

Effect of Non-Uniform Basic Temperature Gradient on the Onset of Rayleigh-Bénard-Marangoni Electro-Convection in a Micropolar Fluid

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ABSTRACT

The effects of electric field and non-uniform basic temperature gradient on the onset of Rayleigh-Bénard-Marangoni convection in a micropolar fluid are studied using the Galerkin technique. The eigenvalues are obtained for an upper free/adiabatic and lower rigid/isothermal boundaries. The microrotation is assumed to vanish at the boundaries. A linear stability analysis is performed. The influence of various micropolar fluid parameters and electric Rayleigh number on the onset of convection has been analysed. Six different non-uniform temperature profiles are considered and their comparative influence on onset is discussed.

Keywords: Rayleigh-Bénard-Marangoni; Non-Uniform Temperature Gradient; Electric Field; Micropolar Fluid; Galerkin Technique

1. Introduction

Convection due to variations of densities between two parallel plates caused by temperature difference and variations of temperature dependence surface tension in a horizontal fluid layer is of great importance because of its applications in science and engineering. Such convection in literature is known as Rayleigh-Bénard and Bénard-Marangoni convection. The developments of convection and corresponding heat transfer are examples of the physical phenomena to be encountered in these types of problems. The flow associated with Rayleigh-Bénard convection and Marangoni convection has been extensively reviewed in the literature. Particularly, Marangoni convection is found to be of importance in crystal growth melts and nucleate boiling. The problem of onset of convection driven by the combined effect of buoyancy and surface tension is also important in understanding many practical problems.

Pearson [1] was the first person to give a detailed mathematical analysis for the onset of convection driven by surface tension gradients. Nield [2] studied the problem of onset of convection driven by the combined effect of buoyancy and surface tension. Later many authors, Rudraiah [3], Maekawa and Tanasawa [4] studied the

onset of convection in a horizontal layer of Newtonian fluid driven by both surface tension variations and buoyancy force by considering the non-deformable surfaces. The corresponding problem with deformable boundaries is studied by Sarma [5,6], Takashima [7], Wilson [8,9], Hashim and Wilson [10].

The application of strong electric field in a poorly conducting fluid can induce bulk motions. This phenomenon is known as **Electro convection**. In recent years, the study of the interaction of electromagnetic fields with fluid stated gain attention with the promise of new applications. Onset of natural convection in the presence of external electric field had been studied by Turnbull [11], Turnbull and Melcher [12], Takashima and Aldridge [13], Stiles [14], Stiles *et al.* [15], Siddheshwar [16], Siddheshwar and Abraham [17,18], Siddheshwar and Chan [19], Shivakumara *et al.* [20], Rudraiah *et al.* [21] and Siddheshwar and Radhakrishna [22].

Most of the previous studies were concerned with convection in Newtonian fluids. However, very less work has been done on convection in micropolar fluid with electric fields. The theory of micropolar fluids, developed by Eringen [23] is the best established theory of fluid with microstructure. Physically, the micropolar fluid consists of small, rigid, cylindrical suspended particles. Many classical flows were re-examined using the micropolar fluid to study the effect of

fluid microstructures. Thermal convection in micropolar fluids with uniform and non-uniform temperature gradients was studied by many authors (Datta and Sastry [24], Siddheshwar and Pranesh [25-27] Pranesh and Riya Baby [28], Pranesh and Arun Kumar [29]). The main result of all these studies is that the micropolar fluid heated from below is more stable compared to the fluid without suspended particles. To the best of our knowledge not much work is done in this field, except the above mentioned.

The main objective of this paper is to provide the theoretical framework for studying the thermo capillary instabilities in the presence of buoyancy instability in earth laboratories for electrically conducting micropolar fluid layers under an AC electric field and non-uniform temperature gradients. Eigen value of the problem is obtained using single term Galerkin method.

2. Mathematical Formulation

Consider an infinite horizontal layer of a Boussinesquian micropolar fluid layer of depth “ d ”. The uniform electric field is directed along z -axis (see **Figure 1**). A Cartesian coordinate system is taken with the origin in the lower boundary and z -axis vertically up-wards. Let ΔT be the temperature difference between lower and upper boundaries of the fluid. The interface at the upper boundary has a temperature dependent surface tension $\sigma(T)$. Expanding $\sigma(T)$ by Taylor series about T_0 , we get

$$\sigma(T) = \sigma_0 - \sigma_1(T - T_0) \tag{1}$$

where, $\sigma_1 = -\left(\frac{d\sigma}{dT}\right)_{T_0}$ and $\sigma_0 = \sigma(T_0)$. The form of $\sigma(T)$ in Equation (1) was used by Pearson.

