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Control Parameters of Magnitude—Seismic Moment Correlation for the Crustal Earthquakes

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ABSTRACT

In connection with conversion from energy class K_R ($K_R = \log_{10} E_R$, where E_R—seismic energy, J) to the universal magnitude estimation of the Tien Shan crustal earthquakes the development of the self-coordinated correlation of the magnitudes (m_b, M_L, M_s) and K_R with the seismic moment M_0 as the base scale became necessary. To this purpose, the first attempt to develop functional correlations in the magnitude—seismic moment system subject to the previous studies has been done. It is assumed that in the expression $M(m_b, M_L, M_s) = k_i + z_i \log_{10} M_0$, the coefficients k_i and z_i are controlled by the parameters of ratio $\log t_0 = a_t + b_t \log_{10} M_0$ (where $t_b = f_0^{-1}$; f_0—corner frequency, Brune, 1970, 1971; M_0, N·m). According to the new theoretical predictions common functional correlation of the advanced magnitudes M_m ($m_{bm} = m_b$, $M_{Lm} = M_L$, $M_{Sm} = M_S$) from $\log_{10} M_0$, $\log_{10} t_0$ and the elastic properties (C_i) can be presented as $M_m = d_i \log_{10} M_0 - 2\log_{10} t_0 + C_i$, where $z_i = d_i - 2b_t$, and $k_i = C_i - 2a_t$, for the averaged elastic properties of the Earth's crust for the m_{bm} the coefficients $C_i = -11.30$ and $d_i = 1.0$, for M_{Lm}: $C_i = -14.12$, $d_i = 7/6$; for M_{Sm}: $C_i = -16.95$ and $d_i = 4/3$. For the Tien Shan earthquakes (1960-2012 years) it was obtained that $\log_{10} t_0 = 0.22\log_{10} M_0 - 3.45$, and on the basis of the above expressions we received that $M_{Sm} = 1.59 m_{bm} - 3.06$. According to the instrumental data the correlation $M_s = 1.57 m_b - 3.05$ was determined. Some other examples of comparison of the calculated and observed magnitude—seismic moment ratios for earthquakes of California, the Kuril Islands, Japan, Sumatra and South America are presented.

Keywords: Magnitude; Seismic Moment; Energy Class; Earthquakes; Frequency

1. Introduction

In world practice, seismological research in assessing the scale of earthquakes magnitude scale of Gutenberg and Richter [1-3] is fundamental. In the countries of the former Soviet Union has been used scale independent energy class K_R, defined as the logarithm of the seismic energy E_R, highlighted by an earthquake, measured in joules ($K_R = \log_{10} E_R$, [4-6]).

For crustal earthquakes Tien Shan when considering the transition to magnitude scale was necessary to develop a self-consistent system of quantitative relationships that justify numerous empirical relationships body-wave magnitude m_b, local magnitude on surface waves M_L, surface wave magnitude for M_S and K_R from seismic moment M_0 (N·m), as the reference scale. In connection with the above purpose is to study the quantitative relationships m_b, M_L, M_S and energy of seismic radiation E_S c M_0 based on the following findings:

1) proportional magnitudes and the maximum amplitude of seismic vibrations [1-3];

2) the statistical dependencies of the average magnitude of displacement along the fault u [7-12] and u functional relationship with the seismic moment, the shear modulus μ and the gap area S [13-14];

3) functional relationship corner period $t_в = f_0^{-1}$ s with M_0, the source radius r_0, speed S—wave v_S and static stress drop $\Delta\sigma$ [15,16], as well as the similarity of the angular frequency f_0 with a fundamental frequency of the acoustic Debye [17] f_D, depending on the amount of source and the elastic properties of the geophysical medium [18].

Our further quantitative construction is based on the following empirical relationship Gutenberg and Richter [3,12]:

$$\log_{10} E_{GR} = K_{GR} = 4.8 + 1.5 M_S \quad (1)$$

$$M_S = 1.59 m_b - 3.97 \quad (2)$$

$$\log_{10} t_0 = 0.32 M_L - 1.4 \quad (3)$$

where E_{GR}—seismic energy according to Getenberg and Richter, J; t_0—fluctuations with a maximum duration of vibration speed A/T in the near field (A—amplitude, T—period), s.

Use the following generalization of Soviet seismologists, which were introduced scale energy class K_R [5], the magnitude of surface waves M_{LH} (IC device) and body waves m_{PV} on device SCM [4,9]:

$$\log_{10} E_R = K_R = 4.0 + 1.8 M_{LH} \quad (4)$$

$$\log_{10} t_m = 0.35 M_{LH} - 1.4 \quad (5)$$

$$m_b = 5.53 + 0.45 (K_R - 14) \quad (6)$$

$$m_{PV} = 0.35 \log_{10} M_0 - 2.75 \quad (7)$$

where E_R—seismic energy according to [5], in J; K_R = $\log_{10} E_R$; t_m—increase the maximum duration of the seismic intensity in the near field, in sec.

The basis of the theoretical constructs are the following functional relations [10,13,15,16,19]:

$$\begin{aligned} M_0 &= \mu \cdot S \cdot u = (16/7) \Delta\sigma \cdot r_0^3 \\ &= (16/7)(2.35/2\pi)^3 \Delta\sigma \cdot V_S^3 \cdot t_b \end{aligned} \quad (8)$$

$$E_{SK} = (\Delta\sigma / 2\mu) M_0 \quad (9)$$

$$r_0 = (2.34/2\pi) \cdot v_S \cdot t_b \quad (10)$$

$$M_W = (2/36) \log_{10} M_0 - 6.07 \quad (11)$$

where r_0—radius of the source, in м; $\Delta\sigma$—static seismic stress drop, in Pa; t_b—corner period, s; M_W—moment magnitude; (E_{SK}, in J; M_0, in N·m; u in m; v_S in m/s); for the constructions made $t_0 = t_b = t_m$.

Many generalizations proved that for a wide range of changes $\log_{10} M_0$ or M_W empirical correlations magnitude m_b, M_L and M_S from M_0 are non-linear, as in Equation (8), as a function of $M_0 \sim f(t_0^n)$ value of n varies from 3 to 6, and is increase $\Delta\sigma$ [7,12,20-24].

However, for individual intervals M_0 or M_W communication between magnitudes relationships and dependencies of the magnitude $\log_{10} M_0$ can be represented as linear relationships.

2. Justification Relations Magnitude—Seismic Moment

Based on the original definition of magnitude on Richter [25], under which the numerical value of the earthquake magnitude is proportional to the logarithm of the maximum oscillation decimal $в_m$, expressed in microns (10^{-6}м), it is assumed that an upgraded body-wave magnitude m_{bm} (equivalent m_b, m_{PV}) is (considering doubling $в_m$ on the ground at the focus):

$$m_{bm} = \log_{10} в_m + 6.3 = \log_{10} u + 6 \quad (12)$$

If $S = \pi r_0^2$ in (8) on the basis Equations (9) and (10) and Equation (12) value m_{bm} equal (M_0, N·m; $t_в$, s; μ, Pa; v_s, m/s):

$$m_{bm} = C_1 + \log_{10} M_0 - 2 \log_{10} t_0 \quad (13)$$

where $C_1 = \log_{10} \left[2\pi (2.34)^{-2} \cdot \mu^{-1} \cdot v_S^{-2} \right] + 6.3$, value C_1 determines the springiness of the geophysical environment at m_{bm}.

Based on generalizations Christensen [26,27] for the crust taken: average density
$\rho = 2830$ kg/m^3, $v_S = 3600$ m/s and
$\mu = \rho \cdot v_S^2 = 36.7$ GPa in what follows, these quantities ρ, v_s and μ taken as the standard.

When these elastic parameters of the geophysical medium expression Equation (13) is transformed to the following form:

$$\begin{aligned} m_{bm} &= \log_{10} M_0 - 2 \log_{10} t_0 - 11.30 \\ &= 1/3 \log_{10} M_0 + 2/3 \log_{10} \Delta\sigma - 4.80 \end{aligned} \quad (14)$$

Seismic energy radiation E_{SK} by Kanamori [19], based on Equations (8) and (9) and Equation (13) is:

$$\begin{aligned} \log_{10} E_{SK} &= K_{SK} = C_2 + 2 \log_{10} M_0 - 3 \log_{10} t_0 \\ &= C_2 - C_1 + 2 m_{bm} + \log_{10} t_0 \end{aligned} \quad (15)$$

where $C_2 = \log_{10} \left[7 \cdot \pi_3 \cdot 4^{-1} \cdot (2.34)^{-3} \cdot \mu^{-1} \cdot v_S^{-3} \right]$.

Taken for the elastic parameters and subject [19]. $E_{SK}/M_0 = \Delta\sigma/2\mu = 5 \times 10^{-5}$ obtain: $\Delta\sigma = 3.67$ MPa and 36.7 bar and the expression Equation (8) can be rewritten in a simple form $\log_{10} t_0 = 1/3 \log_{10} M_0 - 5.43$, then Equation (15) simplifies to:

$$\begin{aligned} K_{SK} &= 2 \log_{10} M_0 - 3 \log_{10} t_0 - 20.61 \\ &= 1.5 M_W + 4.8 = 3 m_{bm} - 3 \end{aligned} \quad (16)$$

On the basis of Equations (13)-(16), reflecting the functional relationship of E_{SK} from M_0, t_0, m_{bm} and μ at $E_{GR} = E_{SK}$ introduced upgraded the magnitude of surface waves M_{Sm} (equivalent of M_S, M_W), while maintaining that the formula Equation (1) Gutenberg and Richter [2,3], with Equation (9), Equations (15) and (16) will be:

$$\begin{aligned} M_{Sm} &= (4/3) \log_{10} M_0 - 2 \log_{10} t_0 + C_S \\ &= 2/3 (\log_{10} M_0 + \log_{10} \Delta\sigma) - 10.45 = (2/3) K_{SK} - 3.2 \end{aligned} \quad (17)$$

where $C_S = (2/3) C_2 - 3.2$.

Taken for ρ and v_S C_S value in Equation (17) is equal to $C_S = -16.95$, and for the special case of $\Delta\sigma = 3.67$ MPa = const and $E_{SK}/M_0 = 5 \times 10^{-5}$ equality: $M_{Sm} = M_W$.

We also introduce a modernized local magnitude on surface waves M_{Lm}—equivalent M_L [18,28], functionally interconnected with $\log_{10}M_0$, $\log t_0$, K_{SK}, m_{bm} and M_{Sm}:

$$M_{Lm} = 0.5\left(m_{bm} + M_{Sm}\right) = \left(7/6\right)\log_{10} M_0 - 2\log t_0 + C_L$$
$$= 0.5\log_{10} M_0 + 2/3\log_{10} \Delta\sigma - 7.62 \tag{18}$$

where C_L = 0.5 (C_1 + C_S): for standard values ρ and v_S value C_L is equal: C_L = –14.12.

