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MHD Mass Transfer Flow past a Vertical Porous Plate Embedded in a Porous Medium in a Slip Flow Regime with Thermal Radiation and Chemical Reaction

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ABSTRACT

This problem presents the effects of thermal radiation and chemical reaction on MHD unsteady mass transfer flow past a semi-infinite vertical porous plate embedded in a porous medium in a slip flow regime with variable suction. A magnetic field of uniform strength is assumed to be applied transversely to the direction of the main flow. Perturbation technique is applied to transform the non-linear coupled governing partial differential equations in dimensionless form into a system of ordinary differential equations. The resulting equations are solved analytically and the solutions for the velocity, temperature and concentration fields are obtained. The effects of various flow parameters on velocity, temperature and concentration fields are presented graphically. For different values of the flow parameters involved in the problem, the numerical calculations for the Nusselt number, Sherwood number and skin-friction co-efficient at the plate are performed in tabulated form. It is seen that chemical reaction causes the velocity field and concentration field to decrease and the chemical reaction decreases the rate of viscous drag at the plate.

Keywords: Porous Medium; Slip Flow; Rarefaction; Viscous Drag; Free Convection

1. Introduction

In recent years, free convective flow and heat transfer problems in the presence of magnetic field through a porous medium have attracted the attention of a number of scholars because of their possible application in many branches of science and technology such as fiber and granular insulation, geothermal system, etc. In engineering science, it finds its application in MHD pumps, MHD bearing, MHD power generators, etc. The phenomena of heat and mass transfer are also very common in theory of stellar structure and observable effects are detectable on the solar structure.

Analytical solutions of the problem of convective flows, which arise in the fluids due to interaction of the force of gravity and density differences caused by simultaneous diffusion of thermal energy and chemical species, have been presented by many authors due to application of such problems in Geophysics and Engineering. Some of them are Bejan and Khair [1], Trevisan and Bejan [2], Acharya *et al*. [3], Rapits and Kafousias [4], and Das *et al*. [5]. The study through porous medium has got its importance because of its occurrence in movement of water. Investigations of such problems also have importance in purification process, petroleum technology and in the field of agricultural engineering. Study of flow problems through porous medium is heavily based on Darcy's experimental law [6]. Wooding [7] and Brinkman [8,9] have modified Darcy's law, which are used by many authors on study of convective flow in porous media. Recently Chaudhary and Jain [10] have studied the combined heat and mass transfer effects on MHD free convective flow through porous medium.

The study of the effect of chemical reaction on heat and mass transfer in a flow is of great practical importance to the engineers and scientists because of its universal occurrence of many branches of science and technology. In processes such as drying, distribution of temperature and moisture over agricultural fields, energy transfer in a wet cooling tower and flow in a cooler heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in many Industries. Many investigations have studied the effect of chemical reaction in different convective heat and mass transfer flows. Chambre and Young [11] have presented a first order chemical reaction in the neighbourhood of horizontal plate. Muthucumaraswamy [12] has presented heat and mass transfer effects on a continuously moving isothermal vertical surface with uniform suction by taking in to account the homogeneous chemical reaction of first order. Muthucumaraswamy and Meenakshisundaram [13] investigated theoretical study of chemical reaction effects on vertical oscillating plate with variable temperature and mass diffusion.

Radiation is a process of heat transfer through electromagnetic waves. Radiative convective flows are encountered in countless industrial and environmental process. For example, heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows etc. Radiative heat and mass transfer play an important role in space related technology. The effect of radiation on various convective flows under different conditions have been studied by many researchers including Hussain and Thakar [14], Ahmed and Sarmah [15], Rajesh and Varma [16], Pal and Mondal [17], Samad and Rahman [18], S. Karthikeyan *et al.* [19], Das *et al.* [20] and Pal *et al.* [21].

At the macroscopic level, it is accepted that the boundary condition for a viscous fluid at a solid wall is one of "no slip". While no-slip boundary condition has been proven experimentally to be accurate for a number of macroscopic flows, it remains an assumption that is based on physical principles. In fact nearly two hundred years ago, Navier's [22] proposes a general boundary condition that incorporates the possibility of fluid slip at a solid boundary. Navier's proposed condition assumes that the fluid slip velocity at a solid surface is proportional to the shear stress at the surface. The mathematical form of the Navier's proposed condition on slip velocity as emphasized by Goldstein [23] is $v=\gamma\left(\frac{\mathrm{d}v}{\mathrm{d}y}\right)$ where γ being the slip coefficient, v the slip velocity and y the normal coordinate. The fluid slippage phenomena at solid boundaries appear in many applications such as in micro-channels or nano channels and in applications where a thin film of light oil is attached to the moving plates or when the surface is coated with special coating such as thick monolayer of hydrophobic octadecyltrichorosilane. Due to practical applications of the fluid slippage phenomemenon at solid boundaries, several scholar have carried out their research work in that literature, the names of whom (Yu & Amed [24], Waltanebe *et al*. [25], Jain and Sharma [26], Khaled and Vafai [27] and Poonia and Chaudhary [28]) are worth meaning.

The present work is concerned with the effect of thermal radiation and chemical reaction on magneto-hydrodynamic convective mass transfer flow of an unsteady viscous incompressible eclectically conducting fluid past a semi-infinite vertical permeable plate embedded in a porous medium in slip flow regime. In this paper, we have generalized the work done by Karthikeyan *et al*. [19] by considering mass transfer with chemical reaction effect. The classical model for radiation effect introduced by Cogley *et al.* [29] is used. Perturbation technique is applied to convert the governing non-linear partial differential equations in to a system of ordinary differential equations which are solved analytically.