The governing equations for the problem are:

Equation of continuity:

$$\nabla \cdot \mathbf{q} = 0 \tag{2}$$

Conservation of Linear Momentum:

$$\rho_0 \left[\frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q} \cdot \nabla) \mathbf{q} \right] = -\nabla p - \rho g \hat{k} + (2\zeta + \eta) \nabla^2 \mathbf{q} + \zeta \nabla \times \boldsymbol{\omega} + (\mathbf{P} \cdot \nabla) \mathbf{E}, \tag{3}$$

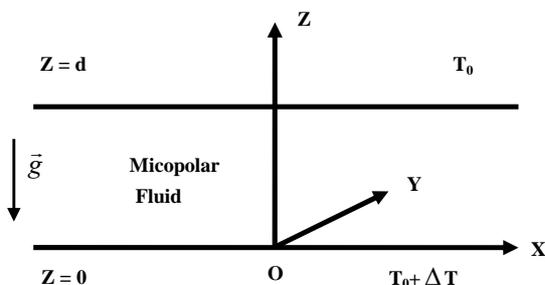


Figure 1. Schematic diagram of Rayleigh-Bénard situation for a fluid with suspended particles.

Conservation of Angular Momentum:

$$\rho_0 I \left[\frac{\partial \boldsymbol{\omega}}{\partial t} + (\mathbf{q} \cdot \nabla) \boldsymbol{\omega} \right] = (\lambda' + \eta') \nabla (\nabla \cdot \boldsymbol{\omega}) + (\eta' \nabla^2 \boldsymbol{\omega}) + \zeta (\nabla \times \mathbf{q} - 2\boldsymbol{\omega}), \tag{4}$$

Conservation of Energy:

$$\rho_0 C_{vE} \left[\frac{\partial T}{\partial t} + \left(\mathbf{q} - \frac{\beta}{\rho_0 C_v} \nabla \times \boldsymbol{\omega} \right) \cdot \nabla T \right] = K_1 \nabla^2 T, \tag{5}$$

Equation of State:

$$\rho = \rho_0 [1 - \alpha(T - T_0)]. \tag{6}$$

Equation of State for Dielectric Constant:

$$\varepsilon_r = \varepsilon_r^0 - e(T - T_0), \tag{7}$$

Faraday’s Law:

$$\nabla \times \mathbf{E} = 0, \quad \mathbf{E} = -\nabla \phi, \tag{8}$$

Equation of Polarization Field:

$$\nabla \cdot (\varepsilon_0 \mathbf{E} + \mathbf{P}) = 0, \quad \mathbf{P} = \varepsilon_0 (\varepsilon_r - 1) \mathbf{E} \tag{9}$$

where, \mathbf{q} is the velocity, $\boldsymbol{\omega}$ is the spin, p is the hydrodynamic pressure, T is the temperature ρ_0 is the density of the fluid at a reference temperature $T = T_0$, ζ is the coupling viscosity coefficient, η is the shear kinematic viscosity coefficient, I is the moment of inertia, λ' and η' are bulk and shear spin viscosity coefficients, β is a Micropolar heat conduction coefficient, c_v is the specific heat, χ is the thermal conductivity, α is the coefficient of thermal expansion, \mathbf{E} is the electric field, \mathbf{P} is the dielectric polarization, ε_0 is the electric permeability of free space and ε_r is the dielectric constant.

The “Equations (2)-(9)” are solved subject to containment conditions appropriate for a rigid and thermally perfect conducting wall on the underside and a free surface on the upper side. This free surface is adjacent to a non-conducting medium and subject to a constant heat flux. Further, the no-spin boundary condition is assumed for micro-rotation. Since the shear stress for a non-classical fluid with suspended particles is no different from that of classical fluids, the boundary conditions for flat free boundaries used by Nield [2] in respect of Newtonian fluids are appropriate for micropolar fluids also.

