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QCD as High Energy Limit of the Scalar Strong Interaction Hadron Theory

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ABSTRACT

This paper is an extension of the book of reference [1] below. QCD Lagrangian is derived from the same equations of motion for quarks used to construct the equations of motion for mesons and baryons in the scalar strong interaction hadron theory that accounts for many basic low energy data not covered by QCD. At high energies, the energetic quarks in a hadron can be far from each other and approximately free. Each quark is associated with a vector in an internal space characterizing its mass and charge. These spaces are interchangeable and provide a new symmetry equivalent to color symmetry in QCD. A quark in a meson has two "colors" and in a baryon three "colors"; the β function of QCD is 61%-92% greater in high energy interactions leading to baryons than that to mesons. This function enters the measurable running coupling constant and this prediction is *testable* against experiment. QCD, successful at high energies, is thus reconciled with the scalar strong interaction hadron theory and both complement each other.

Keywords: QCD at High Energies; Scalar Strong Interaction; Internal Symmetry

1. Introduction

The scalar strong interaction hadron theory [1,2] can account, to a limited extent of varying accuracy, low energy hadronic phenomena of basic importance. The high energy end has been left out. Quantum Chromodynamics (QCD) on the other hand, has proven to be successful at high energies but can basically not account for low energy phenomena.

The situation may be summarized in **Table 1**, which includes the established QED as reference.

In the following, references of the form (x, y, z), Section $x.y$, §$x.y.z$, pxyz, and Table x. y refer to those in [1].

QCD is a quantum field theory for "colored" quarks based on the QCD Lagrangian [3 Reviews.../Standard Model.../Quantum Chromodynamics, Equations (9.1, 2)]. The interquark force is of "color" vectorial nature. However, the equations of motion at the quantum mechanical level obtained from the QCD Lagrangian are in terms of unobservable "colored" quarks and are of no use at low energies. This theory has proven to be successful at high energies.

The equations of motion for ground state mesons (2.4.2) and for baryons (9.3.11) form the basis of the present scalar strong interaction hadron theory. There is no quark wave function in these equations and the interquark force is of scalar nature. These equations and the corresponding Lagrangians can be converted into each other. The theory remains largely at the quantum mechanical level and, as mentioned in §6.4.4, has not been quantized.

The wave functions for mesons in (2.4.2) and for baryons in (9.3.11) have been solved for hadrons at rest. In motion, only dimensional estimates for pseudoscalar meson are given in Section 3.5. Feyman propagator for these hadrons is thus not known and Feynman diagrams in an eventual quantized version cannot be evaluated. More basically, the free hadron wave functions in (2.4.2) and (9.3.11) contain via (3.2.3a) in addition to the laboratory coordinate X, also the relative coordinate x as well as the internal coordinate z and u. The latter ones have no correspondence in classical mechanics and the usual transition rule to quantum mechanics is insufficient. This agrees with the known fact that nonlocal thoeries cannot be quantized.

2. Genaralized Equations of Motion for Quarks

The starting point of the scalar strong interaction meson theory [1,2] is a quark A at space-time point x_I with flavor p interacting scalarly with an antiquark B at point x_{II} having antiflavor r described by

Table 1. Key ingredients in QED, QCD and SSI (scalar strong interaction hadron theory) for comparison.