2. Mathematical Analysis

We consider a two-dimensional unsteady flow of a laminar, incompressible, electrically conducting and heat absorbing fluid past a semi-infinite vertical porous plate embedded in a uniform porous medium. We introduce the coordinate system $\left(\bar{x},\bar{y},\bar{z}\right)$ with X axis is chosen along the plate , Y axis perpendicular to it and directed in the fluid region and Z axis along the width of the plate as shown in the **Figure 1**. A uniform magnetic field of strength B_0 in the presence of radiation is imposed transversely in the direction of Y axis. The induced magnetic field is neglected under the assumption that the magnetic Reynolds number is small. It is assumed that there is no applied voltage which implies the absence of any electrical field. The radiative heat flux in the X direction is considered negligible in comparison to that in Y direction. The governing equations for this study are based on the conservation of mass, linear momentum, energy and species concentration. Taking in to consideration the assumptions made above, these equations in Cartesian frame of reference are given by Equation of continuity:

$$\frac{\partial\bar{v}}{\partial\bar{y}}=0 \tag{1}$$

Momentum equation:

$$\begin{aligned}&\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}+\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\\&=-\frac{1}{\rho}\frac{\partial\bar{p}}{\partial\bar{x}}+g\beta\left(\bar{T}-\bar{T}_\infty\right)+g\bar{\beta}\left(\bar{C}-\bar{C}_\infty\right)\\&\quad+\nu\frac{\partial^2\bar{u}}{\partial\bar{y}^2}-\frac{\sigma B_0^2}{\rho}\bar{u}-\frac{\nu\bar{u}}{K^*}\end{aligned} \tag{2}$$

Energy equation

$$\begin{aligned}&\bar{v}\frac{\partial\bar{T}}{\partial\bar{y}}+\frac{\partial\bar{T}}{\partial\bar{t}}\\&=\frac{k}{\rho C_p}\frac{\partial^2\bar{T}}{\partial\bar{y}^2}-\frac{1}{\rho C_p}\frac{\partial q_r^*}{\partial\bar{y}}+\frac{Q_0\left(\bar{T}_\infty-\bar{T}\right)}{\rho C_p}\end{aligned} \tag{3}$$

Species continuity equation:

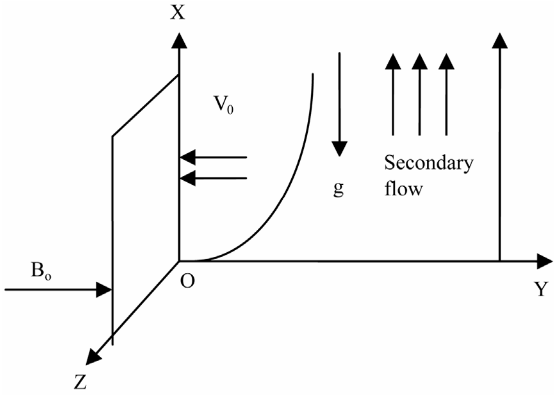

Figure 1. Physical model of the problem.

$$\bar{v}\frac{\partial \bar{C}}{\partial \bar{y}}+\frac{\partial \bar{C}}{\partial \bar{t}}=D_M\frac{\partial^2 \bar{C}}{\partial \bar{y}^2}+\bar{K}\left(\bar{C}_\infty-\bar{C}\right) \tag{4}$$

where $\bar{x}$ and $\bar{y}$ are the dimensional distances along and perpendicular to the plate respectively. $\bar{u}$ and $\bar{v}$ are the components of the dimensional velocities along $\bar{x}$ and $\bar{y}$ respectively. ρ is the density of the medium, g is the acceleration due to gravity, ν is the kinematic viscosity, σ is the fluid electrical conductivity, K^* is the permeability of the porous medium, $\bar{K}$ is first order chemical reaction, β is the coefficient of thermal expansion, $\bar{\beta}$ is the coefficient of mass expansion. $\bar{T}$ is the dimensional temperature of the fluid near the plate, $\bar{T}_\infty$ is the dimensional free stream temperature, $\bar{C}$ is the dimensional concentration of the fluid near the plate, $\bar{C}_\infty$ is the dimensional free stream concentration, k is constant pressure, q_r^* is the radiative heat flux and Q_0 is the dimensional heat absorption coefficient.

Cogley *et al*. [29] showed that, in the optically thin limit for a non-gray gas near equilibrium, the radiative heat flux is represented by the following form:

$$\frac{\partial q_r^*}{\partial \bar{y}}=4\left(\bar{T}-\bar{T}_\infty\right)I^* \tag{5}$$

where $I^*=\int K_{\lambda w}\frac{\partial e_{b\lambda}}{\partial \bar{T}}\mathrm{d}\lambda$, $K_{\lambda w}$ is the absorption coefficient at the wall and $e_{b\lambda}$ is the Planck's function.