In the quiescent state the velocity \mathbf{q} , the spin $\boldsymbol{\omega}$, the temperature T , the electric field \mathbf{E} , the pressure p , the density ρ , the dielectric polarization \mathbf{P} have the following solution:

$$\left. \begin{aligned} \mathbf{q} = 0, \boldsymbol{\omega} = 0, p = p_b(z), \mathbf{E} = E_b(z) \hat{k}, T = T_b(z), \\ \frac{-d}{\Delta T} \frac{dT_b}{dz} = f(z) \end{aligned} \right\} \tag{10}$$

where $f(z)$ is a non-dimensional basic temperature gradient, satisfying the condition $\int_0^1 f(z) dx = 1$.

Since the aim of the present study is to understand control of convection in a micropolar fluid layer by different basic temperature profiles, the various non-uniform basic temperature gradients considered in this study are represented in **Table 1**.

This type of basic temperature gradients arises due to sudden heating or cooling, radiation, through flow, etc.

In order to investigate the stability of the basic solution, infinitesimal disturbances are introduced in the form:

$$\left. \begin{aligned} \mathbf{q} &= \mathbf{q}_b + \mathbf{q}', \quad \boldsymbol{\omega} = \boldsymbol{\omega}_b + \boldsymbol{\omega}', \quad p = p_b + p', \quad \mathbf{E} = \mathbf{E}_b + (E'_1, E'_3), \\ \rho &= \rho_b + \rho', \quad T = T_b + T', \quad \mathbf{P} = \mathbf{P}_b + (P'_1, P'_3). \end{aligned} \right\} \quad (11)$$

where the primes indicate that the quantities are infinitesimal perturbations assumed to be small and subscript 'b' indicates basic state value.

We assume the principal of exchange of stability to be valid and hence deal with only stationary convection. Introducing the electric potential ϕ' and substituting "(11)" into "(2)-(9)" and using the basic state solution (10), we get linearized equation governing the infinitesimal perturbations in the form:

$$\nabla \cdot \mathbf{q}' = 0 \quad (12)$$

$$\rho_o \left[\frac{\partial \mathbf{q}'}{\partial t} \right] = -\nabla p - \rho' g \hat{k} + (2\zeta + \eta) \nabla^2 \mathbf{q}' + (\zeta \nabla \times \boldsymbol{\omega}') + (\mathbf{P}_b \cdot \nabla) \mathbf{E}' + (\mathbf{P} \cdot \nabla) \mathbf{E}_b, \quad (13)$$

$$\rho_o I \left[\frac{\partial \boldsymbol{\omega}'}{\partial t} \right] = (\lambda' + \eta') \nabla (\nabla \cdot \boldsymbol{\omega}') + (\eta' \nabla^2 \boldsymbol{\omega}') + \zeta (\nabla \times \mathbf{q}' - 2\boldsymbol{\omega}'), \quad (14)$$

$$\frac{\partial T'}{\partial t} = \frac{\Delta T}{d} \left(\mathbf{q}' - \frac{\beta}{\rho_o c_v} \nabla \times \boldsymbol{\omega}' \right) f(z), \quad (15)$$

$$\rho' = -\alpha \rho_o T' \quad (16)$$

Table 1. Non-uniform basic temperature gradient profile.

Model	Basic temperature gradients	$f(z)$
1	Linear	1
2	Heating from below	$\begin{cases} \varepsilon^{-1} & 0 \leq z < \varepsilon \\ 0 & \varepsilon < z \leq 1 \end{cases}$
3	Cooling from above	$\begin{cases} 0 & 0 \leq z < 1 - \varepsilon \\ \varepsilon^{-1} & 1 - \varepsilon < z \leq 1 \end{cases}$
4	Step function	$\delta(z - \varepsilon)$
5	Inverted Parabolic	$2(1 - z)$
6	Parabolic	$2z$

The perturbation "(12)-(15)" are non-dimensionalised using the following definitions:

$$\left. \begin{aligned} (x^*, y^*, z^*) &= \frac{(x, y, z)}{d}, \quad W^* = \frac{W'}{(\chi/d)}, \\ \boldsymbol{\omega}^* &= \frac{\boldsymbol{\omega}'}{(\chi/d^2)}, \quad t^* = \frac{t}{(d^2/\kappa)}, \quad T^* = \frac{T'}{\Delta T}, \\ \phi^* &= \frac{\phi'}{\left(\frac{eE_0 \Delta T d}{1 + \chi_e} \right)}, \quad \Omega^* = \frac{\nabla \times \mathbf{w}}{(\chi/d^3)}. \end{aligned} \right\} \quad (17)$$