Scalar Strong Int-SSI	QCD	QED	
Nonlocal theory Cannot be quantized	Loop diagrams Renormalization diff. gauges Asymtotic freedom-1st order nonAbelian self coupling	Loop diagrams Renormalization	QUANTUM FIELD THEORY
At high energies, the quarks are nearly free. The different internal spaces for quarks in SSI provide a new degree of freedom similar to color in QCD, which can largely be taken over (this paper)	Feynman Rules: (FR) quark ≈ gluon ≈ Ghost-nonAbelian self coupling	Feynman Rules: (FR) lepton photon	
Predictive power: Similar to QCD's	Predictive power: Good	Predictive power: Highly accurate	Higher energies, Higher orders in coupling constant
$A^1_{\mu\nu} = \partial_\mu A^1_\nu - \partial_\nu A^1_\mu + q\varepsilon^{lmn} A^m_\mu A^n_\nu$ A^1_μ = like "gluon"	$F^1_{\mu\nu} = \partial_\mu V^1_\nu - \partial_\nu V^1_\mu + g f^{lmn} V^m_\mu V^n_\nu$ V^1_μ = gluon	$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ A_μ = photon	QUANTUM MECHANICS
SSI Lagrangians ↕ Meson Equation of motion (akin to $\lambda\phi^4$ theory) Baryon Equation of motion	QCD Lagrangian (↑FR) does not yield useful Equation of motion	QED Lagrangian ↕ Dirac Equation (↑FR) Maxwell Equation (↑FR)	
Predictive power: Limited but of basic nature, Ok presently	Predictive power: Nearly none	Predictive power: Firmly established	Low energies, 0th, 1st order in coupling constant

$$\left[-i\gamma^\mu\left(\frac{\partial}{\partial x_I^\mu} + iq_{op}(z)A_\mu(x_I)\right) + m_{1op}(z) - V_{SB}(x_I)\right]\psi_A(x_I)z_I^p = 0 \quad (1)$$

$$\left[-i\gamma^\mu\left(\frac{\partial}{\partial x_{II}^\mu} + iq_{op}(z)A_\mu(x_{II})\right) + m_{1op}(z) - V_{SA}(x_{II})\right]\psi_B(x_{II})z_{IIr} = 0 \quad (2)$$

In (1) and (2), following the notations of (A1-A5), ψ is usual Dirac bispinor and $V_{SB}(x_I)$ is the scalar potential (2.1.2) emanating from B acting on A and vice versa (2.1.4) for $V_{SA}(x_{II})$. z^p are p complex variables originally providing a point field for implementing SU(p) transformations [4] and $z_p = (z^p)^*$ in (2.3.4). They [2, Section5] acquire here a more physical role as eigenfunctions of the mass operator [2 (9.6a)], (2.3.26)

$$m_{1op}(z) = m_q\left(z_I^q\frac{\partial}{\partial z_I^q} + z_{Iq}\frac{\partial}{\partial z_{Iq}} + z_{II}^q\frac{\partial}{\partial z_{II}^q} + z_{IIq}\frac{\partial}{\partial z_{IIq}}\right) \quad (3)$$

as well as the charge operator

$$q_{op}(z) = q_r\left(z_I^r\frac{\partial}{\partial z_I^r} - z_{Ir}\frac{\partial}{\partial z_{Ir}} + z_{II}^r\frac{\partial}{\partial z_{II}^r} + z_{IIr}\frac{\partial}{\partial z_{IIr}}\right) = -q_{op}^*(z) \quad (4)$$

generalized from (2.3.14). Repeated indices are summed over. Acting upon z^p in (1), (3) and (4) produce the quark mass m_p and charge q_p, respectively, as eigenvalues. The A_μ's are the associated electromagnetic fields. $\psi_A(x_I)z_I^p$ is regarded as the total wave function for quark A and $\psi_B(x_{II})z_{IIr}$ that of the antiquark B. z_I and z_{II} refer to different internal or flavor spaces in Section 2.1 and are as distinct as x_{II} and x_I. These z's, like the relative coordinates $x = x_{II} - x_I$, are "hidden varaiables" in §2.3.5, p. 327. The above equations are taken as hypothetical, as free quark has not been observed, but are used to construct equations of motion for meson. For this purpose, (1) was originally written in van der Waerden's two spinor form (2.1.1a, 3a) via (2.3.11).

3. Quark Confinement in Meson

The so-constructed equations of motion have been solved for mesons at rest to account for a number of basic problems in the book [1]. After its publication, CP violations in neutral kaon decays have been substantially clarified [5].