Under the assumption, the appropriate boundary conditions for velocity involving slip flow, temperature and concentration fields are given by

$$\begin{aligned}&\bar{u}=\bar{u}_{\text{slip}}=\bar{h}\frac{\partial \bar{u}}{\partial \bar{y}},\ \bar{T}=\bar{T}_w+\varepsilon\left(\bar{T}_w-\bar{T}_\infty\right)\mathrm{e}^{n^*t^*},\\&\bar{C}=\bar{C}_w+\varepsilon\left(\bar{C}_w-\bar{C}_\infty\right)\mathrm{e}^{n^*t^*}\quad \text{at } \bar{y}=0\end{aligned} \tag{6}$$

$$\begin{aligned}&\bar{u}\to\bar{U}_\infty=U_0\left(1+\varepsilon\mathrm{e}^{n^*t^*}\right),\ \bar{T}\to\bar{T}_\infty,\\&\bar{C}\to\bar{C}_\infty\quad \text{as } \bar{y}\to\infty\end{aligned} \tag{7}$$

where $\bar{T}_w$ and $\bar{C}_w$ are the dimensional temperature and species concentration at the wall respectively and $\bar{h}$ is the characteristic dimension of the flow fluid. Science the suction velocity normal to the plate is a function of time only, it can be taken in the exponential form as

$$\bar{v}=-V_0\left(1+\varepsilon A\mathrm{e}^{n^*t^*}\right) \tag{8}$$

where A is a real positive constant, ε and εA are small quantities less than unity and V_0 is a scale of suction velocity which is a non-zero positive constant.

Outside the boundary layer, Equation (2) gives

$$-\frac{1}{\rho}\frac{\mathrm{d}\bar{p}}{\mathrm{d}\bar{x}}=\frac{\mathrm{d}\bar{U}_\infty}{\mathrm{d}\bar{t}}+\frac{\sigma B_0^2}{\rho}\bar{U}_\infty+\frac{\nu}{K^*}\bar{U}_\infty \tag{9}$$

Now we introduce the dimensionless variables as follows

$$\left.\begin{aligned}&u=\frac{\bar{u}}{U_0},v=\frac{\bar{v}}{V_0},y=\frac{\bar{y}V_0}{\nu},U_\infty=\frac{\bar{U}_\infty}{U_0},t=\frac{\bar{t}V_0^2}{\nu},\\&\theta=\frac{\bar{T}-\bar{T}_\infty}{\bar{T}_w-\bar{T}_\infty},\phi=\frac{\bar{C}-\bar{C}_\infty}{\bar{C}_w-\bar{C}_\infty},n=\frac{n^*\nu}{V_0^2},\\&\alpha=\frac{K^*V_0^2}{\nu^2},\Pr=\frac{\mu C_p}{k},M=\frac{\sigma B_0^2\nu}{\rho V_0^2},\\&Gr=\frac{\nu\beta g\left(\bar{T}_w-\bar{T}_\infty\right)}{U_0V_0^2},Gm=\frac{\nu\bar{\beta}g\left(\bar{C}_w-\bar{C}_\infty\right)}{U_0V_0^2},\\&Q=\frac{Q_0\nu}{\rho C_pV_0^2},R=\frac{4\nu I^*}{\rho C_pV_0^2},K=\frac{\bar{K}\nu}{V_0^2},h=\frac{V_0\bar{h}}{\nu}\end{aligned}\right\} \tag{10}$$

where *Pr* is the Prandtl number, *M* is the magnetic field parameter, Gr is Grashof number for heat transfer, Gm is the Grashof number for mass transfer, *Q* is the heat source parameter, α is the permeability parameter, θ is the non dimensional temperature, ϕ is the non dimensional concentration, *R* is radiation parameter, *h* is the rarefaction parameter and *K* is the chemical reaction parameter.

In view of Equations (8) to (10) the governing Equations (2), (3) and (4) reduce the following non-dimensional form:

$$\begin{aligned}&\frac{\partial u}{\partial t}-\left(1+\varepsilon A\mathrm{e}^{nt}\right)\frac{\partial u}{\partial y}\\&=\frac{\mathrm{d}U_\infty}{\mathrm{d}t}+\frac{\partial^2 u}{\partial y^2}+Gr\theta+Gm\phi+N\left(U_\infty-u\right)\end{aligned} \tag{11}$$

$$\frac{\partial \theta}{\partial t}-\left(1+\varepsilon A\mathrm{e}^{nt}\right)\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}-R\theta-Q\theta \tag{12}$$

$$\frac{\partial \phi}{\partial t}-\left(1+\varepsilon A\mathrm{e}^{nt}\right)\frac{\partial \phi}{\partial y}=\frac{1}{Sc}\frac{\partial^2\phi}{\partial y^2}-K\phi \tag{13}$$

where $N=M+\dfrac{1}{\alpha}$

The boundary conditions (6) and (7) in the dimensionless form can be written as

$$u = u_{\text{slip}} = h\frac{\partial u}{\partial y}, \theta = 1 + \varepsilon e^{nt}, \phi = 1 + \varepsilon e^{nt} \quad \text{at } y = 0 \quad (14)$$

$$u \to U_\infty = 1 + \varepsilon e^{nt},\ \theta \to 0,\ \phi \to 0 \quad \text{as } y \to \infty \quad (15)$$

3. Solution of the Problem

Equations (11) to (13) are coupled non-linear partial differential equations and these can be solved in closed form. However, these equations can be reduced to a set of ordinary differential equations, which can be solved analytically. These can be done by representing the velocity, temperature and concentration of the fluid in the neighbourhood of the plate as

$$u = u_0(y) + \varepsilon e^{nt} u_1(y) + O(\varepsilon^2) \quad (16)$$

$$\theta = \theta_0(y) + \varepsilon e^{nt} \theta_1(y) + O(\varepsilon^2) \quad (17)$$

$$\phi = \phi_0(y) + \varepsilon e^{nt} \phi_1(y) + O(\varepsilon^2) \quad (18)$$