Accepted values for ρ and v_S by Equation (8) and Equation (9) the following relationship:

$$\log_{10} \Delta\sigma = \log_{10} M_0 - 3\log_{10} t_0 - 9.74 \tag{19}$$

With the standard values ρ, v_S and $\Delta\sigma$ = 3.67 MPa, based on Equation (14) and Equation (17) we obtain the following theoretical relation:

$$m_{bm} = 2.60 + 0.5M_{Sm} \tag{20}$$

which is within the accuracy of the definitions of the same magnitude satisfactory empirical relation refined body wave magnitude $\hat{m}_b$ of M_W for large earthquakes [19,29] ($m_b \geq 6$):

$$\hat{m}_b = 2.70 + 0.53\, M_W \tag{21}$$

which were used $\hat{m}_b$ to calculate the true maximum oscillation amplitude A_g, taken from seismograms; $A_g \sim f\left(M_0^{0.35}\right)$.

Here it should be emphasized that at a constant value of $\Delta\sigma$ Equation (12) and Equation (14) the value of the maximum amplitude в_{m} is proportional to $\text{M}_0^{1/3}$ or $\text{в}_m \sim f\left(M_0^{0.33}\right)$, that closely coincides with $A_g \sim f\left(M_0^{0.35}\right)$ on [19,29].

In the sequel will be shown $m_{bm} \approx \hat{m}_b$.

Equation (20) agrees satisfactorily with other empirical relationship [9]

($m_{PV} = m_b$ + 0.18):

$$m_{PV} = 2.86 + 0.525M_W \tag{22}$$

The above quantitative ratios indicate that between modernized magnitudes

M_m (m_{bm}, M_{Lm}, M_{Sm}) and $\log_{10}M_0$ may exist linear functional relationship of the form:

$$M_m\left(m_{bm}, M_{Lm}, M_{Sm}\right) = k_i + z_i \log_{10} M_0 \tag{23}$$

in which the coefficients k_i and z_i at the control parameter a_t and в_t in the ratio:

$$\log_{10} t_0 = a_t + \text{в}_t \log_{10} M_0 \tag{24}$$

where $\Delta\sigma$ = const = 3.67 MPa в_t = 1/3 = const and a_t = –5.43, but for other cases в_t is not a constant.

In view of Equations (23) and (24) correlations Equations (14), (17) and Equation (18) for m_{bm}, M_{Sm} and M_{Lm} (standard values ρ and v_S) can be written as follows:

$$m_{bm} = \left(1 - 2\text{в}_t\right)\log_{10} M_0 - 2a_t - 11.30 \tag{25}$$

$$M_{Sm} = \left(4/3 - 2\text{в}_t\right)\log_{10} M_0 - 2a_t - 16.95 \tag{26}$$

$$M_{Lm} = \left(7/6 - 2\text{в}_t\right)\log_{10} M_0 - 2a_t - 14.12 \tag{27}$$

which provide a self-consistent system of semi empirical inter magnitude dependencies. For example, the dependence of $m_{\text{в}m}$ from M_{Sm} based on Equations (25) and (26) can be expressed as:

$$m_{bm} = \frac{1 - 2\text{в}_t}{4/3 - 2\text{в}_t}\left(M_{Sm} + 2a_t + 16.95\right) - 2a_t - 11.30 \tag{28}$$

which is в_t = 0.33 and a_t = −5.43 ransformed into simple formula Equation (20).

3. Discussion of Empirical and Theoretical Relations Magnitude—Seismic Moment

Local magnitude—seismic moment. Since the value of the local magnitude is directly related to the maximum oscillation amplitude of the surface waves and the first inter magnitude connections [2,3] have been developed for California earthquakes, relations M_L – $\log t_0 M_0$ consider according to Thatcher and Hanks [30] in this region ($2 \leq M_L \leq 6.8$).

For this region, the authors have taken ρ = 2700 kg/m^3 and v_S = 3200 m/s, and by (13) and (17) a constant values will be: C_1= −11.09, C_S = −16.5, C_L = −13.72. With known ρ, v_s, $\Delta\sigma/2\mu$ = 5 × 10^{-5} between $\log_{10}t_0$ and $\log_{10}M_0$ would expect the following relationship: $\log_{10}t_0 = \left(1/3\right)\log_{10}M_0 - 5.34$, but the instrumental data obtained (**Figure 1**, N—the number of data, r—correlation coefficient):

$$\log_{10} t_0 = 0.25\left(\pm 0.03\right)\log_{10} M_0 - 3.90\left(\pm 0.43\right) \tag{29}$$

i.e. in accordance with (19) with increasing values of $M_0\log_{10}\Delta\sigma$ increases:

$\log_{10} \Delta\sigma = 0.25\log_{10} M_0 + 2.12$. Therefore, for the considered data characteristic dependence $M_0 \sim f\left(t_0^4\right)$, said Nuttli [12] for mid-plate earthquakes.

If true theoretical Equations (13), (17) and (18), then

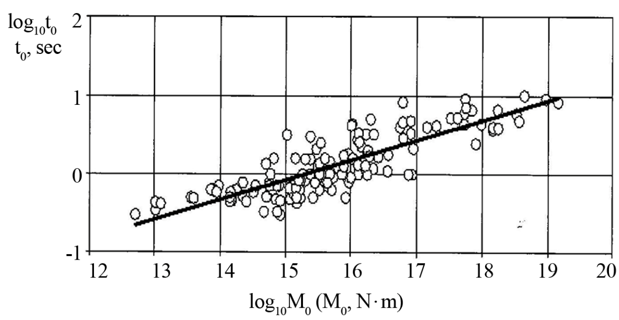

Figure 1. Correlation of $\log_{10}t_0$ from $\log_{10}M_0$ for Southern California earthquakes according to Thatcher and Hanks [30] ($\log_{10}t_0 = 0.25\left(\pm 0.03\right)\log_{10}M_0 - 3.90\left(\pm 0.43\right)$, N = 138, r = 0.84).

Equation (29) and the relationship between M_{Lm} and $\log_{10}t_0$ is given by:

$$\log_{10} t_0 = 0.37 M_{Lm} - 1.68 \quad (30)$$

which is in good agreement with the expression (3) Gutenberg and Richter [2] and Equation (5) Soviet seismologists [31] which allows to consider $t_0 = t_e = t_m$.

In Figure 2 shows the correlation $\log_{10}t_0$ and M_L according to Thatcher [30], which also shows the relationship Equation (3) and Equation (30). The presented data show that the semi-empirical formula Equation (30) is in good agreement with generalizations instrumental data (**Figure 2**). It should also be noted that the $M_L = M_{Lm}$ based on Equation (3) Gutenberg and Richter [2], and Equation (18) can be obtained

$$\log_{10} t_0 = 0.23 \log_{10} M_0 - 3.53 \quad (31)$$

which is in satisfactory agreement with the expression (29).

In **Figure 3** in the range of $0.5 \le M_L \le 6.8$ shows the correlation ratio M_{Lm} of M_L for Southern California earthquakes [30], South-West Germany [32] and Central Japan [33]. In calculations M_{Lm} by Equation (18) for the earthquakes in these regions were considered elastic parameters of the geophysical medium according to these authors. The statistical data confirm the validity of our assumptions on the possible equality M_L and M_{Lm} (**Figure 3**).

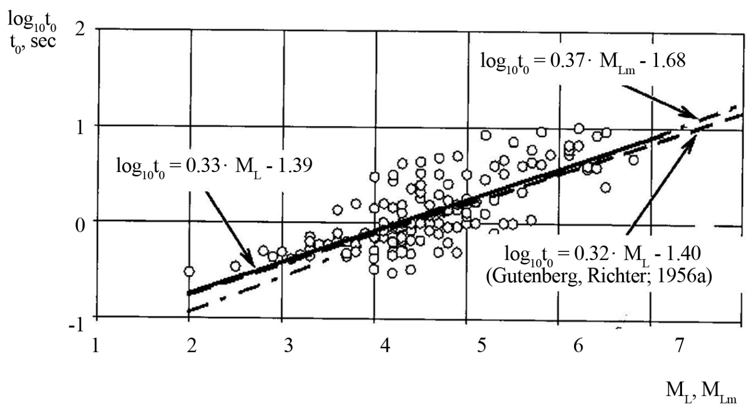

Figure 2. Correlation of $\log_{10}t_0$ from M_L for Southern California earthquakes according to Thatcher and Hanks [30], full line: $\log_{10}t_0 = 0.33(\pm 0.05)M_L - 1.39(\pm 0.23)$, N = 138, r = 0.84. Dashed line—$\log_{10}t_0 = 0.32M_L - 1.40$ by Gutenberg and Richter (1956a); dot-dash line—dependence $\log_{10}t_0$ from M_{Lm}, obtained from correlation $\log_{10}t_0$ from $\log_{10}M_0$ (Figure 1).

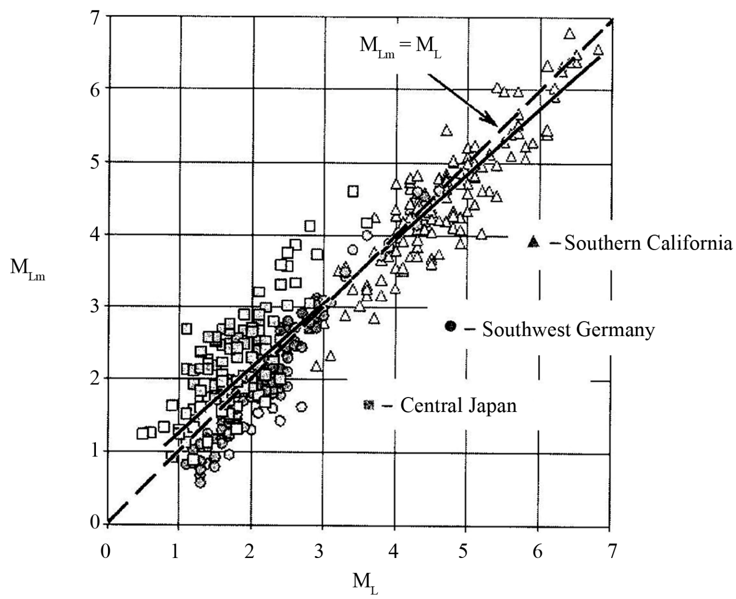

Figure 3. The ratio of calculated M_{Lm} and instrumental M_L for Southern California earthquakes by Thatcher and Hanks [30], South-West Germany (Scherbaum *et al.* 1983) and Cental Japan (Jin *et al.*, 2000). $M_{Lm} = 0.9(\pm 0.03)M_L - 0.28(\pm 0.05)$, N = 384, r = 0.94; dashed line $M_{Lm} = M_L$.

From numerous publications on nonlinear relations $\log_{10}M_0 - M_L$ acceptability of new assumptions considered on the basis of Hasegawa [34] for earthquakes in Eastern Canada. In the range $0 < M_L \leq 6.3$ are two of the interval $0 < M_L \leq 3.9$ and $3.9 \leq M_L \leq 6.3$, which have different dependencies on $\log_{10}t_0$ of M_L and $\log_{10}M_0$ from M_L [34].

For the first group of small earthquakes characterized by the following relationship ($10^5 < \Delta\sigma < 10^6$ Pa): $\log_{10} t_0 = 0.18\log_{10} M_0 - 3.14$, but for another group ($10^6 \leq \Delta\sigma < 5 \times 10^6$ Pa): $\log_{10} t_0 = 0.28\log_{10} M_0 - 4.54$.

