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A Study on Lucas' "Expectations and the Neutrality of Money"—II

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ABSTRACT

My preceding paper on this topic (Otaki [1]) explored whether the equilibrium existence proof in Lucas [2] is truly complete. We showed that the proof is incomplete that some additional conditions are required to complete the job. In this paper, we explore another ambiguity in Lucas's model, which has been pointed out by Grammond (see Lucas [3]): can the model transform the joint probability density function of the exogenous environment into one that which includes market equilibrium information? This problem is peculiar to the signal extraction problem compatible with the market equilibrium condition. The result indicates that although Lucas [3] was fundamentally correct in refuting Grammond's critique, the model contains another crucial assumption concerning the property of the equilibrium function, namely, one-to-one correspondence from the environmental variable to the equilibrium price, which has not been proved by Lucas [2] to date.

Keywords: Signal Extraction Problem Compatible with the Market Equilibrium; Transformation of the Joint Distribution Function; One-to-One Correspondence from the Environmental Variable to the Equilibrium Price

1. Introduction

The essence of Lucas [2] is a signal extraction problem, which is compatible with the market equilibrium condition. This paper attempts to solve a related and very difficult problem. The functional form of equilibrium price is inseparably connected with the results of signal extraction. In other words, whenever individuals try to induce relevant information about their environment through market mechanisms, they need to have exact knowledge of the equilibrium price function.

This fact implies that information on the exogenous environment, which is indispensable for optimal decision making, is never extracted without using the equilibrium price function. Mathematically, although it can never be directly observable, the probability distribution of the economic environment (*i.e.*, money supply per capita and population of young generation) can be fixed by assumption. However, the joint and/or conditional distribution of the environment and equilibrium price cannot be defined without an equilibrium function given a-priori (although it must be consistent with the rational expectations equilibrium (REE)). Thus, as suggested by Grammond, there emerges a room of multiplicity in the endogenously determined environment/equilibrium-price probability distribution.

We have succeeded in showing that the joint distribution of the environment/equilibrium-price necessarily becomes multiple as asserted by Grammond, and that the conditional distribution of these variables is free from the specification of the equilibrium price function. In other words, the unique conditional density function, independent of the shape of the equilibrium price function, is consistently obtained in using Lucas' model [2]. At this point, the transformation between the probability distribution functions in Lucas [2] is verified under an additional assumption, that is, one-to-one correspondence in the equilibrium price function.

This paper is organized as follows. Section 2 clarifies the theoretical problem, and establishes the theorem concerning the existence of the unique conditional probability function. Section 3 contains brief concluding remarks.

2. Necessary Modifications

2.1. Problem to Be Clarified

Lucas' original maximization [2] is expressed as

$$h\left(\frac{mx}{\theta p}\right)\frac{1}{p}=\int V'\left(\frac{mxx'}{\theta p'}\right)\frac{x'}{p'}\mathrm{d}F\left(x',p'\middle|m,p\right),\qquad(1)$$

where the left-hand-side (1) presents the marginal utility derived from current consumption, and the right-hand-side, that from future consumption. That is, (1) is the compounded Euler equation with the market clearing condition in Lucas' model [2]. Each individual maximizes lifetime utility using currently available information (m,p).

Lucas specifies the equilibrium-price function as

$$p=m\phi(z),z\equiv\frac{x}{\theta}.\qquad(2)$$

Substituting (2) into (1) gives the following equation according to Lucas [2]:

$$h\left(\frac{x}{\theta\phi(z)}\right)\frac{x}{\theta\phi(z)}=\int V'\left(\frac{zx'}{\xi\phi(z')}\right)\frac{zx'}{\xi\phi(z')}\mathrm{d}G\left(z',\theta'\middle|z\right),\quad(3)$$

where ξ is the current additional money supply unknowable to individuals. The random variable ξ should be strictly distinguished from x, which means the realized value of ξ [1]. Clearly, the imperfect informational structure of the model, which is the backbone of the Lucas' model [2], requires urgent correction.

The main issue, this article deals with, is whether the transformation from (1) to (3) is independent of the functional form of (2). Since x and θ are assumed to follow independently identically distributed (*i.i.d.*) processes, the problem thus converges to whether the conditional probability distribution function G can be defined independently of the form of the tentatively fixed equilibrium-price function in (2), as expressed by (3).

2.2. Theorem

In this subsection, using Lucas' model [2], we disprove Grammond's critique and provide a positive answer to the question posed in the previous section. That is,

Theorem 1. *The conditional cumulative distribution function* $G\left(\xi,z'\middle|z\right)$ *is invariant with the form of the arbitrarily fixed equilibrium-price function* $\phi(\cdot)$, if $\phi(\cdot)$ is a one-to-one correspondence between Z and $\Re^{1}_{++}$, where Z is the domain of z.

Proof. Since (x',θ') are *i.i.d.* processes and are thereby independent of ξ, we should focus on the relationship between the exogenously given joint density function $J(\xi,z)$ and $\tilde{G}(\xi,z)\equiv\dfrac{G(\xi,z',z)}{f(z')}$. f denotes the density function of z'.

By definition and using the formula of transformation of the distribution function, we obtain

$$\tilde{G}(\xi,z)=\frac{1}{\phi'(p)}J\left(\xi,\phi^{-1}(p)\right).\qquad(4)$$

Thus, the transformation between joint distribution functions depends on the shape of ϕ as suggested by Grammond. However, since the Jacobean of J does not contain ξ, the conditional distribution function of G becomes

$$G\left(\xi\middle|z\right)=\frac{J(\xi,z)}{\int J(\xi,z)\mathrm{d}\xi}.\qquad(5)$$

Hence, as long as the inverse function of $\phi(\cdot)$ is well-defined (*i.e.*, $\phi(\cdot)$ is a one-to-one correspondence), as shown by (5), $G\left(\xi\middle|z\right)$ is independent of the functional form of $\phi(\cdot)$.

Accordingly, we have succeeded in validating Lucas' transformation [2] between (1) and (3).

2.3. Caveat

Despite the validity of Lucas' transformation [2], a caveat is necessary. He defines operator in a Banach space with supnorm $B:T:B\to B$ as

$$T_f\equiv\int G_2\left(G_1\left(\frac{zx'}{\xi}\mathrm{e}^{f(z')}\right)\right)\mathrm{d}J\left(x',z'\right)\mathrm{d}J\left(\xi\middle|z\right),$$

where G_1 is the inverse function of $h(x)x$ and $G_2(x)\equiv V'(x)x$. Under an additional restrictive condition proposed by Otaki [1], T becomes a contraction mapping in B.

Nevertheless, we must note that there is no guarantee that the corresponding image T_f is also invertible, for any fixed invertible f. Consequently, the existence proof of Lucas [2] remains incomplete.

3. Concluding Remarks

This article examined whether Lucas' signal extraction problem [2] under general equilibrium has been properly formulated. The results are as follows.

First, Lucas [2] rigorously induced the conditional distribution function compatible with market equilibrium. This *compatibility* means that the objective conditional distribution function concerning relevant environmental information is not affected by the form of arbitrarily chosen equilibrium price function. In this sense, Grammond's critique, which was also mentioned in Lucas [3], is not a serious problem.

However, the invertibility of the equilibrium-price function is crucial to infer signals correctly. The results also indicate that Lucas [2] failed to consider whether this property is preserved within the functional operator T_f.

[1] As Otaki [1] pointed out, Lucas [2] ultimately equalized ξ to x. This inappropriate calculation leads to a serious incompleteness in the existence proof of the REE. See Otaki [1] for more detail.

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