

# Influence of Cutting and Geometrical Parameters on the Cutting Force in Milling

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## Abstract

This paper presents a numerical modelling of the dynamic behaviour of the Machine-Tool-Part system (MOP) in milling. The numerical study of such complex structure requires the use of sophisticated method like finite elements one. Simulation is employed to predict cutting forces and dynamic response of Machine-Tool-Part system (MOP) during end-milling operation. Finally, spectral analysis is presented to see the influence of feed direction in the vibration.

**Keywords:** Milling, Modelling, Cutting Force, Finite Element

## 1. Introduction

Milling is frequently employed in manufacturing processes for producing parts with desirable dimensions and shape. However, the quality of the obtained surface is mostly influenced by two vibratory phenomena. In the first hand, the grazing provoked mainly by the mechanism of regeneration of the manufactured surfaces leads to chatter vibration. In the second hand, a forced vibration appears owing to the periodicity of the cut. Models describing cutting tool vibrations began to appear in literature about several decades ago. Tobias [1] and Tlustý [2], almost simultaneously, published papers explaining chatter as a regenerative phenomenon which is widely accepted as a cause of self-excited vibrations on machine tools. Then, large number of dynamic models of cutter and part are presented by authors. In their work Montgomery and Altintas [3,4], use a refined time domain simulation model which incorporates the combined horizontal feed and rotation of the cutter to predict cutting force.

Because, major developments have been designed for aeronautic industry where tools are mostly more flexible than part, Altintas and Budak [5] have proposed an analytic method for computing stability lobes. They analysed the geometrical nonlinearities of the milling proc-

ess. After that, they proposed a solution using Fourier series development and by considering the zero order term. In 1998, they prolonged this work by taking account of the behaviour of the part. These authors worked also on the prediction of the profile defects [7,8]. In these works the study relates the thin wall milling where the defects are due to the static deflections of the tool and the piece and the vibrations are not taken in account. Recently, there have been quite a number of published articles on the bifurcation phenomena in interrupted cutting processes. Indeed, Insperger *et al.* [9], Corpus *et al.* [10], Bayly *et al.* [11], Davies *et al.* [12], Peigné *et al.* [13] proposed analytical way of stability loss in milling processes. All results are approved experimentally by Bayly *et al.* [11], Davies *et al.* [12], Peigné *et al.* [13], Mann *et al.* [14,15] and R.Saffar *et al.* [16].

In fact, the large number of machining operations and the factors that influence each operation lead difficult to develop models for predicting the cutting phenomenon. Since, the concept of the undeformed chip thickness has been used by researchers to predict the cutting forces. In reality, it is very difficult to measure the length of the shear line and to find its relation with other measurable variables such as speed or accelerations. Consequently, the most used model is linear cutting and cutting force is proportional to the undeformed chip thick-

ness. In this work, we start by presenting most popular models. Then, comparison between three models is presented. Finally, variation of cutting forces is presented for different cutting parameters and spectral response is also simulated.

### 2. Dynamic Cutting Force Models

In their study M. T. Zaman *et al.* [17] have simulated peripheral milling to develop a three-dimensional analytical model of cutting force. So it is necessary to determine the analytical expressions for  $F_x$ ,  $F_y$  and  $F_z$  which are presented in **Figure 1**.

Cutting force expressions ( $F_x$  and  $F_y$ ) was extracted from Tlustý *et al.* [7] and Bao *et al.* [18] with little modification. In fact, they introduce in their equations horizontal component of tangential force. Thus, cutting force can be written as below:

$$\begin{aligned}
 F_{xi} &= (F_{ti} \cos \psi) \sin \phi_i - F_{ri} \cos \phi_i \\
 F_{yi} &= -(F_{ti} \cos \psi) \cos \phi_i - F_{ri} \sin \phi_i \\
 F_{zi} &= -F_{ti} \sin \psi \\
 F_{ci} &= \sqrt{F_{xi}^2 + F_{yi}^2 + F_{zi}^2}
 \end{aligned}
 \tag{1}$$

By confronting analytical and experimental study they confirm that tangential cutting force arises when cutting depth arises. Despite, this cutting force model can be used to simulate cutting force with an average of 90%.

In general, cutting force model used is the most classical. In general, cutting force depends on cutting depth. However, in dynamic milling cutting depth depends on tool and part position and the profile of machined surface (Cutting force is presented in **Figure 2**).