Using "(16)" in "(13)", operating curl twice on the resulting equation, operating curl on "(14)" and non-dimensionalising the two resulting equations and also "(15)", we get

$$\begin{aligned} R \nabla_1^2 T + (1 + N_1) \nabla^4 W + N_1 \nabla^2 \Omega_z \\ + L \nabla_1^2 T f(z) - L \frac{\partial}{\partial z} (\nabla_1^2 \phi) f(z) = 0, \end{aligned} \quad (18)$$

$$N_3 \nabla^2 \Omega_z - N_1 \nabla^2 W - 2N_1 \Omega_z = 0, \quad (19)$$

$$\nabla^2 T + (W - N_5 \Omega_z) f(z) = 0, \quad (20)$$

$$\nabla^2 \phi - \frac{\partial T}{\partial z} = 0. \quad (21)$$

where the non-dimensional parameters N_1, N_3, N_5, R and L are given by :

$$N_1 = \frac{\zeta}{\zeta + \eta} \quad (\text{Coupling Parameter}),$$

$$N_3 = \frac{\eta'}{(\zeta + \eta) d^2} \quad (\text{Couple Stress Parameter}),$$

$$N_5 = \frac{\beta}{\rho_o C_v d^2} \quad (\text{Micropolar Heat Conduction Parameter}),$$

$$R = \frac{\rho_o \alpha g \Delta T d^3}{\kappa (\zeta + \eta)} \quad (\text{Rayleigh number}),$$

$$L = \frac{\varepsilon_o e^2 E_o^2 \Delta T}{\alpha_1 g \rho_o (1 + \chi_e) h} \quad (\text{Electric Rayleigh number}).$$

The infinitesimal perturbations W, Ω_z, T and ϕ are assumed to be periodic waves and hence these permits and normal mode solution in the following form:

$$\begin{bmatrix} W \\ \Omega_z \\ T \\ \phi \end{bmatrix} = \begin{bmatrix} W(z) e^{i(l_x + m_y)} \\ \Omega_z(z) e^{i(l_x + m_y)} \\ T(z) e^{i(l_x + m_y)} \\ \phi(z) e^{i(l_x + m_y)} \end{bmatrix} \quad (22)$$

where l and m are the horizontal components of the wave number a .

We substituting “(22)” into “(18)-(21)”, then we get

$$(1 + N_1)(D^2 - a^2)^2 W - Ra^2 T + N_1(D^2 - a^2)G \tag{23}$$

$$-La^2 T f(z) + La^2 D\phi f(z) = 0,$$

$$N_3(D^2 - a^2)G - 2N_1 G - N_1(D^2 - a^2)W = 0, \tag{24}$$

$$(D^2 - a^2)T + (W - N_5 G)f(z) = 0, \tag{25}$$

$$(D^2 - a^2)\phi - DT = 0, \tag{26}$$

where $D = \frac{d}{dz}$ and $a^2 = l^2 - m^2$.

In “(23)-(26)”, the asterisks have been omitted.

“(23)” and “(26)” are solved subject to the following boundary conditions:

where,

$$Y_1 = \left[(1 + N_1) \langle T_1 (D^2 - a^2) T_1 \rangle \right] \left[N_3 \langle G_1 (D^2 - a^2) G_1 \rangle - 2N_1 \langle G_1^2 \rangle \right],$$

$$Y_2 = N_1 N_5 \langle G_1 (D^2 - a^2) W_1 \rangle \langle f(z) G_1 T_1 \rangle - \langle f(z) W_1 T_1 \rangle Y_1,$$

$$Y_3 = 2a^2 \langle W_1 D^2 W_1 \rangle - \langle (D^2 W_1)^2 \rangle - a^4 \langle W_1 \rangle,$$

$$Y_4 = \langle f(z) W_1 T_1 \rangle - \frac{\langle W_1 D\phi f(z) \rangle \langle \phi_1 D T_1 \rangle}{\langle \phi_1 (D^2 - a^2) \phi_1 \rangle},$$

$$Y_5 = \frac{\left[N_1^2 \langle T_1 (D^2 - a^2) T_1 \rangle \langle \phi_1 (D^2 - a^2) \phi_1 \rangle \langle W_1 (D^2 - a^2) G_1 \rangle \langle G_1 (D^2 - a^2) W_1 \rangle \right]}{\langle \phi_1 (D^2 - a^2) \phi_1 \rangle}.$$

In “(28)”, $\langle -- \rangle$ denotes integration with respect to z between $z=0$ and $z=1$.