The rest frame pseudoscalar meson is confined by the interquark potential (3.2.8a, 19, 20) via the genealization (2.2.3),

$$V_{SA}(x_{II})V_{SB}(x_I) \to \Phi_m(x_I, x_{II}) \to \Phi_m(\underline{x}) = -\Phi_c(\underline{x}) + \frac{d_m}{r} + d_{m0} \quad (5)$$

$$\Phi_c(r \to \infty) = \frac{g_s^4}{2} r \int_0^\infty \mathrm{d}r' r'^2 \left|\psi_0(r')\right|^2 = \beta_{m0} r \quad (6)$$

where $r = \left|\underline{x}_{II} - \underline{x}_I\right| = \left|\underline{x}\right|$ is the interquark distance, d_m = 0.864 Gev in (5.2.3), d_{m0} = 0.24455 Gev^2 in Table 5.2, g_s^2 is the scalar strong quark-quark coupling in (2.2.1, 4), and $\Phi_c(\underline{x})$ the nonlinear (in ψ) confinement which provides linear confinement at large r. The meson wave fnnction $\psi_0(r)$ is formed from generalization of the product of $\psi_A(x_I)$ in (1) and $\psi_B(x_{II})$ in (2) according to (2.2.1), satisfies (3.2.10) and is given by (4.3.2)

$$\psi_0(r)=\sqrt{\frac{d_m^3}{8\pi\Omega}}\exp(-d_m r/2) \tag{7}$$

where Ω is a large normalization volume in the laboratory space $\underline{X}=(\underline{x}_{II}+\underline{x}_I)/2$. For a free meson, $\Omega\to\infty$ and (7) and (6) $\to 0$ and the quarks are confined by d_m/r in (5) only. From (7), the size of the meson is about 2 fm in (4.7.3); the quarks are tightly bound.

If the same meson is moving, Section 3.5 shows that its wave function is of the plane wave form $\exp(-iEX^0+i\underline{KX})(\psi_0(\underline{x}),\underline{\psi}(\underline{x}))$, where E is the energy and $\underline{K}$ the momentum, and is a four vector in relative space. ψ_0 is the time and large component and the vector part $\underline{\psi}$ makes up the spatial and small components. Equations governing these components have not been solved. In addition, the free meson wave function is seen to be nonlocal and can therefore not be quantized.

For small K, however, approximate forms of ψ_0 and $\underline{\psi}$ have been estimated using dimensional approximations and $\underline{\psi}\propto K$, shown in §3.5.3, 4.

As is discussed in Section 4.5, the above plane wave form is distorted and the corresponding quantity playing the role of Ω no longer $\to\infty$ when the meson is interacting with another particle. Hence, the wave function becomes finite and the linear confinment (6) is called into action in (5); the quarks are always confined.

At higher energies, the quarks also become energetic and the interquark distance r is expected to be large so that the confining term d_m/r in (5) becomes small. In this case, quarks are no longer tightly bound and the nonlinear confinement Φ_c in (6) can still be weak over a large range of r for large enough Ω type of volumes. In this r range, Φ_m in (5) is small and quarks may be considered as approximately *free* so that (1) and (2) with V_{SB}, $V_{SA}\to 0$ via the left member of (5) are applicable.

4. Internal Space Symmetry and Gauge Transformation

In Section 12.8, on p. 272 at the end of the book, it was pointed out that "The internal degrees of freedom in form of the three internal coordinates z_I, z_{II} and z_{III} of Section 9.3 can play some of the roles of the three colors in QCD...". This observation will now be pursued and developed. The two internal spaces z_I and z_{II} in (1) and (2) are not observables and are interchangeable and thus provide a new symmetry analogous to color symmetry in QCD. Noting this, the total quark wave function $\psi_A(x_I)z_I^p$ in (1) is generalized to a column matrix with two elements, one with z_I the other z_{II}. Consequently, the U(1) gauge field A_μ must also be generalized to an SU(2) gauge field. The so-generalized (1) and (2) read