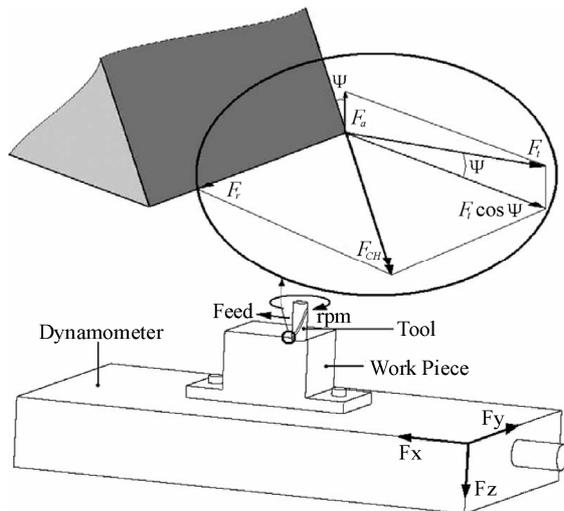


Figure 1. Cutting force component [16].

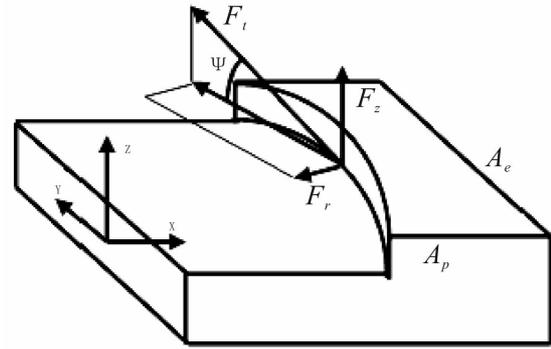


Figure 2. Cutting force model in milling.

$$\begin{aligned}
 F_{ij} &= K_t \cdot a_p \cdot h(\phi_j) \\
 F_{rj} &= K_r F_{ij}
 \end{aligned}
 \tag{2}$$

$h$  is the instantaneous cutting thickness

$$h(\phi_j) = g(\phi_j) [\Delta x \sin(\phi_j) + \Delta y \cos(\phi_j)]
 \tag{3}$$

where

$$\begin{aligned}
 \Delta x &= x(t) - x(t - \tau) \\
 \Delta y &= y(t) - y(t - \tau)
 \end{aligned}$$

And 
$$g(\phi_j) = \begin{cases} 0 & \text{si } \phi_e < \phi_j, \phi_j < \phi_s \\ 1 & \text{si } \phi_e > \phi_j > \phi_s \end{cases}$$

$\phi_j$  is angle of cutter rotation of a point on the cutting edge.

**Figure 3** represents tool position compared with part position,  $e$  is distance between tool and part axis,  $\phi_e$  is entry angle,  $\phi_s$  is leaving angle.

Cutting phase is defined by angles  $\phi_e$  and  $\phi_s$  in the metal. These angles are calculated compared with nominal position. Thus, transitory phases of beginning and end machining are not modeled. However, if the cutting is discontinued, the tool can leave the metal.

In this case, cutting depth is null and cutting forces are also null. This non linearity permits to amplify vibration amplitude.

Angle  $\phi_e$  and  $\phi_s$  are defined as below:

$$\begin{aligned}
 \cos \phi_e &= \frac{A_e + 2e}{D} \\
 \cos \phi_s &= \frac{A_e - 2e}{D}
 \end{aligned}
 \tag{4}$$

### 3. Dynamic Response

The first step consists to meshing the s Machine-Tool-Part structure. So, voluminal elements are generated for modelling all components of milling machine,

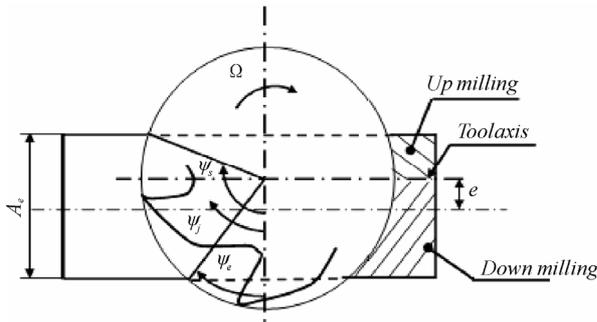


Figure 3. Tool position compared with part one.

tool and part. Figure 4 shows the photograph and the finite elements modeling of used milling machine.