Substituting (16) to (18) in Equations (11) to (13) and equating the harmonic and non harmonic terms and neglecting the coefficient of $O(\varepsilon^2)$ we get the following pairs of equations for (u_0, θ_0, ϕ_0) and (u_1, θ_1, ϕ_1).

$$u_0'' + u_0' - Nu_0 = -N - Gr\theta_0 - Gm\phi_0 \quad (19)$$

$$\begin{aligned} &u_1'' + u_1' - (N+n)u_1 \\ &= -Au_0' - Gr\theta_1 - Gm\phi_1 - (N+n) \end{aligned} \quad (20)$$

$$\theta_0'' + \Pr\theta_0' - \Pr(R+Q)\theta_0 = 0 \quad (21)$$

$$\theta_1'' + \Pr\theta_1' - \Pr(R+Q+n)\theta_1 = -A\Pr\theta_0' \quad (22)$$

$$\phi_0'' + Sc\phi_0' - KSc\theta_0 = 0 \quad (23)$$

$$\phi_1'' + Sc\phi_1' - Sc(K+n)\theta_1 = -ASc\phi_0' \quad (24)$$

where the primes denote the differentiation with respect to y

The corresponding boundary conditions can be written as

$$\begin{aligned} &u_0 = hu_0',\ u_1 = hu_1',\ \theta_0 = 1, \\ &\theta_1 = 1,\ \phi_0 = 1, \phi_1 = 1 \quad \text{at } y = 0 \end{aligned} \quad (25)$$

$$\begin{aligned} &u_0 = 1,\ u_1 = 1,\ \theta_0 \to 0,\ \theta_1 \to 0, \\ &\phi_0 \to 0, \phi_1 \to 0 \quad \text{at } y \to \infty \end{aligned} \quad (26)$$

The solutions of Equations (18) to (23) which satisfy the boundary conditions (24) and (25) are given by

$$u_0(y) = 1 + D_5 e^{-m_5 y} + D_4 e^{-m_3 y} + D_3 e^{-m_1 y} \quad (27)$$

$$\begin{aligned} u_1(y) &= 1 + D_{11}e^{-m_6 y} + D_6 e^{-m_1 y} - D_7 e^{-m_2 y} \\ &+ D_8 e^{-m_3 y} - D_9 e^{-m_4 y} + D_{10} e^{-m_5 y} \end{aligned} \quad (28)$$

$$\theta_0(y) = e^{-m_1 y} \quad (29)$$

$$\theta_1(y) = (1 - D_1)e^{-m_2 y} + D_1 e^{-m_1 y} \quad (30)$$

$$\phi_0(y) = D_2 e^{-m_3 y} \quad (31)$$

$$\phi_1(y) = (1 - D_2)e^{-m_4 y} + D_2 e^{-m_3 y} \quad (32)$$

where,

$$m_1 = \frac{\Pr + \sqrt{\Pr^2 + 4\Pr(R+Q)}}{2},$$

$$m_2 = \frac{\Pr + \sqrt{\Pr^2 + 4\Pr(R+Q+n)}}{2},$$

$$m_3 = \frac{Sc + \sqrt{Sc^2 + 4KSc}}{2},$$

$$m_4 = \frac{Sc + \sqrt{Sc^2 + 4Sc(K+n)}}{2},$$

$$m_5 = \frac{1 + \sqrt{1+4N}}{2},$$

$$m_6 = \frac{1 + \sqrt{1 + 4(N+n)}}{2}$$

$$D_1 = \frac{A\Pr m_1}{m_1^2 - \Pr m_1 - \Pr(\Pr + Q + n)},$$

$$D_2 = \frac{AScm_3}{m_3^2 - Scm_3 - Sc(K+n)}, \quad D_3 = \frac{-Gr}{m_1^2 - m_1 - N},$$

$$D_4 = \frac{-Gm}{m_2^2 - m_2 - N},$$

$$D_5 = \frac{-(1 + D_3 + D_3 hm_1 + D_4 + D_4 hm_3)}{1 + m_5 h},$$

$$D_6 = \frac{AD_3 m_1 - GrD_1}{m_1^2 - m_1 - (N+n)},$$

$$D_7 = \frac{-Gr(1 - D_1)}{m_2^2 - m_2 - (N+n)},$$

$$D_8 = \frac{AD_4 m_3 - GmD_2}{m_3^2 - m_3 - (N+n)}, D_9 = \frac{-Gm(1 - D_2)}{m_4^2 - m_4 - (N+n)},$$

$$D_{10} = \frac{AD_5 m_5}{m_5^2 - m_5 - (N+n)},$$

$$D_{11} = \frac{\left(-(1 + D_6 - D_7 + D_8 - D_9 + D_{10}) + h(-D_6 m_1 + D_7 m_2 - D_8 m_3 + D_9 m_4 - D_{10} m_5)\right)}{1 + hm_6}$$