$$\left[-i\gamma^\mu\left(\frac{\partial}{\partial x_I^\mu}+iq_{op}(z)\overline{\underline{\sigma}\underline{A}_\mu}(x_I)\right)+m_{1op}(z)\right]\Psi_A^p=0,\quad \Psi_A^p=\begin{pmatrix}\psi_A(x_I)z_I^p\\ \psi_A(x_I)z_{II}^p\end{pmatrix} \tag{8}$$

$$\left[-i\gamma^\mu\left(\frac{\partial}{\partial x_{II}^\mu}-iq_{op}(z)\left\{\overline{\underline{\sigma}\underline{A}_\mu}(x_I)\right\}_{I\leftrightarrow II}\right)+m_{1op}(z)\right]\Psi_{Br}=0,\quad \Psi_{Br}=\begin{pmatrix}\psi_B(x_{II})z_{IIr}\\ \psi_B(x_{II})z_{Ir}\end{pmatrix} \tag{9}$$

observing the definitions

$$A=\begin{pmatrix}A_{11}&A_{12}\\A_{21}&A_{22}\end{pmatrix},\quad \overline{A}=\begin{pmatrix}A_{11}z_I^p\partial/z_I^p & A_{12}z_I^p\partial/z_{II}^p\\ A_{21}z_{II}^p\partial/z_I^p & A_{22}z_{II}^p\partial/z_{II}^p\end{pmatrix} \tag{10}$$

The stepping operators of the from $z_I\partial/\partial z_{II}$ has been introduced earlier [6] in connection with the $W^\pm$ bosons. As the $\overline{A}$ type of matices in (8) and (9) operates on z_I and z_{II},

$$\overline{A}^2=\begin{pmatrix}A_{11}z_I^p\partial/z_I^p & A_{12}z_I^p\partial/z_{II}^p\\ A_{21}z_{II}^p\partial/z_I^p & A_{22}z_{II}^p\partial/z_{II}^p\end{pmatrix}^2=A\overline{A}=\overline{A^2},\quad \overline{A}^n=A^{n-1}\overline{A}=\overline{A^n} \tag{11}$$

because the second order deivatives of the form $\partial^2/\partial z_I^2$, $\partial^2/\partial z_I\partial z_{II}$ in (11) vanish. Therefore, all higher order deivatives in z_I and z_{II} also drop out. Also, $z_I\partial/\partial z_I$ and $z_{II}\partial/\partial z_{II}$ in $\overline{A}$ may be dropped without affecting the results.

Analogous to the usual field tensor [7 (18.8)], define the gauge field tensor

$$\overline{\underline{\sigma}\underline{A}_{\mu\nu}}(x_I)=\frac{\partial}{\partial x_I^\mu}\overline{\underline{\sigma}\underline{A}_\nu}(x_I)-\frac{\partial}{\partial x_I^\nu}\overline{\underline{\sigma}\underline{A}_\mu}(x_I)+iq_{op}(z)\left[\overline{\underline{\sigma}\underline{A}_\mu}(x_I),\overline{\underline{\sigma}\underline{A}_\nu}(x_I)\right] \tag{12}$$

Let

$$U_2(x_I)=\exp(i\underline{\sigma}\underline{\vartheta}(x_I)),\quad \overline{U_2}(x_I)=\exp\left(i\overline{\underline{\sigma}\underline{\vartheta}}(x_I)\right) \tag{13}$$

Under the SU(2) gauge transformations

$$\Psi_A^p\to\Psi_A'^p=\begin{pmatrix}\psi_A(x_I)'z_I^p\\ \psi_A(x_I)'z_{II}^p\end{pmatrix}=\overline{U_2}(x_I)\Psi_A^p \tag{14a}$$

$$\overline{\underline{\sigma}\underline{A}_\mu}(x_I)\to\overline{\underline{\sigma}\underline{A}_\mu}(x_I)'=\overline{U_2}(x_I)\overline{\underline{\sigma}\underline{A}_\mu}(x_I)\overline{U_2^{-1}}(x_I)-\frac{i}{q_{op}(z)}\overline{U_2}(x_I)\frac{\partial}{\partial x_I^\mu}\overline{U_2^{-1}}(x_I) \tag{14b}$$