Generally the dynamic analysis of the system by finite elements is reduced to the resolution of a linear system of the second order differential equations whose rigidity matrix depends on time:

$$[M]\{\ddot{X}(t)\} + [C]\{\dot{X}(t)\} + [K]\{X(t)\} = \{F(t)\} \quad (5)$$

Where  $M$ ,  $C$  and  $K$  are respectively mass matrix, damping matrix and stiffness matrix,  $X$  is generalized displacement vector and  $F(t)$  is cutting force vector. All coefficients of these matrixes are determined from finite element modelisation of the total system tool-machine-part.

$$\begin{aligned} [\phi_r]^T [M][\phi_r]\{\ddot{q}(t)\} + \\ [\phi_r]^T [K][\phi_r]\{q(t)\} = [\phi_r]^T \{F(t)\} \end{aligned} \quad (6)$$

The matrix  $[M]$ ,  $[K]$  and  $[C]$  projected on the basis of the eigenmodes leads to matrix  $[I]$ ,  $[\Lambda]$  and  $[D]$  by:

$$\begin{aligned} [\phi_r]^T [K][\phi_r] &= [\Lambda] \\ [\phi_r]^T [M][\phi_r] &= [I] \\ [\phi_r]^T [C][\phi_r] &= [D] \end{aligned} \quad (7)$$

So dynamic equation projected on the basis of the eigenmodes was reduced to the system of equations in the following form:

$$[I]\{\ddot{q}(t)\} + [C_{ii}]\{\dot{q}(t)\} + [\Lambda]\{q(t)\} = \{f(t)\} \quad (8)$$

### 4. Numerical Results

The resolution of dynamic Equation (6) by numerical method of resolution like Newmark and Newton Raphson permits the determination of the displacement of the tool in the x-y directions. Besides, cutting force variation in time can be predicted.

To validate the developed modelling of milling opera-

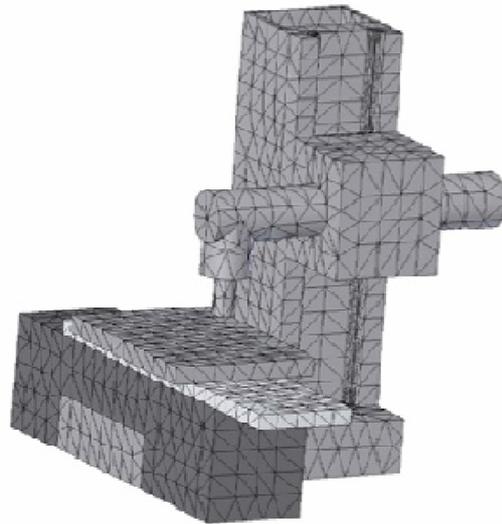
tion, validation test is performed. So, comparison with results of J. Saffar et al. [16] is presented in Figure 5.

In Figure 5 variation of cutting force according to rotation angle is presented. We remark a good agreement with J.Saffar's results.

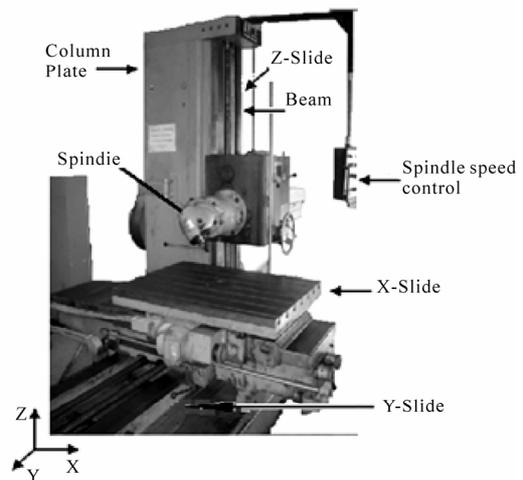
### 4.1. Comparison of Different Cutting Force Models

Table 1 presents three different model used by authors to quantify cutting force.

In Figure 6 variation cutting force is presented for three different models. We remark that for the first and third model Cutting component in y and z direction are almost equal. But, for the cutting component in x direction the difference between three models is visible.



Finite elements modeling



Reel machine

Figure 4. Modeling of milling machine.