On the basis of these empirical formulas for Equation (18) and Equation (24) with $C_L = -14.21$ ($\rho = 2800$ kg/m^3 and $v_s = 3800$ m/s) **Figures 4** and **5** shows the calculated dependences of $\log_{10}t_0$ from M_{Lm} and $\log_{10}M_0$ from M_{Lm}, which in satisfactory agreement with the relations $\log_{10}t_0 - M_L$ and $\log_{10}M_0 - M_L$ (**Figures 4** and **5**) by Hasegawa [34].

Finally, for the Southern California Earthquake Equation (18) and Equation (29) we can obtain the following relationship: $M_{Lm} = 0.67\log_{10} M_0 - 5.92$, which coincides with the ratio of [30]:

$$M_L = 0.67\log_{10} M_0 - 6.0 \tag{32}$$

According to Equations (23) and (24) and Equation (27) if $в_t = 0.25$ we get $z_i = 7/6 - 2в_t = 0.67$, which indicates the acceptability of the proposed relations.

From Equation (32) it follows that $b_t = 0.25$ in Equation (24) the values of M_L and M_{Lm} magnitude M_W corresponds to Equation (11). Probably, the presence of the form Equation (29) between $\log_{10}t_0$ and $\log_{10}M_0$ explains equality $M_L = M_W$ for earthquakes with $M_W \leq 7.0$ North-West Europe [35], New Zealand [36], western Canada [37] and about Taiwan [38].

3.1. Ratio $m_b - \log_{10}M_0$: Design and Data Tools

As in the case of search based $M_L - \log_{10}M_0$, for body-wave magnitude m_b consider empirical relationships

According to Zapolsky [31], Gutenberg [1], specifically examining the relationship between the energy of focal radiation and earthquake magnitude according to

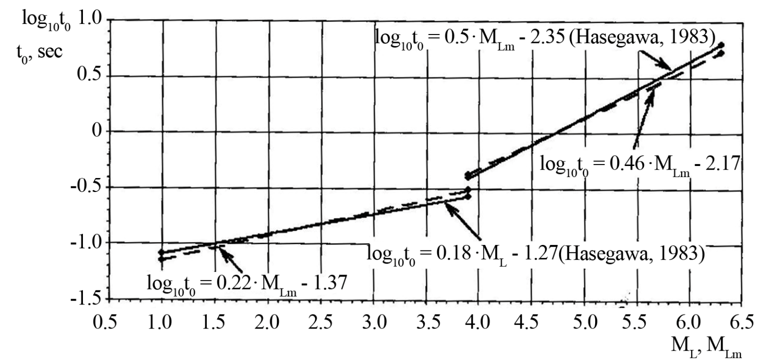

Figure 4. Correlation $\log_{10}t_0$ from M_L (full line—Hasegawa [34] and from calculated M_{Lm} (dashed line, see the text) for East Canada earthquakes.

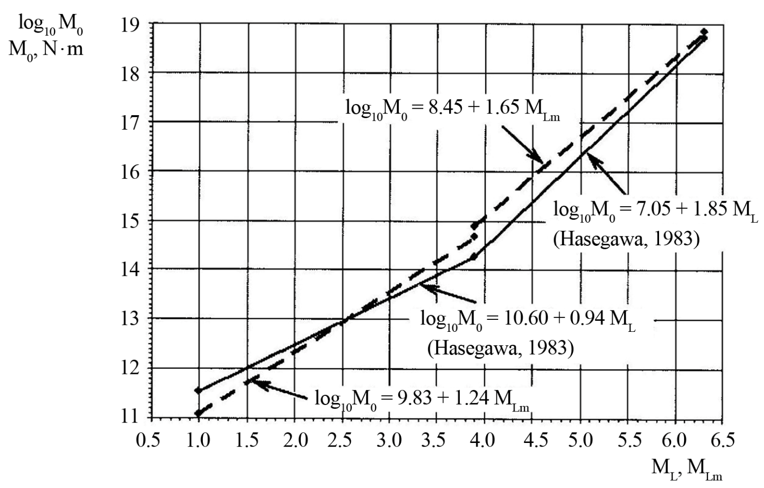

Figure 5. Correlation $\log_{10}M_0$ from M_L (full line—Hasegawa [34] and from calculated M_{Lm} (dashed line, see the text) for East Canada earthquakes.

the observations in the epicentral area, showed that the duration t_0, determine the energy of the oscillations with the maximum intensity depends strongly on the magnitude and 2.5-fold increases with increasing magnitude of m_b on unit [31].

$$\log_{10} t_0 = 0.4 m_b - 1.9 \quad (33)$$

A little-known empirical formula Equation (32) Gutenberg [1] is a key for further generalizations of our constructions on relations $m_b - \log_{10}M_0$, and $m_b - M_S$.

On the basis of (13) and (29) with $C_1 = -11.09$, we can get:

$$\log_{10} t_0 = 0.5 m_b - 2.26 \quad (34)$$

Substitution $\log_{10} t_0 = 0.23 \log_{10} M_0 - 3.53$ in Equation (31) into (13) leads to the following formula:

$$\log_{10} t_0 = 0.42 m_b - 1.84 \quad (35)$$

which is in good agreement with (33) provided $m_b = m_{bm}$.

Graphic expressions Equations (33)-(35) are shown in **Figure 6**, from which it can be assumed about the close convergence of these relations and the possible equality $m_b = m_{bm}$ (**Figure 6**). At equality $m_b = m_bm$—based Equations (13) and (33) for the standard ρ and v_S can obtain the expression:

$$\log_{10} t_0 = 0.22 \log_{10} M_0 - 3.57 \quad (36)$$

which is in good agreement with Equations (29) and (31), which may indicate the consistency of our constructions relating m_b, m_{bm}, M_L, M_{Lm} and $\log_{10}t_0$ with $\log_{10}M_0$ for earthquakes in California, despite the fact that the conclusions are based on statistical formulas in which the correlation coefficients are not equal to unity (r = 0.75 - 0.90)

If we use the Equation (36), on the basis of Equation (24) with $в_t = 0.22$ and Equations (25) and (26) for the

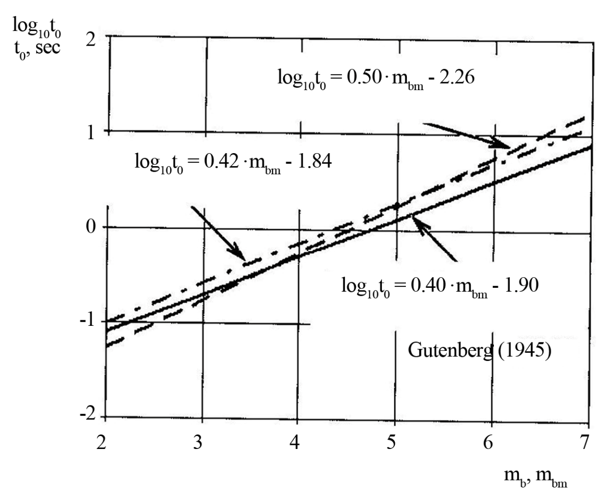

Figure 6. Correlation $\log_{10}t_0$ and m_b (full line), $\log_{10}t_0$ and m_{bm} (dashed line, see the text).

standard values ρ and v_S, M_{Sm} dependence on m_{bm} can be expressed as:

$$M_{Sm} = 1.59\ m_{bm} - 3.20 \quad (37)$$

which almost corresponds to the classical formula Equation (2) Gutenberg and Richter (1956в) and for which the equality $M_{Sm} = m_{bm}$ complied with M_{sm} = 5.40, which coincides closely with generalizations Chen [7], Gusev [9], Nuttli [12] and Utsu [24].

In **Figure 7** shows the correlation of $\log_{10}t_0$ from $\log_{10}M_0$ for earthquakes in the world (1981-1991) by the Catalogue Choy [39], for which the value of t_0 was taken from the Global CMT Catalogue. The ratio of $\log_{10}t_0$ from $\log_{10}M_0$ for these data is given by (**Figure 7**):

$$\log_0 t_0 = 0.30(\pm 0.01)\log_{10} M_0 - 4.88(\pm 0.01) \quad (38)$$

for which the range $17 \le \log_{10}M_0 \le 21$ value of $\log_{10}\Delta\sigma$ by Equation (19) increases from 6.60 to 7.10.

Substituting (38) in (13) leads to ($C_1 = -11.30$):

$$m_{bm} = 0.40 \log_{10} M_0 - 1.54 \quad (39)$$

which agrees closely with the empirical formula:

$$m_b = 0.22(\pm 0.02)\log_{10} M_0 + 1.85(\pm 0.02) \quad (40)$$

shown on **Figure 8**.

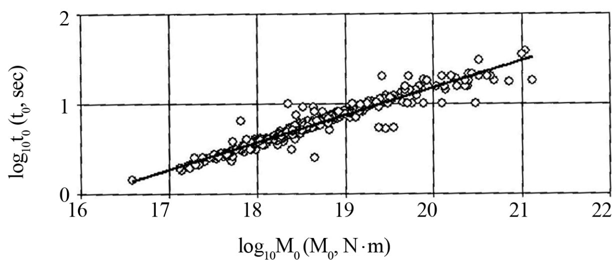

Figure 7. Correlation dependence $\log_{10}t_0$ from $\log_{10}M_0$ for major earthquakes of the world (1981-1991) by Choy's Catalogue [39]. $\log_{10}t_0 = 0.30(\pm 0.01)\log_{10}M_0 - 4.88(\pm 0.01)$, N = 379, r = 0.96.

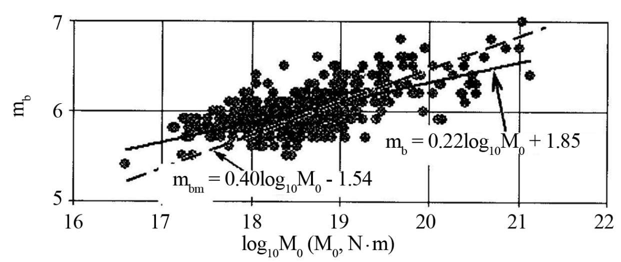

Figure 8. Correlation dependence m_b from $\log_{10}M_0$ (full line) for major earthquakes of the world (1981-1991) by Choy's Catalogue *et al.* (1995) for 1981-1991.
$m_b = 0.22(\pm 0.02)\log_{10}M_0 - 1.85(\pm 0.02)$, N = 362, r = 0.67, dashed line—calculated dependence:
$m_{bm} = 0.40\log_{10}M_0 - 1.54$.

Equation (39) is in good agreement with the dependence on m_b from $\log_{10}M_0$ for Sumatra island earthquake ($\varphi = -10^\circ + 10^\circ$, $\lambda = +90^\circ + 100^\circ$) for 1993-2012 (**Figure 9**).

Table 1 shows a comparison of the magnitude $\hat{m}_b$ obtained by the true maximum amplitude [19,29,40] and the calculated value m_{bm} (**Table 1**) for a number of large earthquakes in 1960-1984. The presented data suggest that for most of the earthquakes characterized by the following inequality: $\hat{m}_b \geq m_{bm} > m_b$.

