most often influences the value of α (Frazier, 1981; Knoebel et al., 1986; Lejeune, 1994; Leduc et al., 2001).

Conclusion

In this study, models have been developed to assess marketable teak resource produced by private teak plantations. The main advantage of modeling parameter with stand characteristics is the compatibility between the characteristics of the observed populations and those obtained through theoretical distributions. Results indicated that the three methods compared were generally suitable for modeling the distribution of marketable products. However, the relative performance of each method depends on its ability to predict the observed girth size class frequencies. The method based on moment (*MOM*) appears to be the most appropriate.

Distribution models for marketable products were developed for coppices and for high forests using stand variables and *MOM*.

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