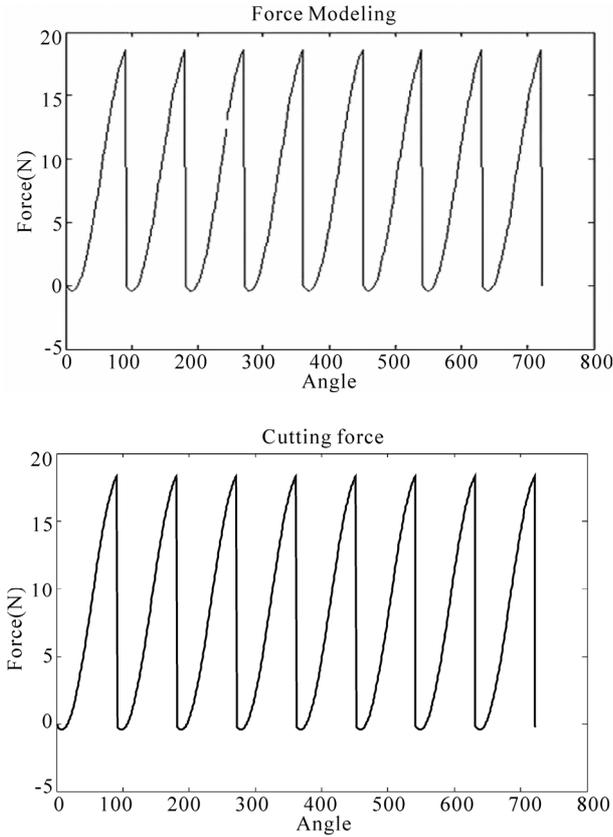


Figure 5. Comparison with saffar results.

4.2. Influence of Feed

It is clear from equation that cutting forces are proportional to feed. Figure 7 represents variation of maximal cutting force according to feed and we remark that cutting forces rise when feed rises. In addition we note that  $F_y$  component rise with an important rate. Besides, Figure 8 represents variation of instantaneous cutting force. The only difference detected is the amplitude of cutting forces.

$N = 500 \text{ revs. min}^{-1}$  ( $A_e = 40 \text{ mm}$ ,  $A_p = 1 \text{ mm}$ ) for feed ( $f_1 = 0.1 \text{ mm. rev}^{-1}$ ,  $f_2 = 0.2 \text{ mm rev}^{-1}$ ,  $f_3 = 0.4 \text{ mm. rev}^{-1}$ ,  $f_4 = 0.8 \text{ mm. rev}^{-1}$ ).

4.3. Influence of Cutting Depth

Figure 9 and Figure 10 show that cutting force increase

Table 1. Different model of cutting force.

Author	Model	
Kienzle (1950)	$F_i = a_p \cdot K_{cl,i} \cdot h^{1-m}$	
(Altintas, 1998)	$F_t = a_p \cdot K_{tc} \cdot h + a_p \cdot K_{te}$	$F_r = a_p \cdot K_{rc} \cdot h + a_p \cdot K_{re}$
(Insperger T., 2000)	$F_i = a_p \cdot K_r \cdot h^{vf}$	