When $\log_{10}\Delta\sigma \geq 7.1$ value of $\hat{m}_b$ is close to the m_{bm} same as for Great Chilean earthquake $\hat{m}_b = 7.57$ and $m_{bm} = 7.71$, for Tangshan (1976) $\hat{m}_b = 6.9$ and $m_{bm} = 6.92$, Yanyuan (1976). $\hat{m}_b = 6.5$, $m_{bm} = 6.18$, and if $6.36 \leq \log_{10}\Delta\sigma < 7.0$ value of $\hat{m}_b$ more then m_{bm} (**Table 1**).

Table 2 presents a comparison of calculated m_{bm} and $\hat{m}_b$ (21) for 80 major earthquakes of the world for 2000-2012 for calculations m_{bm}, $\hat{m}_b$ and M_{Sm} used data from Global CMT Catalogue (**Table 2**). When comparing $\log_{10}\Delta\sigma$ from **Table 1** to **Table 2** shows that with increasing $\log_{10}M_0$ from 19.15 to 22.72 for the 2000-2012 earthquakes $\log_{10}\Delta\sigma$ value ranges from 6.75 - 7.58 with an average of 7.16, that is, much higher than for earthquakes 1960-1984 (**Tables 1** and **2**) and higher than the standard $\log\Delta\sigma = 6.56$.

For such high values $\Delta\sigma$ values m_{bm} closely coincide with the design $\hat{m}_b$, and for values M_{Sm} characterized by inequality: $M_{Sm} > M_W$ (**Table 2**) confirmed that conclusion is the relation m_{bm} − $\hat{m}_b$—for earthquakes in Japan and the Kuril Islands ($\varphi = 30^\circ + 40^\circ$, $\lambda = 140^\circ + 150^\circ$) for the 1993-2012 shown in **Figure 10**.

Thus for large earthquakes 1960-1984 and 1993-2012 at $\log\Delta\sigma > 7.1$ m_{bm} values coincide closely with the magnitude $\hat{m}_b$ calculated from the true maximum amplitude (A_g) of seismic vibrations, the magnitude of which is proportional to the seismic moment: $A_g \sim f\left(M_0^{0.35}\right)$ to Houston [29] and Kanamori [19]. Consequently, the m_{bm} value is proportional to the $\log_{10}A_g$.

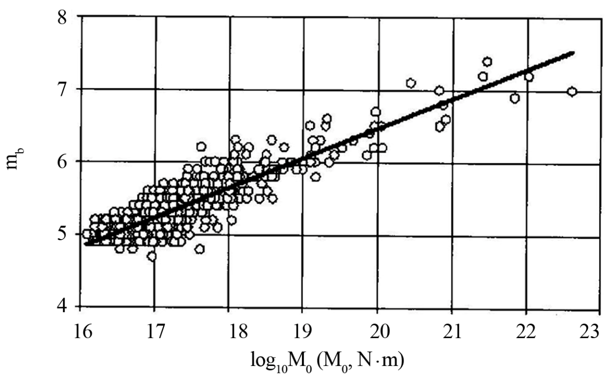

Figure 9. Correlation dependence m_b from $\log_{10}M_0$ for the earthquakes Sumatra region (1993-2012 years). $m_b = 0.41(\pm 0.02)\log_{10}M_0 - 1.76(\pm 0.02)$, **$N$ = 631, r = 0.88.**

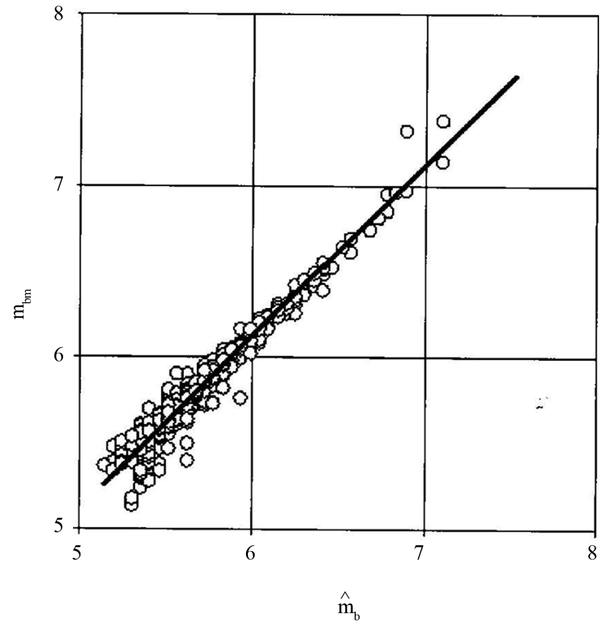

Figure 10. Correlation calculated magnitudes m_{bm} and $\hat{m}_b$ for the earthquakes in Japan and Kuril Islands for 1992-2012 years. $m_{bm} = 1.0(\pm 0.02)\hat{m}_b + 0.14(\pm 0.003)$, **$N$ = 521, r = 0.97. Values of $\hat{m}_b$ were calculated according to the formula (21).**

The ratio of $M_S - \log_{10}M_0$. In Mamyrov's papers [18], [28] have shown that in the range of $16 \leq \log_{10}M_0 < 21.0$ if $\log_{10}\Delta\sigma \leq 7.0$ at the rated M_{Sm} closely coincides with M_S and M_W, and for high $\Delta\sigma \geq 10^7$ Pa following inequality $M_{Sm} > M_S$, as shown in **Table 2**.

In **Figure 11** shows the correlation of M_S from $\log_{10}M_0$ for earthquakes of the world for 1981-1991 according to the Catalog Chou *et al.* [39]:

$$M_S = 0.73(\pm 0.03)\log_{10}M_0 - 7.47(\pm 0.02) \quad (41)$$

which is in satisfactory agreement with the dependence $M_{Sm} = 0.73\log_{10}M_0 - 7.19$ (**Figure 11**, dashed line), derived from Equations (38) and (26). These relations with $M_S = M_{Sm}$ with $\log_{10}M_0$ are in good agreement with the generalization of Perez [41] for crustal earthquakes of the world for the years 1950-1997: $\log_{10}M_0 = 1.33M_S + 10.22$.

In **Figure 12** shows the correlation M_S with $\log_{10}M_0$ (solid line) for the earthquakes in Japan and the Kuril Islands in 1993-2012:

$M_S = 0.77(\pm 0.02)\log_{10}M_0 - 8.23(\pm 0.02)$, here, we show the same relationship

M_{Sm} from $\log_{10}M_0$ (**Figure 12**, dashed line):

$M_{Sm} = 0.69\log_{10}M_0 - 6.09$, obtained with ($N$ = 521, r = 0.99):

Table 1. A comparison of the magnitude $\hat{m}_b$ (Houston, Kanamori, 1986; Zhuo, Kanamori, 1987) and settlement m_{bm} for several major earthquakes of the world.

№№	Date	Time	φ	λ	Depth h, km	$\log_{10}M_0$, N·m	$\log_{10}t_0$ t_0, sec	m_b	$\hat{m}_b$	m_{bm}	M_S	M_W	M_{sm}	$\log_{10}\Delta\sigma$, $\Delta\sigma$, Pa	Region
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1960/5/22	19:11:17.5	−38.29	−73.05	35	23.35	2.17		7.57	7.71	8.5	9.6	9.76	7.10	Great Chilean
2	1963/10/13	5:17:55.1	44.76	149.80	26	21.85	1.88		7.23	6.79	8.1	8.5	8.35	6.47	Great Alaska
3	1964/3/28	3:36:12.7	61.02	−147.63	6	22.96	2.15		7.64	7.36	8.4	9.2	9.29	6.77	Rat Island
4	1965/2/4	5:01:21.7	51.21	178.50	29	22.15	1.94		7.19	6.97	8.2	8.7	8.63	6.59	Kurile Isl.
5	1967/7/22	16:56:55.3	40.63	30.74	4	20.20	1.32		6.38	6.26	7.1	7.4	7.28	6.50	Tyrkey
6	1968/5/16	0:49:0.4	40.90	143.35	26	21.45	1.71		7.18	6.73	8.1	8.3	8.16	6.58	Tokachi-oki
7	1969/8/11	21:27:37.6	43.48	147.82	46	21.34	1.71		6.90	6.62	7.8	8.2	8.01	6.47	Kurile Isl.
8	1971/2/9	14:00:41.0				19.08	0.85	6.2	6.41	6.08	6.7	6.6	6.73	6.79	San Fernando
9	1974/10/3	14:21:34.5	−12.25	−77.52	36	21.18	1.65		7.0	6.68	7.6	8.1	7.92	6.49	Haicheng, China
10	1975/2/4	11:36:7.1	40.67	122.65	16	19.61	1.05		6.76	6.21		7.0	7.03	6.72	Peru
11	1976/2/4	9:01:7.2	15.14	−89.78	16.3	20.31	1.14	6.2	6.66	6.73	7.5	7.5	7.78	7.15	Gua temala
12	1976/5/29	12:23:29.9	24.39	98.65	15	19.09	0.72	6.1	6.5	6.35	6.9	6.7	7.00	7.19	Longlin, China
13	1976/5/29	14:00:33.2	24.29	98.58	15	19.05	0.73	6.0	6.5	6.29	7.0	6.6	6.93	7.12	Longlin, China
14	1976/7/27	19:42:11.1	39.52	118.03	15	20.44	1.11	6.3	6.9	6.92	7.9	7.6	8.01	7.37	Tangshan, China
15	1976/7/28	10:45:45.9	39.75	118.78	15	19.55	0.88	6.3	6.7	6.49	7.4	7.0	7.29	7.47	Tangshan, China
16	1976/8/16	14:6:55.0	32.63	104.42	15	19.11	0.73	6.1	6.9	6.35	6.9	6.7	7.01	7.18	Songpan, China
17	1976/8/16	16:11:38.7	7.07	123.75	33	21.04	1.34	6.4	7.26	7.06	7.9	8.0	8.35	7.28	Mindanao
18	1976/8/21	21:49:57.8	32.37	104.29	15.3	18.50	0.52	6.1	6.7	6.16	6.4	6.3	6.61	7.20	Songpan China
19	1976/8/23	3:30:11.5	32.11	104.21	19.6	18.66	0.58	6.2	6.6	6.20	6.7	6.4	6.71	7.18	Songpan China
20	1976/11/6	18:4:16.0	27.50	101.40	22.7	18.56	0.54	5.8	6.5	6.18	6.5	6.3	6.65	7.20	Yanyuan, China
21	1976/11/15	13:53:7.2	39.45	117.71	15	18.63	0.56	6.0	6.3	6.21	6.3	6.4	6.71	7.21	Tangshan, China
22	1976/11/24	12:22:25.3	38.88	43.96	15	19.62	0.90	6.1	6.58	6.52	7.3	7.0	7.34	7.16	Sumbawa
23	1977/8/19	6:9:33.1	−11.14	118.23	23.3	21.55	1.48	7.0	7.47	7.29	7.9	8.3	8.75	7.37	Iran
24	1978/9/16	15:36:13.5	33.37	57.02	11	20.12	1.34	6.5	6.9	6.14	7.4	7.3	7.13	6.36	Oaxaca
25	1978/11/29	19:53:2.9	16.22	−96.56	16.1	20.72	1.36	6.4	6.87	6.70	7.7	7.7	7.89	6.90	Tyrkey
26	1979/3/14	11:7:31.1	17.78	−101.37	26.7	20.23	1.27	6.5	6.71	6.39	7.6	7.4	7.41	6.69	Petatlan
27	1979/10/15	23:17:0.8	32.62	−115.57	12	18.86	0.78	5.7	5.92	6.00	6.9	6.5	6.57	6.78	Imperial Yalley
28	1979/12/12	8:00:7.0	2.32	−78.81	19.7	21.23	1.35	6.4	6.91	7.23	7.7	8.1	8.58	7.44	Colymbia
29	1980/1/1	16:42:49.8	38.80	−27.74	10	19.45	1.00	6.0	6.3	6.15	6.7	6.9	6.92	6.71	Azores Isl.
30	1980/2/7	10:49:26.3	−54.29	158.43	15	19.36	1.01	6.1	6.2	6.04	6.5	6.8	6.79	6.59	Macguarie Isl.
31	1980/7/17	19:43:3.1	−12.44	165.94	34	20.68	1.24	5.8	6.79	6.90	7.9	7.8	8.07	7.22	Eureka
32	1980/10/10	12625:25.5	36.14	1.41	12	19.70	1.00	6.5	6.5	6.40	7.3	7.1	7.32	6.36	Santa Grus Isl.
33	1980/11/8	10:27:45.9	41.14	−124.36	15	20.05	1.00	6.2	6.7	6.75	7.2	7.3	7.72	7.31	El Asnam, Algeria