Substituting equations (27)-(32) in Equations (16)-(18), we obtain the velocity, temperature and concentration distributions in the boundary layer as follows:

$$u(y,t) = 1 + D_5 e^{-m_5 y} + D_4 e^{-m_3 y} + D_3 e^{-m_1 y} + \varepsilon e^{nt}\left(1 + D_{11} e^{-m_6 y} + D_6 e^{-m_1 y} - D_7 e^{-m_2 y} + D_8 e^{-m_3 y} - D_9 e^{-m_4 y} + D_{10} e^{-m_5 y}\right) \quad (33)$$

$$\theta(y,t) = e^{-m_1 y} + \varepsilon e^{nt}\left((1-D_1)e^{-m_2 y} + D_1 e^{-m_1 y}\right) \quad (34)$$

$$\phi(y,t) = e^{-m_3 y} + \varepsilon e^{nt}\left((1-D_2)e^{-m_4 y} + D_1 e^{-m_3 y}\right) \quad (35)$$

4. Skin Friction

The non dimensional skin friction at the plate is given by:

$$C_f = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left(\frac{\partial u_0}{\partial y} + \varepsilon e^{nt}\frac{\partial u_1}{\partial y}\right)_{y=0} = -\left(m_5 D_5 + m_3 D_4 + m_1 D_3\right) + \varepsilon e^{nt}\left(-m_6 D_{11} - m_1 D_6 + m_2 D_7 - m_3 D_8 + m_4 D_9 - m_5 D_{10}\right) \quad (36)$$

5. Nusselt Number

The non-dimensional form of the rate of heat transfer in terms of Nusselt number at the plate is given by:

$$N_u = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = -\left(\frac{\partial \theta_0}{\partial y} + \varepsilon e^{nt}\frac{\partial \theta_1}{\partial y}\right)_{y=0} = m_1 + \varepsilon e^{nt}\left(m_2(1-D_1) + m_1 D_1\right) \quad (37)$$

6. Sherwood Number

The non-dimensional form of the rate of mass transfer in terms of Sherwood number at the plate is given by:

$$S_h = -\left(\frac{\partial \phi}{\partial y}\right)_{y=0} = -\left(\frac{\partial \phi_0}{\partial y} + \varepsilon e^{nt}\frac{\partial \phi_1}{\partial y}\right)_{y=0} = m_3 + \varepsilon e^{nt}\left(m_4(1-D_2) + m_3 D_2\right) \quad (38)$$

7. Results and Discussion

In order to get physical insight in to the problem, we have carried out numerical calculations for non-dimensional velocity field, temperature field, concentration field, co-efficient of skin friction C_f at the plate, the rate of heat transfer Nu and the rate of mass transfer in terms of Sherwood number Sh by assigning specific values to the different values to the parameters involved in the problem, viz., Magnetic parameter *M*, Chemical reaction parameter *K*, Radiation parameter *R*, Grashof number for heat transfer Gr, Grashof number for mass transfer Gm, permeability parameter α, heat source parameter *Q* and rarefaction parameter *h*. Throughout our investigation the value of Prandtl number *Pr* is kept constant at 0.71 which corresponding to air at $20^{\circ}C$. The Schmidt number *Sc* are taken in such a way that they represent the diffusing chemical species of common interest in air (for example $Sc = 0.30$ for He, Sc = 0.30 for H_2O and $Sc = 0.78$ for NH_3), time $t = 1$, $n = 0.1$, $A = 1$ and the values of other parameters are chosen arbitrarily. The numerical results are demonstrated through different graphs and table and their results are interpreted physically.

Figure 2 plots the velocity profiles against the spanwise coordinate y for different magnetic parameters. This illustrates that velocity decreases as the existence of magnetic field becomes stronger. This conclusion agrees with the fact that the magnetic field exerts retarding force on the free-convection flow.

Figure 3 illustrates the effect of radiation on the velocity. It is seen from this figure that there is a steady increase in the velocity with the increase in radiation parameter *R*. The increase in this parameter *R* leads to increase the boundary layer thickness and to reduce the

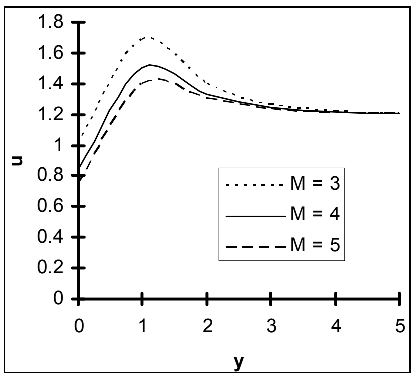

Figure 2. Velocity *u* versus *y*, under the effect of *M*, for *R* = 2, *K* = 1, *Q* = 1, *h* = 0.3, *ε* = 0.2, *Sc* = 0.6, *Gr* = 6, *Gm* = 4, *α* = 1.

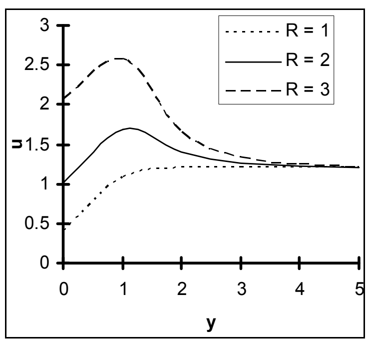

Figure 3. Velocity *u* versus *y*, under the effect of *R*, for *M* = 3, *K* = 1, *Q* = 1, *h* = 0.3, *ε* = 0.2, *Sc* = 0.6, *Gr* = 6, *Gm* = 4, *α* = 1.

heat transfer rate in the presence of thermal buoyancy force.

The change of velocity profile due to different chemical reaction parameters is plotted in **Figure 4**. This figure shows that the fluid motion is retarded on account of chemical reaction. This shows that the consumption of chemical species leads to fall in the concentration field which in turn diminishes the buoyancy effects due to concentration gradients. Consequently, the flow field is decelerated.