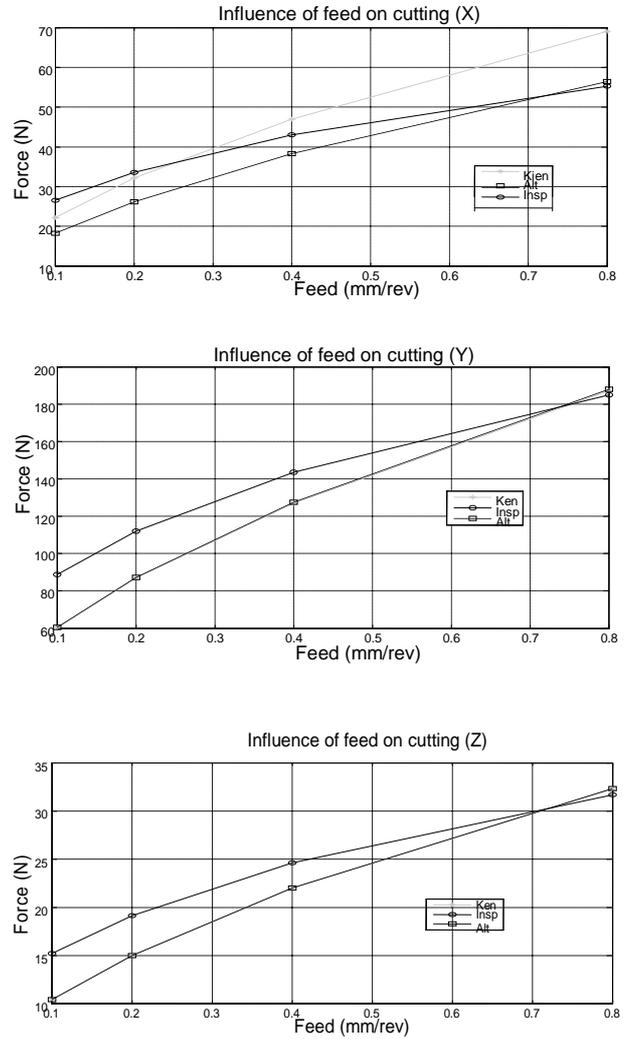


Figure 6. Comparison of different cutting force models.

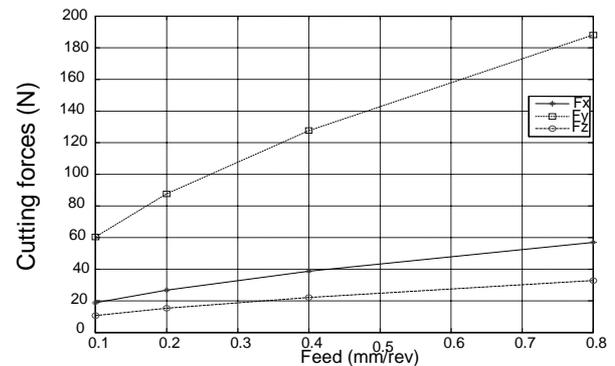


Figure 7. Influence of feed on maximal cutting forces.

when cutting depth increases. This raise is justified by the increase of the width of chip. In fact, it is the largest parameter acting on cutting forces.