(8) and (12) are invariant; these invariances are the same as the conventional ones without the z's in (13) and in (8) and (12), noting that (3) and (4) are invariant under $z_I\leftrightarrow$

z_{II} and can respectively be replaced by m_p and q_p. $\overline{\underline{A}_\mu}$ corresponds to the SU(2) gluons in QCD. There are only two degrees of freedom represented by z_I and z_{II} corresponding to two colors for quarks in a meson.

5. Derivation of QCD Lagrangian

The quark Lagrangian density L_Q is obtained by multiplying (8) from the left by $\overline{\Psi}_{Ap}$ and integrating over the angles in z_I^p and z_{II}^p. Limiting ourselves to light quarks u, d and s corresponding respectively to flavor p = 1, 2 and 3, which have about the same quark mass m_p in Table 5.2, one obtains

$$\begin{aligned}
&\overline{\Psi}_{Ap} = \left(\overline{\psi}_A(x_I) z_{Ip}, \overline{\psi}_A(x_I) z_{IIp}\right) \\
&L_Q = \int \mathrm{d}v_{z_I p} \mathrm{d}v_{z_{II} p} \\
&\cdot\left\{\overline{\Psi}_{Ap}\left[-i\gamma^\mu\left(\frac{\partial}{\partial x_I^\mu} + iq_p \overline{\underline{\sigma}\underline{A}_\mu}(x_I)\right) + m_p\right]\Psi_A^p\right\} \\
&= C_N\left(\overline{\psi}_A(x_I), \overline{\psi}_A(x_I)\right) \\
&\cdot\left[-i\gamma^\mu\left(\frac{\partial}{\partial x_I^\mu} + iq_p \underline{\sigma}\underline{A}_\mu(x_I)\right) + m_p\right]\begin{pmatrix}\psi_A(x_I) \\ \psi_A(x_I)\end{pmatrix}, \\
&C_N = N_p^2 \frac{\pi^6}{3}
\end{aligned} \tag{15}$$

Here, (2.3.9, 6b) have been employed and N_p is a constant in (2.3.9). The quark charge q_p becomes here an unrenormalized coupling constant.

The SU(2) "gluon" Lagrangian density L_G is similarly obtained

$$\begin{aligned}
&L_G \\
&= \int \mathrm{d}v_{z_I p} \mathrm{d}v_{z_{II} p}\left\{z_{Kp}\left(-\frac{1}{2}\right)\mathrm{trace}\left[\overline{\underline{\sigma}\underline{A}_{\mu\nu}}(x_I)\overline{\underline{\sigma}\underline{A}_{\mu\nu}}(x_I)\right]z_K^p\right\} \\
&= C_N\left(-\frac{1}{2}\right)\mathrm{trace}\left[\underline{\sigma}\underline{A}_{\mu\nu}(x_I)\underline{\sigma}\underline{A}_{\mu\nu}(x_I)\right]
\end{aligned} \tag{16}$$

where K runs from I to II and (4) has been used in (12). Note that quarks having different "colors" or z_K's have the same space time wave function in (15), as it should. Adding (15) and (16) yields the total "two color" Lagragian density for p = 1, 2 or 3.

$$L_Q + L_G = C_N L_{QCD2} \tag{17}$$

The conventional QCD Lagragian density L_{QCD2} [7, 6 line below (18.6)] for two colors is thus recovered.

The antiqauark Equation (9) can be treated analogously leading to the same results.