Figure 5 depicted the effect of heat source parameter on velocity field. It is seen from this figure that the heat source parameter Q leads the fluid motion to retard.

Figure 6 indicates the fact that an increase in Schmidt number Sc decelerates the fluid flow. In other words, mass diffusivity causes the fluid velocity to increase.

It is observed from **Figure 7** that an increase in Grashof number for heat transfer leads to a rise in the values of velocity u due to enhancement in buoyancy force.

The plot of velocity profile for different values of Grashof number for mass transfer is given in **Figure 8**. It is observed that velocity increases for the increasing values of Grashof number for mass transfer.

The change in velocity profile due to different permeability of porous medium is plotted in **Figure 9**. Here it is seen that due to increase of porosity of the medium fluid motion is accelerated. Moreover **Figures 10** and **11** displays that the velocity u increases as ε and rarefaction parameter h are increased indicating the fact that slips at the surface accelerates the fluid motion.

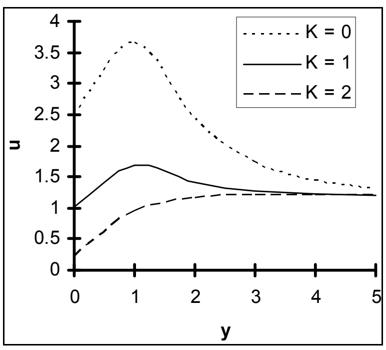

Figure 4. Velocity *u* versus *y*, under the effect *K*, for *R* = 2, *M* = 3, *Q* = 1, *h* = 0.3, *ε* = 0.2, *Sc* = 0.6, *Gr* = 6, *Gm* = 4, *α* = 1.

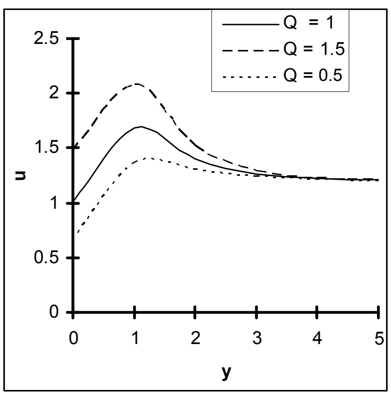

Figure 5. Velocity *u* versus *y*, under the effect *Q*, for *R* = 2, *M* = 3, *K* = 1, *h* = 0.3, *ε* = 0.2, *Sc* = 0.6, *Gr* = 6, *Gm* = 4, *α* = 1.

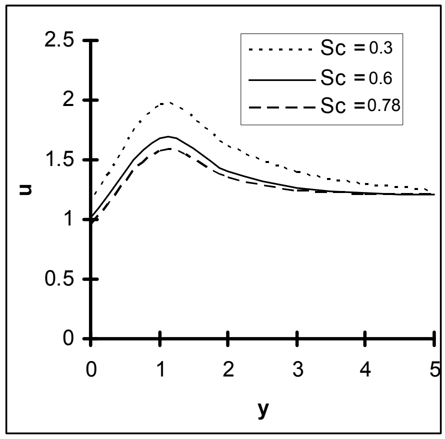

Figure 6. Velocity *u* versus *y*, under the effect *Sc*, for *R* = 2, *M* = 3, *Q* = 1, *h* = 0.3, *ε* = 0.2, *K* = 1, *Gr* = 6, *Gm* = 4, *α* = 1.

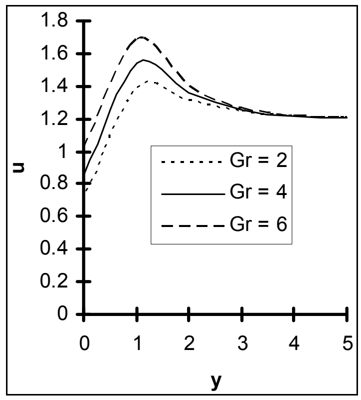

Figure 7. Velocity *u* versus y, under the effect of *Gr*, for *R* = 2, *M* = 3, *K* = 1, *Q* = 1, *h* = 0.3, *ε* = 0.2, *Gm* = 4, *α* = 1.

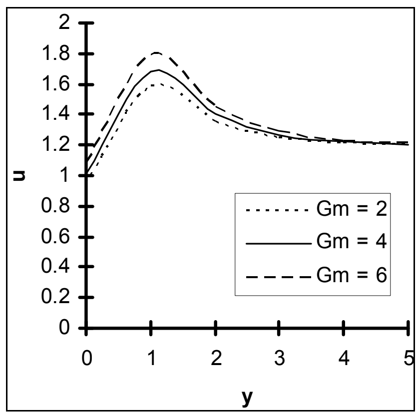

Figure 8. Velocity *u* versus *y*, under the effect of *Gm*, for *R* = 2, *M* = 3, *K* = 1, *Q* = 1, *h* = 0.3, *ε* = 0.2, *Gr* = 6, *α* = 1.

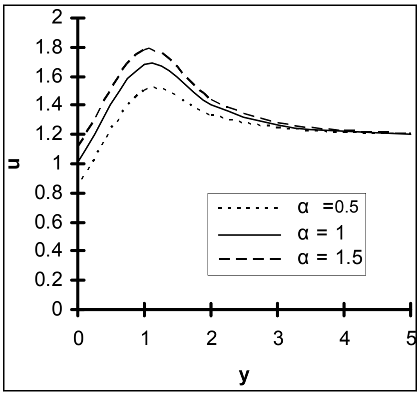

Figure 9. Velocity *u* versus *y*, under the effect of *α*, for *R* = 2, *M* = 3, *K* = 1, *Q* = 1, *h* = 0.3, *ε* = 0.2, *Gr* = 6, *Gm* = 4.