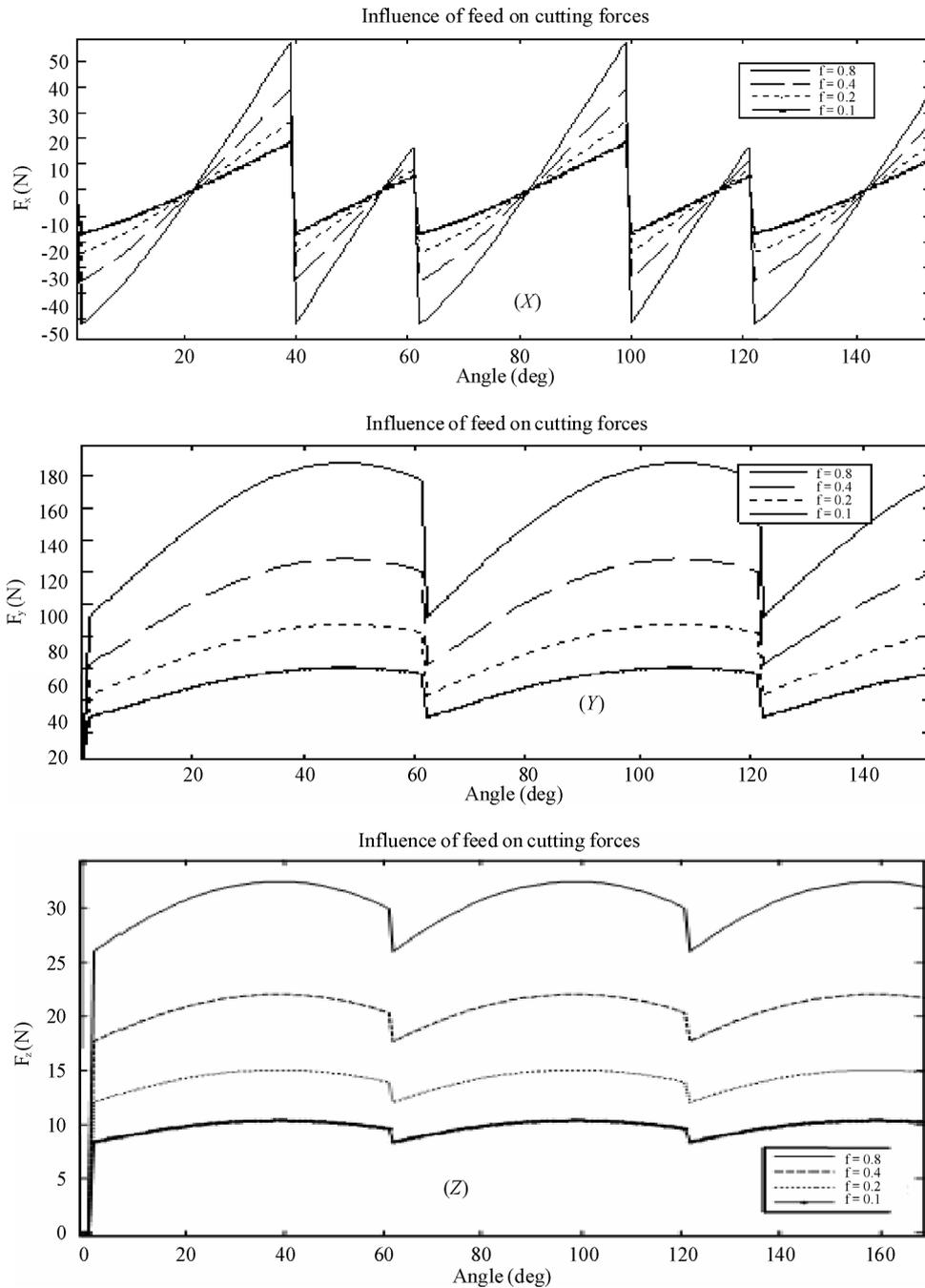


Figure 8. Influence of feed on cutting forces.

Figure 11 shows that rising cutting depth leads to rising vibration amplitude and can lead to chatter vibration. In fact, the dynamic response for Kienzel model with 5 mm cutting depth is amplified almost ten times and the response is divergent. For Atlintas model the amplitudes is less than for other model.

#### 4.4. Influence of the Beginning Angle $\varphi_e$

Increasing the spacing between the tool and the piece

caused a slight increase in the cutting force. Figure 12 shows the influence of the angle of entry on the cutting forces because the period of machining for each tooth has increased (for  $e = 20$  mm and  $e = -20$  mm).

#### 4.5. Influence of Teeth Number in Beginning

Figure 13 shows the influence of numbers of teeth in attack that affects the shapes of the cutting forces and