$M(W_1, G_1, T_1, \phi_1)$ in “(28)” is a functional and Euler-Lagrange equations for the extremisation of M “(23)-(26)”.

The trial functions satisfying “(27)” are

$$\left. \begin{aligned} W_1 &= z^2(1 - z^2), \\ T_1 &= z(2 - z), \\ G_1 &= z(1 - z), \\ \phi_1 &= z^2(3z - 2z^2). \end{aligned} \right\} \tag{29}$$

such that they satisfy all the boundary conditions “(27)” except the one given by $D^2 W + a^2 M T = 0$ at $z=1$, but the residual from this is included in the residual from the differential equations. Substituting “(29)” in “(28)” and performing the integration, we can calculate the critical Marangoni number M_c which attains its minimum at a_c^2 .

$$\left. \begin{aligned} W = DW = T = G = 0, & \quad \text{at } z = 0. \\ W = D^2 W + a^2 M T = DT = G = 0, & \quad \text{at } z = 1. \end{aligned} \right\} \tag{27}$$

where M is the Marangoni number.

“(27)” indicates the use of rigid, isothermal lower boundary and upper, free, thermally insulating boundary (with respect to perturbation). The condition on G is spin-vanishing boundary condition.

We now use the single term Galerkin expansion technique to find the critical eigenvalue. Multiplying “(23)” by W , “(24)” by G , “(25)” by T and “(26)” by ϕ , integrating the resulting equations by parts with respect to z from 0 to 1, using the boundary conditions “(27)” and using $W = AW_1$, $G = BG_1$, $T = CT_1$ and $\phi = E\phi_1$ in which A, B, C and E are constants and W_1, G_1, T_1 and ϕ_1 are trial functions, yield the following equation for the Marangoni number M :

$$M = \frac{[Y_1 Y_3 + Ra^2 \langle W_1 T_1 \rangle Y_2 + La^2 Y_4 Y_2 - Y_5]}{a^2 D W_1(1) T_1(1) (1 + N_1) Y_2} \tag{28}$$

3. Result and Discussion

As per the motivation stated in the introduction to the paper we consider the effects of electric Rayleigh number, micropolar fluid parameters and non-uniform basic temperature distribution on the onset of Rayleigh-Bénard-Marangoni convection in a micropolar fluid layer. These three effects are respectively represented by the L , $(N_1, N_3$ and $N_5)$ and $f(z)$ in the Eigen value “(28)”. Due to the thermodynamics restrictions the following values of N_1, N_3 and N_5 are chosen:

$$0 \leq N_1 \leq 1; \quad 0 \leq N_3 \leq m; \quad 0 \leq N_5 \leq n$$

where, m and n are finite, positive real numbers (see Siddheshwar and Pranesh [29]) and the values of L are those chosen for Newtonian fluid problems. One linear and five non-linear basic temperature gradients are considered in the study (see **Table 1**). The results obtained in the study are depicted by the **Figures 2-5**. It is observ-

ed that for the critical Marangoni number, M_c , the following inequality holds for the six models chosen in the study:

$$M_{c_4} < M_{c_3} < M_{c_6} < M_{c_2} < M_{c_1} < M_{c_5},$$

i.e., step function is the most destabilizing temperature profile and inverted parabolic temperature is the most stabilizing temperature profile. In the case of piecewise linear and step function profiles, the critical Marangoni number, M_c , depends on the thermal depth, ε , in addition to depending on the parameters of the problem. In the

case of piecewise linear profile heating from below, cooling from above and step function profiles, the minimum value of M_c is attained at $\varepsilon = 0.95$, $\varepsilon = 0.45$ and $\varepsilon = 0.75$ respectively. With the above background and with the motive specified in the introduction we now discuss the results presented by the **Figures 2-5**.