Continued

34	1981/1/23	21:13:55.6	30.86	101.35	10	18.86	0.78	5.7	6.5	6.00	6.8	6.5	6.57	6.78	Daofu, China
35	1981/2/24	20:53:49.2	38.07	23.04	10	18.95	0.78	5.9	6.6	6.09	6.7	6.6	6.69	6.97	Greece
36	1981/4/24	21:50:14.3	−13.51	166.43	44.4	19.55	0.90	6.1	6.2	6.45	6.9	6.8	7.25	7.11	Yanuati Isl.
37	1981/4/27	18:17:40.0	−57.58	147.86	10	18.91	0.78	5.7	6.1	6.05	6.5	6.5	6.64	6.83	Macguarie Isl.
38	1981/7/28	17:22:43.6	30.01	57.8	15.2	19.95	1.15	5.7	6.9	6.35	7.1	7.2	7.28	6.76	Kurile Isl.
39	1991/9/3	5:35:50.1	42.97	147.87	35.7	18.88	0.70	6.6	6.5	6.18	6.6	6.5	6.76	7.04	Papua, New Guinea
40	1981/11/6	16:47:51	−3.18	143.72	15	18.96	0.84	6.2	6.2	5.98	6.9	6.6	6.59	6.70	Loayltu Isl.
41	1981/11/24	23:30:41.9	−22.19	170.32	23.3	19.13	0.87	5.7	6.2	6.09	6.7	6.7	6.75	6.79	Kermades Isl.
42	1981/12/24	5:33:33.3	−29.81	−177.55	19.4	19.32	1.08	6.1	6.4	5.86	6.8	6.8	6.58	6.34	Iran
43	1982/8/5	20:33:2.0	−12.52	166.01	23.9	19.50	0.98	6.2	6.3	6.24	7.1	6.9	7.02	6.82	Santa Grus Isl.
44	1982/12/19	17:44:21.8	−24.31	175.0	29.2	20.30	1.33	6.0	6.4	6.34	7.7	7.5	7.39	6.57	Tonga Isl.
45	1983/4/3	2:50:26.4	8.85	−83.25	28	20.26	1.27	6.5	6.6	6.42	7.2	7.4	7.52	6.71	Panama
46	1983/5/2	23:43:44.7	36.42	−120.66	14.5	18.60	0.79	6.2	6.0	5.72	6.5	6.3	6.25	6.49	Akito Oki, Japan
47	1983/5/26	3:0:18.3	40.44	138.87	12.6	20.66	1.30	6.8	7.2	6.76	7.7	7.7	7.93	7.02	Coolinga
48	1983/10/4	18:52:37.8	−26.01	−70.56	38.7	20.53	1.46	6.4	6.8	6.31	7.3	7.6	7.50	6.41	Chili
49	1983/10/28	14:6:22.5	44.35	−113.98	13.7	19.49	1.00	6.2	6.6	6.19	7.3	6.9	6.97	6.75	Idaho
50	1983/11/16	16:13:5.9	19.40	−155.59	11	19.03	0.78	6.3	6.7	6.17	6.6	6.6	6.90	6.95	Hawaii
51	1983/11/30	17:46:28.9	−6.35	71.75	10	20.61	1.23	6.6	7.1	6.85	7.5	7.7	8.00	7.18	Chugos Arch
52	1984/2/7	21:33:36.1	−9.81	160.42	21.9	20.40	1.33	6.5	6.7	6.44	7.5	7.5	7.52	6.67	Solomon Isl.
53	1984/3/19	20:28:39	40.38	63.37	15	19.55	0.78	6.5	6.7	6.69	7.0	7.0	7.49	7.47	Uzbekistan

Notice: for the earthquakes N1—10 $\log_{10}t_0$ calculated according to $\Delta\sigma$ Kasachara (1984), Purcaru and Berkhemer (1982); for the rest earthquakes N11-52 all data have been taken according to Global CMT Cataloge, for the earthquakes N53—data have been taken from USSR's catalogue, 1984.

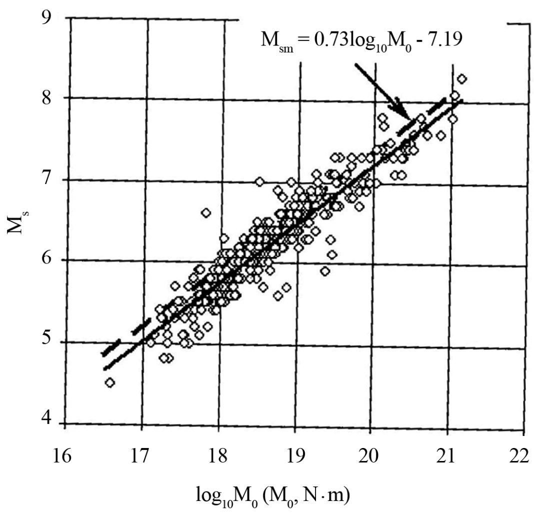

Figure 11. Correlation of magnitudes M_S and $\log_{10}M_0$ (full line) by Catalogue of major earthquakes of the world Choy [39]: $M_S = 0.73(\pm0.03)\log_{10}M_0 - 7.47(\pm0.02)$, N = 372, r =0.93. Dashed line—calculated dependence. $M_S = 0.73\log_{10}M_0 - 7.19$.

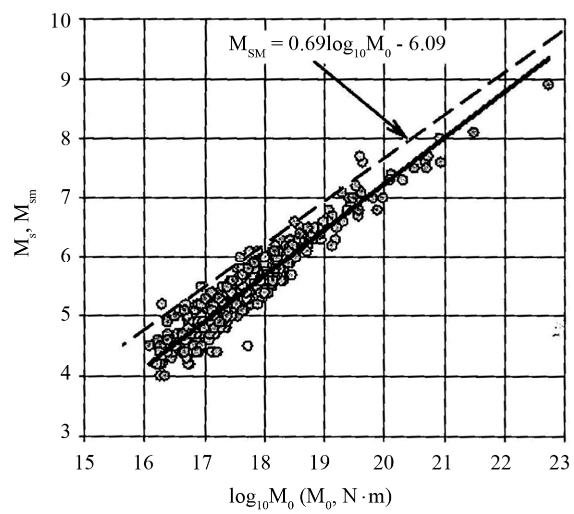

Figure 12. Correlation dependence of magnitude M_S from $\log_{10}M_0$ for the earthquakes in Japan and Kuril Islands for 1993-2012 years. $M_S = 0.77(\pm0.03)\log_{10}M_0 - 8.23(\pm0.02)$, N = 514, r = 0.95. Dashed line—calculated dependence. $M_{Sm} = 0.69\log_{10}M_0 - 6.09$.

Table 2. Comparison of calculated (m_{bm}, M_{Sm}) and instrumental (m_b, M_S, M_W) for a number of magnitude large earthquakes of the world 2000-2012 years.