Baryon consists of three quarks and makes use of three of (1) associated with three internal spaces z_I, z_{II} and z_{III} in (9.3.1) or three internal degrees of fredom or "colors". At rest, two quarks merge to form a diquark via (9.2.12) and (9.3.4) and there are only two "colors". The quark-diquark confinement potential (10.1.6a, 8) differs from (5) in that $d_{b2}r^2$ there provides confinement independent of the nonlinear confinement $\Phi_{bc}(\underline{x})$ there; the quark and the diquark are always confined irrespective whether there is another particle nearby.

When interacting with another particle at higher energies, the quarks also become energetic and the diquark is expected to break up so that there are three interquark distances instead of r. The corresponding form of $\Phi_b(\underline{x})$ is unknown but these three distances, like r in high energy mesons, are expected to be large so that the three quarks may be regarded as approximately *free* in some ranges of the three distances. Again, the nonlinear confinement corresponding to $\Phi_{bc}(\underline{x})$, which provides cubic form of confinement at large r in the quark-diquark configuration (10.2.5b), renders the quarks to be confined. In this case, the above treatment of quarks in meson can straightforwardly be extended to apply to quarks in baryons. The Pauli matrices $\underline{\sigma}$ above is replaced by the Gell-Mann matrices $\underline{\lambda}$ and $\underline{\theta}$ has now eight components. The conventional L_{QCD3} [7, 6 line below (18.6)] for three colors is recovered.

The whole development of L_{QCD2} and L_{QCD3}, including quantization, choice of gauges, renormalization, Feynman rules, asymtotic freedom, etc can be taken over. The QCD beta function $\propto 11C_2 – 2n_f$ [7 (18.146)] where C_2 is 3 for baryon and 2 for meson and the number of flavors n_f is 2-5. This function is thus 61%-92% greater in high energy interactions leading to baryons than that to mesons. As this function enters the measurable running coupling constant, this prediction should be *testable* against experiment.

This effect appears to be effective at large quark separations r for confinement which is taken over by the scalar strong interaction confinement d_m/r in (5) and d_b/r in (10.1.6a) for small r.

In this way, QCD is reconciled with the scalar strong interaction hadron theory and complement each other; the former holds at high energies while the latter accounts for, so far to limited extent of basic nature, data at the low energy end of elementary particle theory. The intermediate energy range remains not covered.

6. On Exact and Broken Symmetry

The known symmetries, C, P, T, and electroweak SU(2) gauge, are all broken. Because they reside in space-time and isospace, the degrees of symmetry breaking can be measured. The SU(2) internal or "color" gauge symmetry here is however *exact* and cannot be broken. The associated gauge transformation act on two abstract, internal spaces z_I and z_{II}, which have been created artifcially [2 Section 5], Section 2.3 to accommodate different quark flavor vectors. These two spaces are identical in structure and there is no quantity in the formalism that distinguish them from each other, *i.e.*, can break this $z_I \leftrightarrow z_{II}$ symmetry. Even if some such quantity will appear later, it cannot be measured to determine the degree of symmetry breaking because z_I and z_{II} are unobservable "hidden" variables mentioned below (4). Such a symmetry breaking would appear as nonexisting. As the "gluons" in (8, 9) depend upon the unobservable quark coordinates x_I and x_{II} as well as z_I and z_{II}, they can also not be observed.

7. Conclusion

QCD has hitherto been based upon the assumption that a quark has three colors. This has its root in the Pauli exclusion principle. This principle has been confirmed for freely observable fermions; but quarks are not freely observable and hence do not have to obey this principle and the above assumptions become *ad-hoc*. In the scalar strong interaction theory [1], the quark wave function is generalized to include an internal part in Section 2.3 characterized by z_I, and z_{II} in (1) and (2), which is needed to specify its mass and charge via its flavor. It is shown here that this internal degree freedom plays an analogous role as color does in QCD and "justifies" the assumption of colored quarks, with the difference that quarks in mesons have only two "colors". This difference is experimentally testable. QCD is thus reconciled with the scalar strong interaction hadron theory.

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