Figure 2 is the plot of critical Marangoni number M_c versus coupling parameter N_1 for different non uniform temperature gradients and for different electric Rayleigh number L . Clearly M_c increases with N_1 . Increase in N_1 indicates increase in the concentration of micro elements. These elements consume the greater part of the energy of

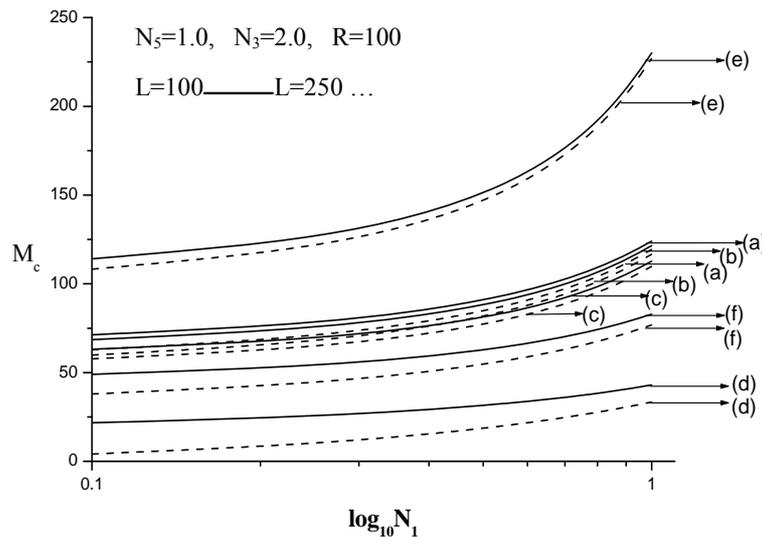


Figure 2. Plot of critical Maragoni number M_c versus coupling parameter N_1 for different non -uniform temperature gradient and different electric Rayleigh Number L . (a) Linear, (b) Heating from below, (c) Cooling from above, (d) Step function, (e) Parabolic, (f) Inverted parabolic.

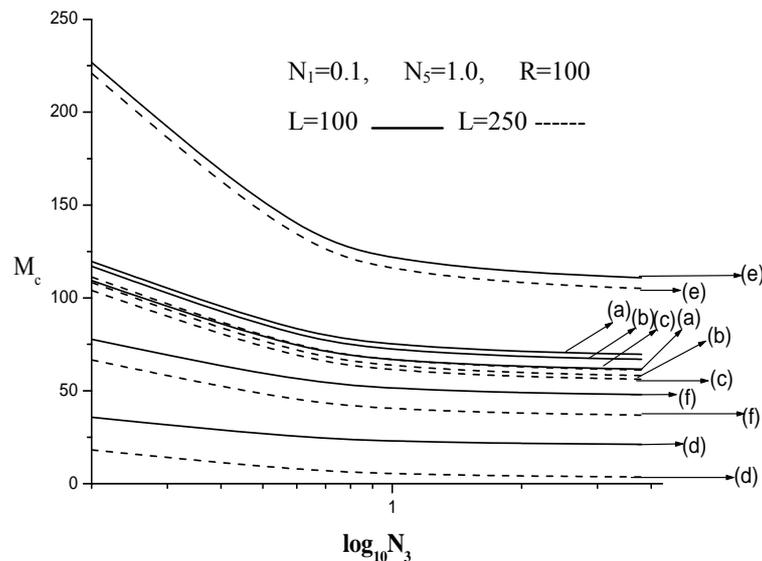


Figure 3. Plot of critical Maragoni number M_c versus coupling parameter N_3 for different non-uniform temperature gradient and different Electric Rayleigh number L . (a) Linear, (b) Heating from below, (c) Cooling from above, (d) Step function, (e) Parabolic, (f) Inverted parabolic.