№№	Date	Time	φ	λ	Depth H, км	$\log_{10}M_0$ M_o, Н·м	$\log_{10}t_0$ t_0, с	m_b	$\hat{m}_b$	m_{bm}	M_S	M_W	M_{sm}	$\log_{10}\Delta\sigma$, $\Delta\sigma$, П*a*	Region
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	2000/1/8	16:47:30.2	−16.84	−173.81	162.4	19.84	0.99	6.5	6.52	6.56	6.6	7.2	7.46	7.13	Tonga Isl.
2	2000/2/25	1:44:5.2	−19.55	174.17	16.8	19.70	0.95	6.1	6.46	6.50	7.1	7.1	7.35	7.11	Vanuatu Isl.
3	2000/5/4	4:21:33.4	−1.29	123.59	18.6	20.39	1.18	6.7	6.67	6.73	7.5	7.5	7.81	7.11	Sulawesi
4	2000/6/4	16:28:46.5	−4.73	101.94	43.9	20.87	1.46	6.8	6.83	6.65	8.0	7.8	7.89	6.75	Sumatra
5	2000/6/18	14:44:27.6	−13.47	97.17	15.0	20.90	1.41	6.8	6.89	6.78	7.8	7.9	8.03	6.93	Indian Ocean
6	2000/11/16	4:55:36.5	−4.56	152.79	24.0	21.09	1.34	6.0	6.94	7.11	8.2	8.0	8.42	7.33	New Ireland
7	2000/11/16	7:42:44.5	−5.03	153.17	31.2	20.81	1.36	6.2	6.83	6.79	7.8	7.8	8.01	6.99	New Ireland
8	2000/11/17	21:2:20.1	−5.26	152.34	17.0	20.75	1.33	6.2	6.83	6.79	8.0	7.8	7.99	7.02	New Britain
9	2000/12/6	17:11:14.7	39.60	54.87	33.0	19.59	0.94	6.7	6.41	6.41	7.5	7.0	7.22	7.03	Turkmenia
10	2001/1/1	6:57:24.0	6.73	127.07	44.0	20.24	1.08	6.4	6.62	6.78	7.2	7.4	7.81	7.26	Mindanao
11	2001/1/26	3:16:54.9	23.63	70.24	19.8	20.53	1.38	6.9	6.73	6.47	8.0	7.6	7.59	6.65	India
12	2001/6/23	20:34:23.3	−17.28	−72.71	29.6	21.67	1.63	6.7	7.15	7.11	8.2	8.4	8.61	7.04	Peru
13	2001/8/21	6:52:14.3	−36.70	179.08	59.0	19.71	1.01	6.4	6.41	6.39	7.1	7.0	7.24	6.94	New Zealand
14	2001/10/12	15:2:23.3	12.88	145.08	42.0	19.57	0.92	6.7	6.41	6.43	7.3	7.0	7.24	7.07	Mariana Isl.
15	2002/9/8	18:44:38.3	−3.27	143.38	19.5	20.47	1.26	6.5	6.73	6.65	7.8	7.6	7.75	6.95	Papua
16	2002/10/10	10:50:41.9	−1.79	134.30	15.0	20.41	1.18	6.5	6.67	6.75	7.7	7.5	7.83	7.13	Java Isl.
17	2002/11/02	1:26:25.9	2.65	95.99	23.0	19.95	1.09	6.2	6.52	6.47	7.6	7.2	7.40	6.94	Sumatra
18	2002/11/03	22:13:28.0	63.23	−144.89	15.0	20.87	1.37	7.0	6.83	6.83	8.5	7.8	8.07	7.02	Alaska
19	2003/03/17	16:36:26.6	51.33	177.58	27.0	19.62	0.89	5.9	6.41	6.54	6.7	7.0	7.36	7.21	Aleutian Isl.
20	2003/08/21	12:12:59.5	−45.01	166.87	31.8	19.87	0.98	6.6	6.52	6.61	7.5	7.2	7.52	7.19	New Zealand
21	2003/09/25	19:50:38.2	42.21	143.84	28.2	21.48	1.52	6.9	7.10	7.14	8.1	8.3	8.58	7.18	Hokkaido
22	2003/09/27	11:33:36.2	50.02	87.86	15.0	19.97	1.01	6.5	6.52	6.65	7.5	7.2	7.59	7.20	Siberia, Russia
23	2004/12/23	14:59:30.9	−49.91	161.25	27.5	21.21	1.43	6.5	7.00	7.05	7.7	8.1	8.40	7.58	Macquarie Isl.
24	2004/12/26	1:1:9.0	3.09	94.26	28.6	22.60	1.98	7.0	7.47	7.34	8.9	9.0	9.15	6.92	Sumatra
25	2005/03/28	16:10:31.5	1.67	97.07	25.8	22.02	1.69	7.2	7.26	7.34	8.4	8.6	8.96	7.21	Sumatra
26	2005/10/08	3:50:51.5	34.38	73.47	12.0	20.47	1.18	6.9	6.73	6.81	7.7	7.6	7.91	7.19	Pakistan
27	2005/11/14	21:38:59.3	38.22	144.97	18.0	19.57	0.86	6.7	6.41	6.55	6.8	7.0	7.36	7.25	Honshu
28	2006/02/22	22:19:15.0	−21.20	33.33	12.0	19.62	0.90	6.5	6.41	6.52	7.5	7.0	7.34	7.18	Mozambique
29	2006/04/20	23:25:17.6	60.89	167.05	12.0	20.48	1.18	6.8	6.73	6.82	7.6	7.6	7.93	7.20	Siberia, Russia
30	2006/05/03	15:27:3.7	−20.39	−173.47	67.8	21.05	1.37	7.2	6.94	7.01	7.9	8.0	8.31	7.20	Tonga Isl.
31	2006/11/15	11:15:8.0	46.71	154.33	13.5	21.54	1.54	6.6	7.10	7.16	8.3	8.3	8.62	7.18	Kuril Isl.
32	2007/01/13	4:23:48.1	46.17	154.80	12.0	21.25	1.44	7.3	7.00	7.07	8.2	8.1	8.43	7.19	Kuril Isl.
33	2007/04/01	20:40:38.9	−7.79	156.34	14.1	21.20	1.42	6.8	7.00	7.06	8.1	8.1	8.41	7.20	Solomon Isl.
34	2007/01/21	11:28:1.0	1.10	126.21	22.2	20.30	1.12	6.7	6.67	6.77	7.5	7.5	7.81	7.20	Molucca Sea
35	2007/08/15	23:41:57.9	−13.73	−77.04	33.8	21.05	1.37	6.7	6.94	7.01	8.0	8.0	8.31	7.20	Peru
36	2007/09/12	11:11:15.6	−3.78	100.99	24.4	21.83	1.63	6.9	7.20	7.27	8.5	8.5	8.82	7.20	Sumatra
37	2007/09/12	23:49:35.3	−2.46	100.13	43.1	20.91	1.32	6.6	6.89	6.97	8.1	7.9	8.22	7.21	Sumatra
38	2007/11/14	15:41:11.2	−22.64	−70.62	37.6	20.68	1.25	6.7	6.78	6.88	7.7	7.7	8.05	7.19	Chile

Continued

39	2008/10/05	15:53:1.1	39.50	73.64	12.0	19.15	0.73	6.4	6.25	6.59	6.9	6.7	7.06	7.22	Kyrgyzstan
40	2008/11/16	17:2:43.8	1.50	122.05	29.2	20.12	1.06	6.5	6.57	6.70	7.3	7.3	7.69	7.20	Sulavesi
41	2009/01/03	22:33:44.9	−0.58	133.48	18.2	20.15	1.07	6.7	6.62	6.71	7.4	7.4	7.71	7.20	Java Isl.
42	2009/01/15	17:49:48.3	46.97	155.39	45.2	20.18	1.08	6.9	6.62	6.72	7.5	7.4	7.73	7.20	Kuril Isl.
43	2009/05/28	8:25:4.8	16.50	−87.17	12.0	20.11	1.05	6.7	6.57	6.71	7.3	7.3	7.70	7.22	Honduras
44	2009/07/15	9:22:49.6	−45.85	166.26	23.5	20.76	1.28	6.5	6.83	6.90	7.8	7.8	8.10	7.18	New Zealand
45	2009/08/10	19:56:5.0	14.16	92.94	22.0	20.29	1.12	6.9	6.67	6.75	7.6	7.5	7.80	7.19	Andaman Isl.
46	2009/09/02	7:55:7.5	−8.12	107.33	53.2	19.56	0.87	6.8	6.41	6.52	7.0	7.0	7.32	7.21	Java Isl.
47	2009/09/29	17:48:26.8	−15.13	−171.97	12.0	21.22	1.43	7.1	6.99	7.06	8.1	8.1	8.41	7.19	Samoa Isl.
48	2009/09/30	10:16:17.4	−0.79	99.67	77.8	20.44	1.17	7.1	6.73	6.8	7.5	7.6	7.89	7.19	Sumatra
49	2009/10/07	22:3:28.9	−12.59	166.27	44.2	20.51	1.19	6.4	6.73	6.83	7.7	7.6	7.95	7.20	Vanuatu Isl.
50	2009/10/07	22:19:15.3	−11.86	166.01	41.7	20.82	1.30	6.4	6.83	6.92	7.9	7.8	8.14	7.18	Santa Crus Isl.
51	2009/10/07	23:14:0.6	−13.12	166.37	42.5	20.22	1.09	6.4	6.62	6.74	7.4	7.4	7.76	7.21	Vanuatu Isl.
52	2010/01/03	22:36:42.4	−8.88	157.21	12.0	19.76	0.94	6.4	6.46	6.58	7.1	7.1	7.45	7.20	Solomon Isl.
53	2010/02/27	6:35:14.5	−35.98	−73.15	23.2	22.27	1.78	7.2	7.36	7.41	8.3	8.8	9.11	7.19	Chile
54	2010/04/06	22:15:19.1	2.07	96.74	17.6	20.82	1.29	7.0	6.83	6.94	7.9	7.8	8.16	7.21	Sumatra
55	2010/08/10	5:23:53.9	−17.57	167.81	31.9	20.00	1.02	6.4	6.57	6.66	7.3	7.3	7.61	7.20	Vanuatu Isl.
56	2010/10/25	14:42:59.8	−3.71	99.32	12.0	20.83	1.30	6.5	6.83	6.93	7.8	7.8	8.15	7.19	Sumatra
57	2010/12/21	17:19:53.6	27.10	143.76	15.6	20.24	1.10	7.0	6.62	6.74	7.5	7.4	7.77	7.20	Bonin Isl.
58	2010/12/25	13:16:51.4	−19.67	168.04	16.6	20.05	1.04	6.8	6.57	6.67	7.4	7.3	7.64	7.19	Vanuatu Isl.
59	2011/01/02	20:20:26.6	−38.71	−73.84	19.4	19.80	0.95	6.6	6.46	6.60	7.1	7.1	7.48	7.21	Chile
60	2011/01/18	20:23:31.8	28.61	63.90	52.3	19.94	1.00	6.7	6.52	6.64	7.2	7.2	7.57	7.20	Pakistan
61	2011/03/09	2:45:32.0	38.56	142.78	14.1	20.09	1.05	6.4	6.53	6.69	7.3	7.3	7.67	7.20	Honshu Isl.
62	2011/03/11	5:47:32.8	37.52	143.05	20.0	22.72	1.84	7.2	7.52	7.74	8.9	9.1	9.59	7.46	Honshu Isl.
63	2011/03/11	6:15:58.7	35.92	141.38	29.0	20.93	1.33	6.8	6.89	6.97	6.8	7.9	8.23	7.20	Honshu Isl.
64	2011/03/11	6:26:12.6	38.27	144.63	21.1	20.49	1.19	7.1	6.73	6.81	7.5	7.6	7.95	7.18	Honshu Isl.
65	2011/04/07	14:32:50.6	38.82	141.85	53.3	19.77	0.94	6.9	6.46	6.59	7.1	7.1	7.49	7.21	Honshu Isl.
66	2011/07/06	19:3:32.5	−29.22	−175.83	22.3	20.47	1.18	7.0	6.73	6.81	7.8	7.6	7.98	7.19	Kermadec Isl.
67	2011/07/10	0:57:16.3	37.98	143.33	22.0	19.60	0.89	6.6	6.41	6.52	7.0	7.0	7.34	7.25	Honshu Isl.
68	2011/10/23	10:41:28.4	38.64	43.40	12.0	19.80	0.95	6.9	6.46	6.60	7.3	7.1	7.48	7.21	Turkey
69	2012/01/10	18:37:13.3	2.59	92.98	23.7	19.88	0.98	6.6	6.52	6.62	7.2	7.2	7.53	7.20	Sumatra
70	2012/02/02	13:34:49.2	−17.69	167.11	20.5	19.64	0.90	6.5	6.41	6.54	7.1	7.0	7.37	7.20	Vanuatu Isl.
71	2012/03/20	18:2:54.9	16.60	−98.39	15.4	20.30	1.12	6.6	6.67	6.76	7.6	7.5	7.81	7.20	Guerrero
72	2012/03/25	22:37:20.9	−35.31	−72.41	33.8	19.78	0.94	6.5	6.46	6.60	7.1	7.1	7.48	7.22	Chile
73	2012/04/11	8:39:31.4	2.35	92.82	45.6	21.96	1.67	7.4	7.26	7.29	8.6	8.6	8.92	7.21	Sumatra
74	2012/04/11	10:43:38.2	0.90	92.31	54.7	21.46	1.51	7.2	7.05	7.14	8.2	8.2	8.57	7.19	Sumatra
75	2012/04/12	7:16:4.6	28.57	−112.76	15.8	19.66	0.91	6.2	6.41	6.54	7.0	7.0	7.38	7.19	Mexico
76	2012/08/27	4:37:38.2	11.91	−89.18	12.0	20.07	1.04	6.5	6.57	6.69	7.3	7.3	7.66	7.21	Salvador
77	2012/08/31	12:47:43.0	11.02	127.00	46.1	20.52	1.20	6.5	6.73	6.82	7.6	7.6	7.94	7.18	Philippine
78	2012/09/05	14:42:23.7	9.87	−85.54	30.8	20.49	1.18	6.8	6.73	6.83	7.6	7.6	7.94	7.21	Costa Rica
79	2012/10/28	3:4:39.2	52.47	132.13	15.0	20.71	1.26	6.2	6.78	6.86	7.7	7.7	8.07	7.19	Charlotte Isl.
80	2012/11/07	16:35:55.2	13.93	−92.47	28.7	20.11	1.06	6.6	6.57	6.69	7.4	7.3	7.68	7.19	Guatemala

$$\log_{10} t_0 = 0.32(\pm 0.003)\log_{10} M_0 - 5.43(\pm 0.003) \quad (42)$$

From the data that the value M_{Sm} an average of 0.5 more than the M_S, because according to the relation $\log_{10}t_0$ with $\log_{10}M_0$ (from 42) with growth $\log_{10}M_0$ from 16 to 22 on the basis of (19), the value increases from 7.19 $\log\Delta\sigma$ to 7.43 (**Figure 12**), and using equation (38) in the same size ranges of $\log_{10}M_0$ the value of $\log_{10}\Delta\sigma$ increases from 6.5 to 7.10. It is likely that for most crustal earthquakes before 1993 was characterized by the above limits to growth $\log_{10}\Delta\sigma < 7.10$.