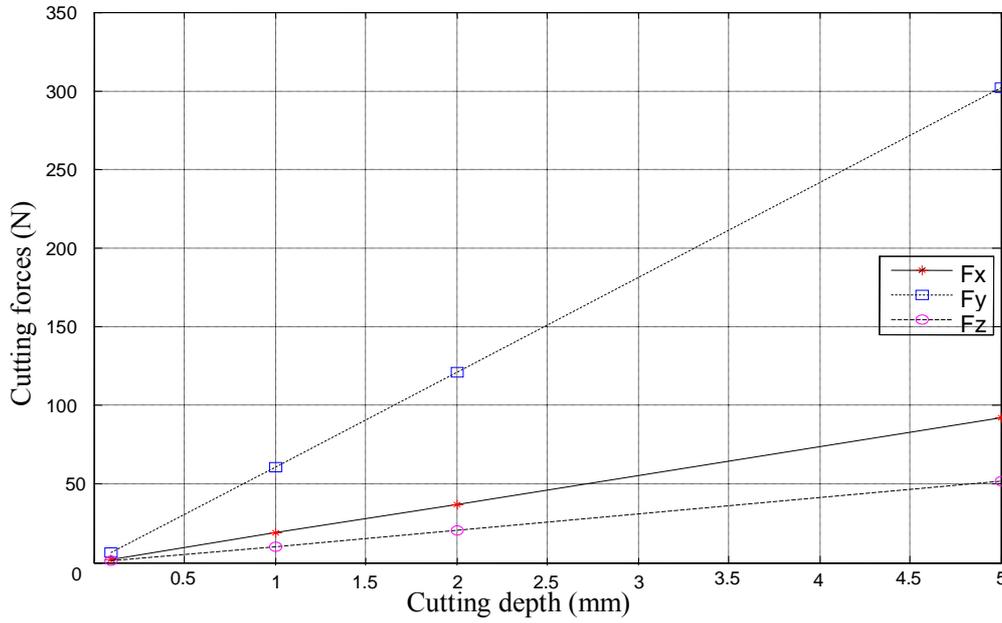
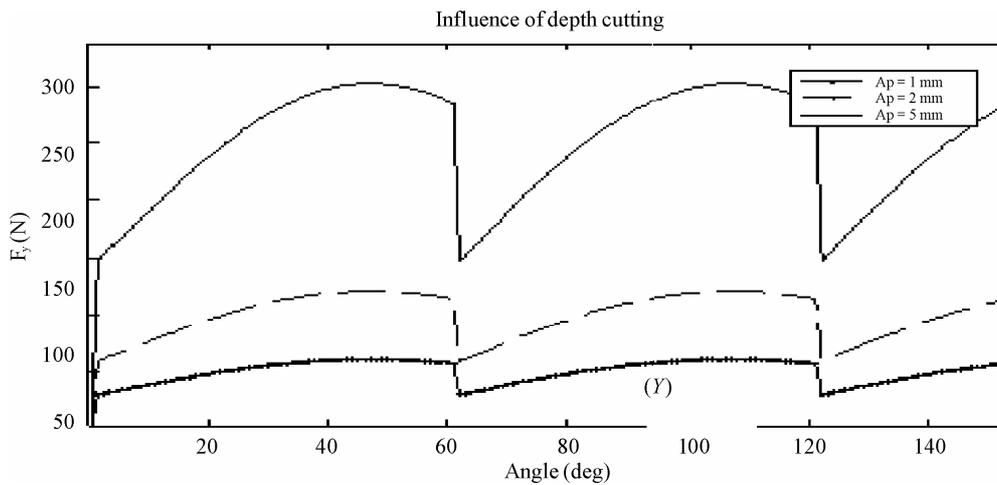
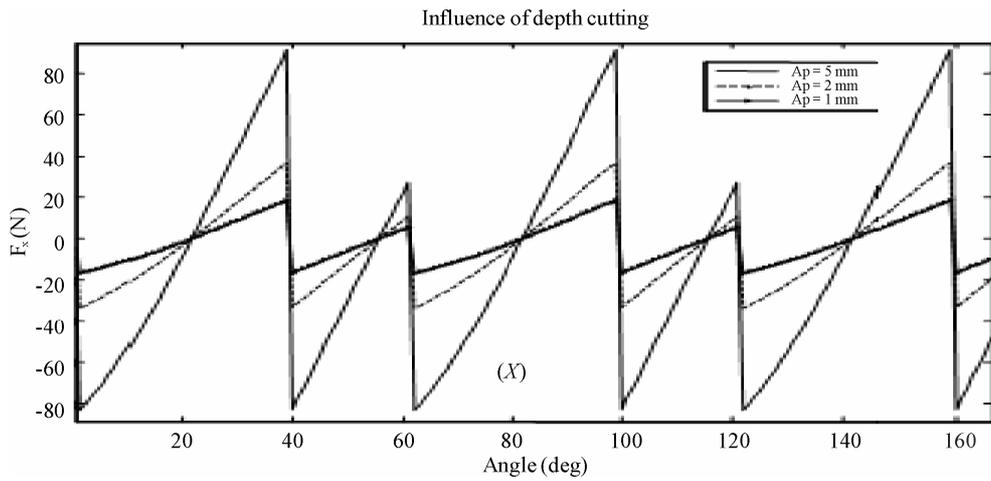


Figure 9. Influence of cutting depth on cutting force  $f = 0.1 \text{ mm/rev}$ ;  $N = 500 \text{ rev/min}$  and  $Z = 6 \text{ teeth}$ .