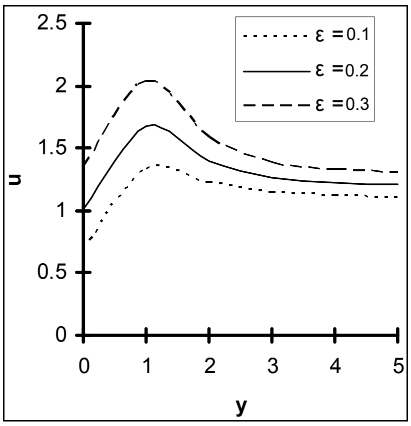

Figure 10. Velocity *u* versus *y*, under the effect of *ε*, for *R* = 2, *M* = 3, *K* = 1, *Q* = 1, *h* = 0.3, *Gm* = 4, *Gr* = 6, *α* = 1.

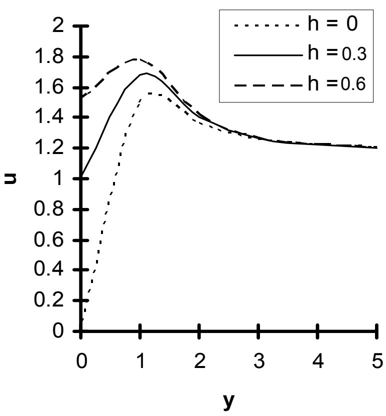

Figure 11. Velocity *u* versus *y*, under the effect of *h*, for *R* = 2, *M* = 3, *K* = 1, *Q* = 1, *ε* = 0.2, *Gm* = 4, *Sc* = 0.6, *Gr* = 6, *α* = 1.

The effects of Radiation parameter *R*, heat source parameter *Q* and ε on temperature field against *y* are displayed in **Figures 12-14**.

It is observed from **Figure 12** that the temperature θ decreases as the radiation parameter *R* increases. This result qualitatively agrees with expectation, since the effect of radiation is to decrease the rate of energy transport to the fluid, thereby decreasing the temperature of the fluid.

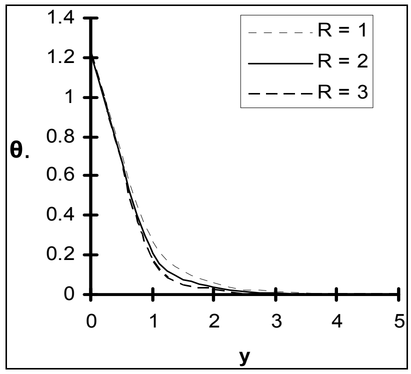

Figure 12. Temperature for different radiation parameter *R* with *Q* = 1, *ε* = 0.2.

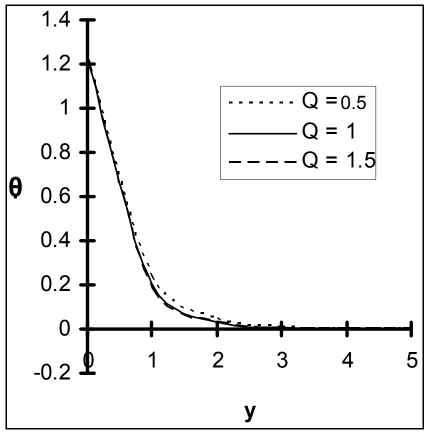

Figure 13. Temperature for different heat source parameters *Q* with *R* = 2, *ε* = 0.2.

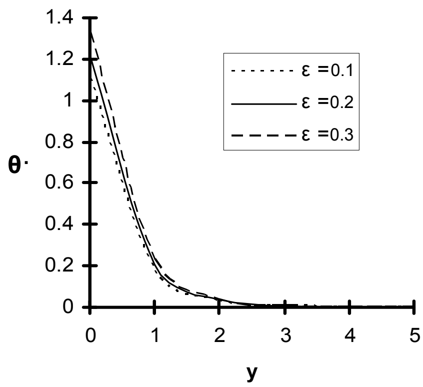

Figure 14. Temperature for different radiation parameter *ε* with *Q* = 1, *R* = 2.

It is seen from **Figure 13** that increases in the heat source parameter decreases the temperature profile.

Moreover from **Figure 14** it reveals that the temperature increases as ε increases.

Figures 15-17 exhibit the variation of species concen-

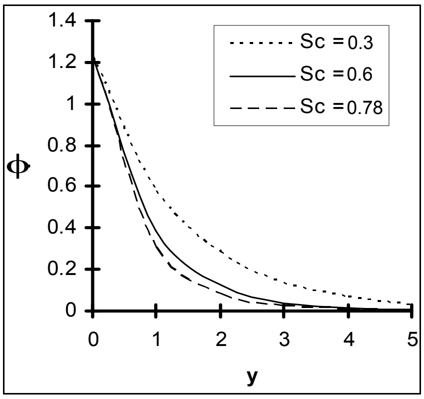

Figure 15. Concentration profile for different *Sc* with *Q* = 1, *ε* = 0.2, *K* = 1.