Ratio $m_b - M_S$ и $m_{bm} - M_{Sm}$. In **Figure 13** shows the correlation ratio $m_b - M_S$ for crustal earthquakes of the Kuril Islands and Japan for 1993-2011:

$$m_b = 0.52(\pm 0.03)M_S + 2.78(\pm 0.02) \quad (43)$$

which is in good agreement with the expression:

$$m_{bm} = 0.52\, M_{Sm} + 2.74 \quad (44)$$

derived from (42) and (28) for $в_t = 0.32$ и $a_t = -5,43$ (**Figure 13**).

Figure 14 shows the correlation ratio $m_b - M_S$ for crustal earthquakes in South America for the years 1993-2012, ($\varphi = -40° - 0°$, $\lambda = -85° - 65°$) by Global CMT Catalogue:

$$m_b = 0.52(\pm 0.03)M_S + 2.64(\pm 0.02) \quad (45)$$

for this region was obtained ($N = 576$, $r = 0.99$):

$$\log_{10} t_0 = (0.32 \pm 0.004)\log_{10} M_0 - 5.48(\pm 0.003) \quad (46)$$

the substitution of which in (26), $в_t = 0.32$ and $a_t = -5.48$ leads to the formula

$$m_{bm} = 0.52 M_{Sm} + 2.77 \quad (47)$$

Equations (43)-(46) are in good agreement with Equations (21) and (22).

Figure 15 shows the correlation $\log_{10}t_0$ of $\log_{10}M_0$ for earthquakes of the Tien Shan ($\varphi = 38.5° - 45°$, $\lambda = 63° - 96°$) for 1960-2012 in interval $13.0 \le \log_{10}M_0 \le 21.5$ ($N = 684$, $r = 0.85$):

$$\log_{10} t_0 = 0.22(\pm 0.01)\log_{10} M_0 - 3.45(\pm 0.01) \quad (48)$$

which closely coincides with Equations (29), (31) and (36) typical for earthquakes in California (**Figures 1** and **15**).

Therefore, we can expect that the relationship between magnitudes $m_b - M_S$ for earthquakes of the two regions may be similar in this range of seismic moment. Indeed, the data in **Figure 16** confirmed these assumptions and empirical relationship of M_S from m_b for Tien Shan's earthquakes is expressed by the following relation ($N = 1183$, $r = 0.95$, **Figure 16**):

$$M_S = 1.57(\pm 0.03)m_b - 3.05(\pm 0.02) \quad (49)$$

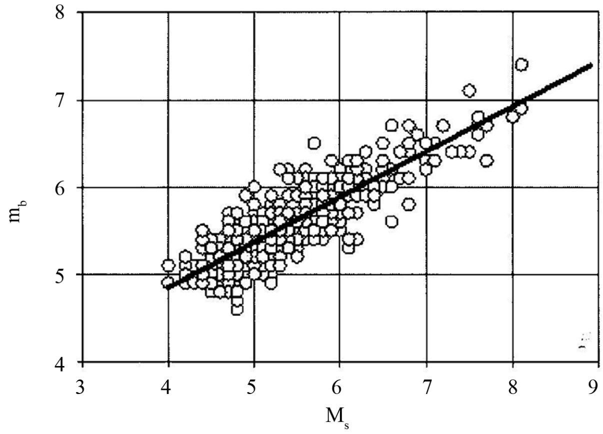

Figure 13. Correlation of magnitudes m_b and M_S for the earthquakes in Japan and Kuril Islands for 1993-2012 years. $m_b = 0.52(\pm 0.03)M_S + 2.78(\pm 0.02)$, $N = 514$, $r = 0.84$. Calculated dependence $m_{bm} = 0.52M_{Sm} + 2.74$ if $\log_{10}t_0 = 0.32\log_{10}M_0 - 5.43$ (see text).

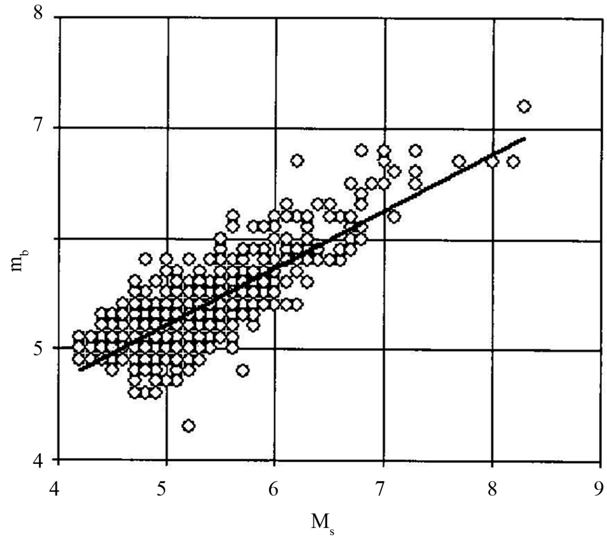

Figure 14. Correlation of magnitudes m_b and M_S for the earthquakes in South America 1993-2012 years. $m_b = 0.52(\pm 0.03)M_S - 2.64(\pm 0.02)$, $N = 547$, $r = 0.82$. Calculated dependence $m_{bm} = 0.52M_{Sm} - 2.77$ if $\log_{10}t_0 = 0.32\log_{10}M_0 - 5.48$ (see text).

Calculated dependence of M_{Sm} from m_{bm} based on Equations (25), (26) and (47) for the elastic parameters of the standard as follows:

$$M_{Sm} = 1.59 m_{bm} - 3.06 \quad (50)$$

which is in good agreement with Equations (2), (37) and (49).

Therefore, we have adopted model of the relationship of linear relations between M (m_b, M_L, M_S) and $\log_{10}t_0$ with $\log_{10}M_0$ explains many existing empirical formulas.

For a wide range $6 \le \log_{10}M_0 \le 23$ changing $\log_{10}t_0$, to a first approximation, can be described by a nonlinear dependence of ($A_0 = \log_{10}M_0$):

$$\log_{10} t_0 = 0.167A_0 - 2.83 + 1/3\exp\left(2.166A_0 - 0.045A_0^2 - 25.09\right) \quad (51)$$

in which the first two terms describes the linear growth $\log_{10}t_0$ in the range $6 \le A_0 \le 15$. On the basis of Equations (25)-(27) and (51) in **Figure 17** shows estimates nonlinear dependence m_{bm}, M_{Lm} and M_{Sm} from M_W to (11) for crustal earthquakes. From **Figure 17** shows that in the

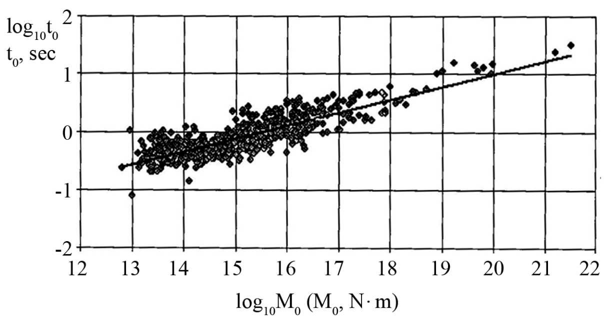

Figure 15. Correlation dependence $\log_{10}t_0$ from $\log_{10}M_0$ for Tien Shan earthquakes (1960-2012 years).
$\log_{10}t_0 = 0.22(\pm0.01)\log_{10}M_0 - 3.45(\pm0.01)$, $N = 684$, $r = 0.85$.

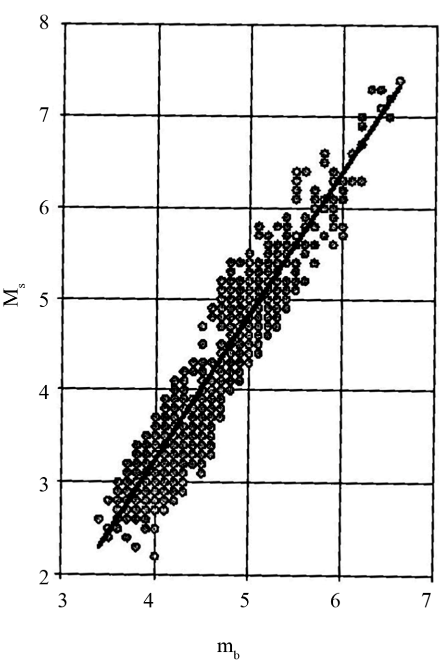

Figure 16. Correlation of magnitudes M_S and m_b for Tien Shan earthquakes (1902-2012 years).
$M_S = 1.57(\pm0.03)m_b - 3.05(\pm0.02)$, $N = 1183$, $r = 0.95$.
Calculated dependence $M_{Sm} = 1.59m_{bm} - 3.06$ **if** $\log_{10}t_0 = 0.22\log_{10}M_0 - 3.45$ **(see the text).**

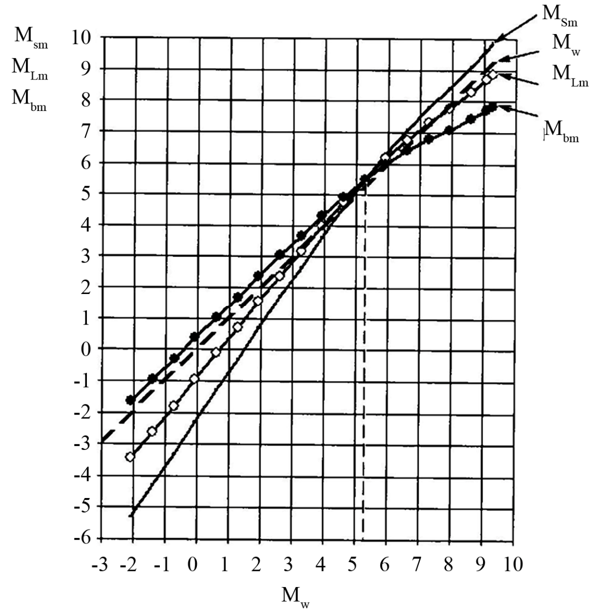

Figure 17. Averaged according M_{Sm} M_{Lm} and m_{bm} from M_W for crustal earthquakes (see text), the dashed line represents the intersection of the curves $M_{Sm} \approx M_{Lm} \approx m_{bm} \approx M_W \approx 5.26 - 5.50$.

interval $4 \le M_W \le 6,5$ numerical values of magnitudes $m_{bm} \approx m_b$, $M_{Lm} \approx M_L$, $M_{Sm} \approx M_S$ and M_W within the accuracy of these parameters are close. In accordance with Equations (19) and (51) in the interval $6.0 < A \le 23.0$ $\log_{10}\Delta\sigma$ value increases from 1.75 to 7.53, and the most intense increase in this parameter is in the range $6.0 \le A_0 \le 15.0$.