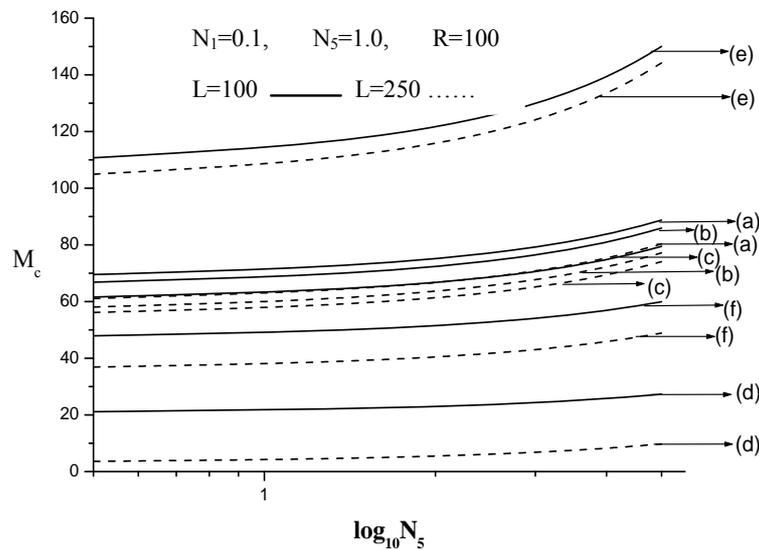


Figure 4. Plot of critical Maragoni number M_c versus coupling parameter N_5 for different non-uniform temperature gradient and different Electric Rayleigh number L . (a) Linear, (b) Heating from below, (c) Cooling from above, (d) Step function, (e) Parabolic, (f) Inverted parabolic.

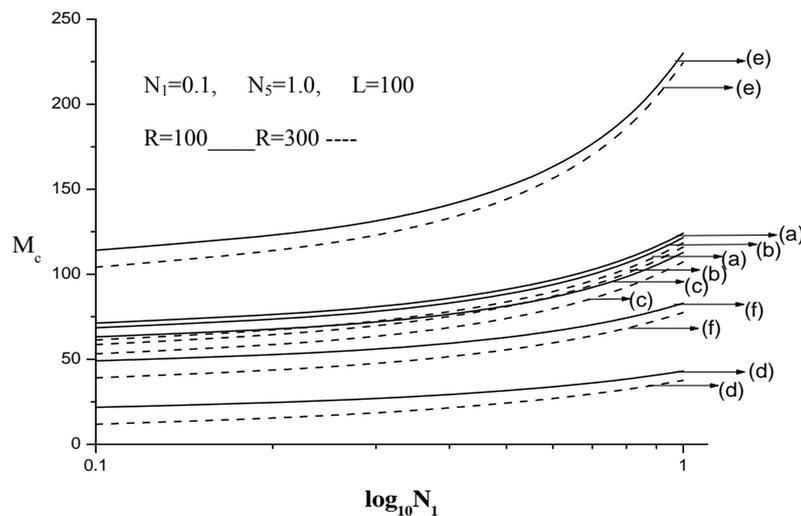


Figure 5. Plot of critical Maragoni number M_c versus coupling parameter N_1 for different non-uniform temperature gradient and different Rayleigh number R . (a) Linear, (b) Heating from below, (c) Cooling from above, (d) Step function, (e) Parabolic, (f) Inverted parabolic.

the system in developing the gyrotational velocity of the fluid and as a result the onset of convection is delayed. Therefore, the increase N_1 is to stabilize the system. It is also observed that increase in electric Rayleigh number decreases the M_c . Electric Rayleigh number L is the ratio of electric force to gravitational force. Thus, L has a destabilizing effect on the system.

Figure 3 is the plot of M_c versus coupling stress parameter N_3 for different non uniform temperature gradients and for different electric number L . Clearly M_c decreases with increase in N_3 . Increase in N_3 , decreases the couple stress of the fluid which causes a decrease in mi-

crorotation and hence makes the system more unstable.

Figure 4 is the plot of M_c versus micropolar heat conduction parameter N_5 for different non uniform temperature profiles and for different electric number L . Clearly M_c increases with N_5 . When N_5 increases, the heat induced in the fluid due to these microelements also increases which reduces the heat transfer from bottom to top. The decrease in heat transfer is responsible for delaying the onset of instability. Thus increase in N_5 is to stabilize the system.

Figure 5 is the plot of M_c versus coupling parameter N_1 for different non uniform temperature gradients and

for different Rayleigh number R . Clearly M_c increases with N_1 . It is also observed that the increase in R decreases the M_c and ultimately we get the case of convection dominated by buoyancy force.