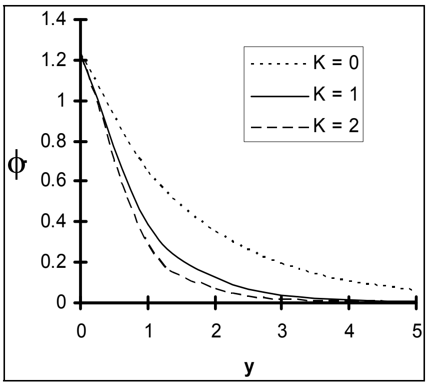

Figure 16. Concentration profile for different *K* with *Q* = 1, *ε* = 0.2, *Sc* = 0.6.

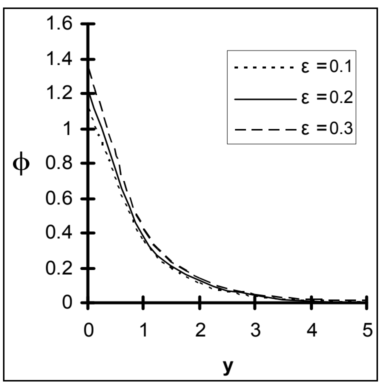

Figure 17. Concentration profile for different with *ε*, *Q* = 1, *K* = 1, *Sc* = 0.6.

tration against spanwise co-ordinate y under the influence of chemical reaction parameter K, Schmidt number Sc and ε. It is seen from **Figure 15** that the concentration level of the fluid drops due to increasing Schmidt number indicating the fact that the mass diffusivity raises the concentration level steadily. Further it is observed from **Figures 15** and **16** that concentration falls under the effect of chemical reaction parameter K whereas it rises due to the effect of ε.

The numerical values for skin-friction, Nusselt number and Sherwood number are computed for various values of the parameters M, Sc, K, α, Q, h, Gr, R and Gm. These results are presented in table 1. It is seen from this table that the effect of increasing values of M, Sc and K is to decrease skin-friction coefficient whereas increasing values α, Q, Gr, Gm, h and R increases skin-friction coefficient.

It is observed from the **Table 1** that there is no effect of M, Sc, K, α, h, Gr and Gm is seen on Nusselt number. But Nusselt number decreases with increase in Q and R.

Similarly, no effect of M, R, α, h, Gr and Gm is seen on Sherwood number whereas it is decrease with the increasing values of Sc and K respectively.

8. Conclusions

Our investigation of the problem setup leads to the following conclusions:

- The fluid velocity decreases as the existence of the magnetic field parameter becomes stronger.
- The fluid velocity is decelerated in the region adjacent to the plate, due to the effects of Schmidt number as well as chemical reaction.
- The fluid velocity is accelerated under the effects of thermal radiation, Grashof number for heat and mass transfer, heat source parameter, ε, rear fraction parameter h and porosity of the medium.
- There is a steady drop in temperature for high radiation and chemical reaction.
- The mass diffusivity raises the concentration level steadily, *i.e.*, the concentration level of the fluid, falls due to increasing Schmidt number.
- Increase in chemical reaction decreases the temperature whereas temperature increases as ε increases.
- The viscous drag at the plate in the direction of the buoyancy force may be successfully inhibited on application of strong magnetic field in operation.
- An increase in Grashof number for heat and mass transfer, thermal radiation, heat source parameter, rear fraction, ε and porosity of the medium results in a growth in the drag force and it falls under the effects of chemical reaction and Schmidt number.
- The rate of heat transfer (from the plate to the fluid) decreases due to the effects of thermal radiation and heat source parameter.

The mass flux from the plate to the fluid is reduced under the influence of Schmidt number and chemical reaction.

9. Acknowledgements

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Table 1. Skin friction, Nusselt number and Sherwood number for various values of *M*, *R*, *Gr*, *Gm*, *Sc*, *K*, *α* with Pr = 0.7, *n* = 0.1, *A* = 1, *ε* = 0.2.

Sc	*M*	*K*	*α*	*Q*	*h*	*Gr*	*R*	*Gm*	Cf	Nu	Sh
	3								3.194506	2.358945	1.394154
0.6	5	1	1	1	0.3	6	2	4	2.679543	2.358945	1.394154
	7								2.389400	2.358945	1.394154
0.3									3.6053425	2.358945	0.885951
0.6	3	1	1	1	0.3	6	2	4	3.1945066	2.358945	1.394154
0.78									2.9820064	2.358945	1.670695
		0							7.9651191	2.358945	0.755311
0.6	3	1		1	0.3	6	2	4	3.1945066	2.358945	1.394154
		2							0.5904764	2.358945	1.766132
			0.5						2.6795643	2.358945	1.394154
0.6	3	1	1	1	0.3	6	2	4	3.1945066	2.358945	1.394154
			10.5						3.4687398	2.358945	1.394154
				0.5					2.1014135	2.2108416	1.394154
0.6	3	1	1	1	0.3	6	2	4	3.1945066	2.3589459	1.394154
				10.5					4.6123157	2.4960342	1.394154
						2			2.2527005	2.358945	1.394154
0.6	3	1	1	1	0.3	4	2	4	2.7236035	2.358945	1.394154
						6			3.1945066	2.358945	1.394154
								2	2.9747134	2.358945	1.394154
0.6	3	1	1	1	0.3	6	2	4	3.1945066	2.358945	1.394154
								6	3.4142988	2.358945	1.394154
					0				4.772427	2.358945	−1.394154
0.6	3	1	1	1	0.3	6	2	4	3.194506	2.358945	−1.394154
					0.6				2.400774	2.358945	−1.394154

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