4. Conclusions

1) A broad range of local Richter magnitude M_L, m_b, and M_S crustal earthquakes in different regions shows a possible functional relationship with the seismic moment magnitude, corner frequency, voltage and depressurized seismic elastic parameters of the geophysical environment. These links justify numerous empirical relationships with magnitudes of seismic moment.

2) It is assumed that an upgraded body-wave magnitude m_{bm} for large earthquakes is proportional to the logarithm of the average displacement along the fault $\log_{10}u$, $\hat{m}_b$, the true magnitude and the maximum amplitude of seismic vibrations A_g; magnitude M_{Sm} is proportional to the logarithm of the square average displacement along the fault ($2\log_{10}u$) and local magnitude proportional $1.5\log_{10}u$.

3) Control parameters of the quantitative relations with seismic moment magnitudes are coefficients depending on the change in corner period of seismic stress drop or discharged from the seismic moment, which provide a self-consistent system of equations between the main source parameters of crustal earthquakes.

REFERENCES

[1] B. Gutenberg, "Amplitudes of P, PP, an S waves and Magnitude of Shallow Earthquakes," *Bulletin of the Seismological Society of America*, Vol. 35, No. 2, 1945, pp. 57-69.

[2] B. Gutenberg and C. F. Richter, "Earthquake Magnitude, Intensity, Energy and Acceleration," *Bulletin of the Seismological Society of America*, Vol. 46, No. 2, 1956, pp. 105-145.

[3] B. Gutenberg and C. F. Richter, "Magnitude and Energy of Earthquakes," *Annali di Geofisica*, Vol. 9, No. 1, 1956, pp. 1-15.

[4] N. V. Kondorskaya, A. I. Zakharova and L. S. Chepkunas, "The Quantitative Characteristics of Earthquake Sources as Determined in the Seismological Practice of the U.S.S. R.," *Tectonophysics*, Vol. 166, No. 1-3, 1989, pp. 45-52. doi:10.1016/0040-1951(89)90204-7

[5] T. G. Rautian, "Energy of Earthquakes," In: Y. V. Riznichenko, Ed., *Methods for the Detailed Study of Seismicity*, Akademii Nauk SSSR, Moscow, 1960, pp. 75-114.

[6] T. G. Rautian, V. J. Khalturin, K. Fujita, K. G. Mackey, *et al*., "Origins and Methodology of the Russian Energy K-Class System and Its Relationship to Magnitude Scales," *Seismological Research Letters*, Vol. 78, No. 6, 2007, pp. 579-590. doi:10.1785/gssrl.78.6.579

[7] P. Chen and H. Chen, "Scaling Law and Its Applications to Earthquake Statistical Relations," *Tectonophysics*, Vol. 166, No. 1-3, 1989, pp. 53-72. doi:10.1016/0040-1951(89)90205-9

[8] L. B. Grant, "Paleoseismology, International Handbook of Earthquake and Engineering Seismology, Part A," Academic Press, Waltham, 2002.

[9] A. A. Gusev and V. N. Melnikova, "Relations between Magnitudes: Global and Kamchatka Data," *Volkanology and Seismology*, Vol. 6, 1990, pp. 55-63.

[10] K. Kasahara, "Earthquake Mechanics," 1985.

[11] P. Mai and G. C. Beroza, "Source Scaling Potpies from Finite-Fault-Rupture Models," *Bulletin of the Seismological Society of America*, Vol. 90, No. 3, 2000, pp. 604-615. doi:10.1785/0119990126

[12] O. W. Nuttli, "Average Seismic Source-Parameters Relation for Mid-Plate Earthquakes," *Bulletin of the Seismological Society of America*, Vol. 73, No. 2, 1983, pp. 519-535.

[13] K. Aki, "Generation and Propagation of G-Waves from the Niigata Earthquake of June 16, 1964. Part 2. Estimation of Earthquake Moment, Released Energy, and Stress-Strain Drop from the G-Wave Spectrum," Bulletin of the Earthquake Research Institute, Tokyo, 1966.

[14] K. Aki and P. A. Richards, "Quantitative Seismology. Theory and Methods, v. t., Moskau: Mir," 1983.

[15] J. N. Brune, "Tectonic Stress and the Spectra of Seismic Shear Waves from Earthquakes," *Journal of Geophysical Research*, Vol. 75, No. 26, 1970, pp. 4997-5009. doi:10.1029/JB075i026p04997

[16] V. Keilis-Borok, "On Estimation of the Displacement in an Earthquake Source and Dimension," *Annales Geophysicae*, Vol. 12, No. 1-4, 1959, pp. 205-214.

[17] P. Debye, "Zur Theorie der Spezifischen Wärmen," *Annalen der Physik*, Vol. 344, No. 14, 1912, pp. 789-839. doi:10.1002/andp.19123441404

[18] E. Mamyrov, "Relations among Earthquake Source Parameters Derived from Debye Solid-Body Model," *Journal of Geodynamics*, Vol. 22, No. 1, 1996, pp. 137-143. doi:10.1016/0264-3707(96)00005-1

[19] H. Kanamori, "The Energy Released in Great Earthquakes," *Journal of Geophysical Research*, Vol. 82, No. 20, 1977, pp. 2981-2987. doi:10.1029/JB082i020p02981

[20] G. A. Bollinger, M. C. Chapman and M. S. Sibol, "A Compassion of Earthquake Damage Areas a Function of Magnitude across the United States," *Bulletin of the Seismological Society of America*, Vol. 83, 1993, pp. 1064-1080.

[21] D. M. Boore, "The Richter Scale: Its Development and Use for Determining Earthquake Source Parameters," *Tectonophysics*, Vol. 166, 1989, pp. 1-14. doi:10.1016/0040-1951(89)90200-X

[22] T. S. Hanks and D. M. Boore, "Moment-Magnitude Relations in Theory and Practice," *Journal of Geophysical Research*, Vol. 89, No. B7, 1984, pp. 6229-6235. doi:10.1029/JB089iB07p06229

[23] E. M. Scordilis, "Empirical Global Relations Converting M_S and m_e to Moment Magnitude," *Journal of Seismology*, Vol. 10, 2006, pp. 225-236. doi:10.1007/s10950-006-9012-4

[24] T. Utsu, "Relationships between Magnitude Scales, International Handbook of Earthquake and Engineering Seismology, Part A," Academic Press, Waltham, 2002.

[25] C. F. Richter, "An Instrumental Earthquake Magnitude Scale," *Bulletin of the Seismological Society of America*, Vol. 25, No. 1, 1935, pp. 1-32.

[26] N. I. Christensen, "Poisson's Ratio and Crustal Seismology," *Journal of Geophysical Research*, Vol. 101, No. B2, 1996, pp. 3139-3156. doi:10.1029/95JB03446

[27] N. I. Christensen and W. D. Mooney, "Seismic Velocity Structure and Composition of the Continental Crust: A Global View," *Journal of Geophysical Research*, Vol. 100, No. B6, 1995, pp. 9761-9788. doi:10.1029/95JB00259

[28] E. Mamyrov, "New System of Quantitative Correlations between Seismic Energy, Magnitude and Energy of Seismic Radiation of the Crust Earthquakes in Tien Shan," *The 33rd General Assembly of the European Seismological Commission*, Moscow, 19-24 August 2012, pp. 29-30.

[29] H. Houston and H. Kanamori, "Source Spectra of Great Earthquakes: Teleseismic Constraints on Rupture Process and Strong Motion," *Bulletin of the Seismological Society of America*, Vol. 76, No. 1, 1986, pp. 19-42.

[30] W. Thatcher and C. Hahks, "Source Parameters of Southern California Earthquakes," *Journal of Geophysical Research*, Vol. 78, No. 35, 1973, pp. 8547-8575. doi:10.1029/JB078i035p08547

[31] K. K. Zapolskii, J. L. Nersesov, T. G. Rautian and V. I. Halturin, "Physical Basis of Magnitude Classification of Earthquakes," 1974.

[32] F. Scherbaum and D. Stoll, "Source Parameters and Scal-

ing Laws of the 1978 Schwabian Jura (Southwest Germany) Aftershocks," *Bulletin of the Seismological Society of America*, Vol. 73, No. 5, 1983, pp. 1321-1343.

[33] A. Jin, C. A. Moya and M. Ando, "Simultaneous Determination of Site Responses and Source Parameters of Small Earthquakes along the Atotsugawa Fault Zone, Central Japan," *Bulletin of the Seismological Society of America*, Vol. 90, No. 6, 2000, pp. 1430-1445. doi:10.1785/0119990140

[34] H. S. Hasegawa, "Lg-Spectra of Local Earthquake Recorder by the Eastern Canada Telemeters Network and Spectral Scaling," *Bulletin of the Seismological Society of America*, Vol. 73, 1983, pp. 1041-1061.

[35] G. Grunthal and R. Wahlstrom, "An M_W Based Earthquake Catalogue for Central, Northern and North-Western Europe Using a Hierarchy of Magnitude Conversions," *Journal of Seismology*, Vol. 7, No. 4, 2003, pp. 507-531. doi:10.1023/B:JOSE.0000005715.87363.13

[36] J. G. Ristau, G. C. Rogers and F. Cassidy, "Moment Magnitude-Local Magnitude Calibration for Earthquake in Western Canada," *Bulletin of the Seismological Society of America*, Vol. 95, No. 5, 2005, pp. 1994-2000. doi:10.1785/0120050028

[37] J. Ristau, "Comparison of Magnitude Estimates for New Zealand Earthquakes: Moment Magnitude, Local Magnitude, and Teleseismic Body-Wave Magnitude," *Bulletin of the Seismological Society of America*, Vol. 99, No. 3, 2009, pp. 1841-1852. doi:10.1785/0120080237

[38] Y. M. Wu, R. M. Allen and C. W. Wu, "Revised M_L Determination for Crustal Earthquake in Taiwan," *Bulletin of the Seismological Society of America*, Vol. 95, No. 6, 2005, pp. 2517-2524. doi:10.1785/0120050043

[39] G. L. Choy and J. Boatwright, "Global Patterns of Radiated Seismic Energy and Apparent Stress," *Journal of Geophysical Research*, Vol. 100, No. B9, 1995, 18205-18228. doi:10.1029/95JB01969

[40] Y. Zhuo and H. Kanamori, "Regional Variation of the Short-Period (1 to 10 Second) Source Spectrum," *Bulletin of the Seismological Society of America*, Vol. 77, No. 2, 1987, pp. 514-529.

[41] O. J. Perez, "Revised World Seismicity Catalog (1950-1997) for Strong ($M_S \geq 6$) Shallow ($h \leq 70$ km) Earthquakes," *Bulletin of the Seismological Society of America*, Vol. 89, No. 2, 1999, 335-341.