From the **Table 2** it is clear that with an increase in N_1 , M_c becomes zero for higher values of R compared to Newtonian fluids. Similar results have been obtained in respect to the influences of N_3 and N_5 . Treating Marangoni number as the critical parameter we find from the **Table 3** that any particular infinitesimal disturbance can be destabilized with electric field. The results of the **Table 3** are in respect of linear temperature gradient. This general observation on the destabilizing influence of electric field, however holds good for the other five temperature gradients as well. From **Table 3** as well as **Table 2**,

we also note that as R increases M_c decreases. From **Table 4**, it is found that the critical wave number is in general insensitive to the changes in the micropolar parameters but is influenced by the electric field. This indicates that, the convection cell at the onset increases in size with the increase in electric field. In the limit L tends to zero, we recover the results of Siddheshwar and Pranesh [29] for Chandrasekhar number tends to zero from the present study.

The above result indicates that externally applied electric field is an effective means of controlling, Rayleigh-Bénard-Marangoni convection in micropolar fluids. The result also suggests that Rayleigh-Bénard-Marangoni convection in Newtonian fluids may be delayed by adding micron sized suspended particles. Further by creating

Table 2. Critical Marangoni number (M_c) _{j} ($j = 1$ to 6) for different values of L, R and $N_1, L = 100, N_3 = 2.0, N_5 = 1.0$.

N_1	R	M_{c_1}	M_{c_2}	M_{c_3}	M_{c_4}	M_{c_5}	M_{c_6}
0.5	0	94.4581	83.1598	69.7583	17.9915	148.788	66.3832
	100	90.8073	79.509	75.8051	14.3407	145.137	62.7324
	492.78	76.4677	65.1694	61.4655	0.001156	130.797	48.3928
	500	76.2042	64.9058	61.2019	-0.26243	130.534	48.1292
	1000	57.9502	46.6518	42.9479		112.28	29.8753
	1818.29	28.0761	16.7778	13.0738		82.4057	0.001178
	2176.39	15.0026	3.70426	0.0003483		69.3322	
	2277.85	11.2985	0.000167			65.6281	
	2500	3.18829				57.5179	
	2587.32	0.0004127				54.33	
4075.47					0.0007194		
1.0	0	126.791	117.888	111.67	32.9602	228.56	85.6961
	100	124.053	115.15	108.932	30.2221	225.822	82.958
	500	113.101	104.197	97.9798	19.2698	214.87	72.0056
	1000	99.4104	90.5069	84.2893	5.57929	201.179	58.3151
	1203.75	93.8315	84.9281	78.7105	0.000422	195.601	52.7363
	2500	58.3389	49.4355	43.2179		160.108	17.2437
	3129.69	41.0974	32.194	25.9764		142.867	0.002182
	4078.39	15.1211	6.21768	0.0000763		116.89	
	4305.47	8.90344	0.0000154			110.673	
	4630.59	0.0013457				101.77	
8347.39					0.0009143		

Table 3. Critical Marangoni number M_c for different values of L and R with $f(z) = 1, N_1 = 0.5, N_3 = 2.0, N_5 = 1.0$.

	$L = 0$	$L = 100$	$L = 200$	$L = 500$
R	M_c	M_c	M_c	M_c
0	98.5737	94.4581	90.3407	77.9774
100	94.9229	90.8073	86.6899	74.3266
200	91.2721	87.1565	83.0392	70.6758
1000	62.0657	57.9502	53.8328	41.4694
2135.89	20.5967	16.4812	12.3638	0.000432
2474.48	8.2355	4.11997	0.002584	
2587.33	4.11558	0.00004762		
2700	0.002234			

Table 4. The values of critical wave number a_c^2 for various values of N_1, N_3, N_5 and L with $f(z) = 1$ and for all values of R .

$L = 100, R = 100$				$L = 200, R = 100$			
N_1	N_3	N_5	a_c^2	N_1	N_3	N_5	a_c^2
0.1	2	1	5.85319	0.1	2	1	5.7939
0.5			5.85469	0.5			5.81864
1			5.75689	1			5.73666
0.1	2	1	5.85319	0.1	2	1	5.7939
	4		5.85167		4		5.7939
	6		5.85103		6		5.7939
0.1	2	0.5	5.85319	0.1	2	0.5	5.7939
		1	5.85319			1	5.7939
		1.5	5.85319			1.5	5.7939

conditions for an appropriate basic temperature gradient, we can also make an appropriate decision delaying the convection. It has been observed that parabolic temperature profile is the most stabilizing temperature profile and step function is the most destabilizing temperature profile.

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