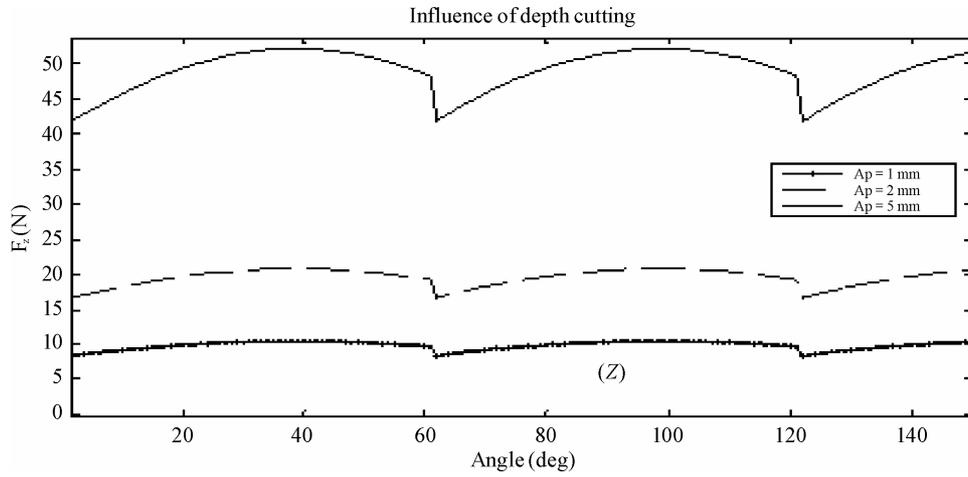
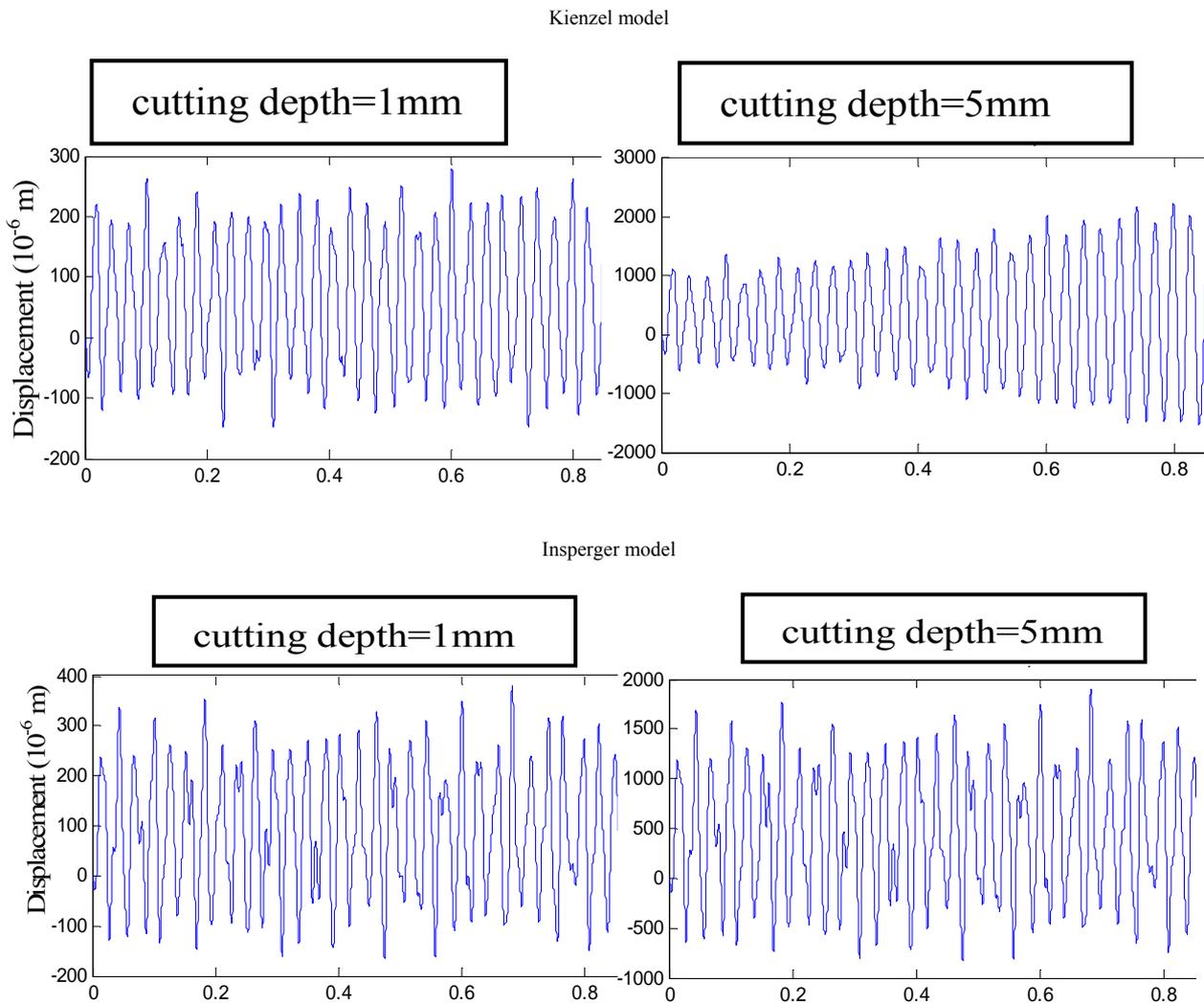


Figure 10. Influence of cutting depth on cutting force.



Atlintas model

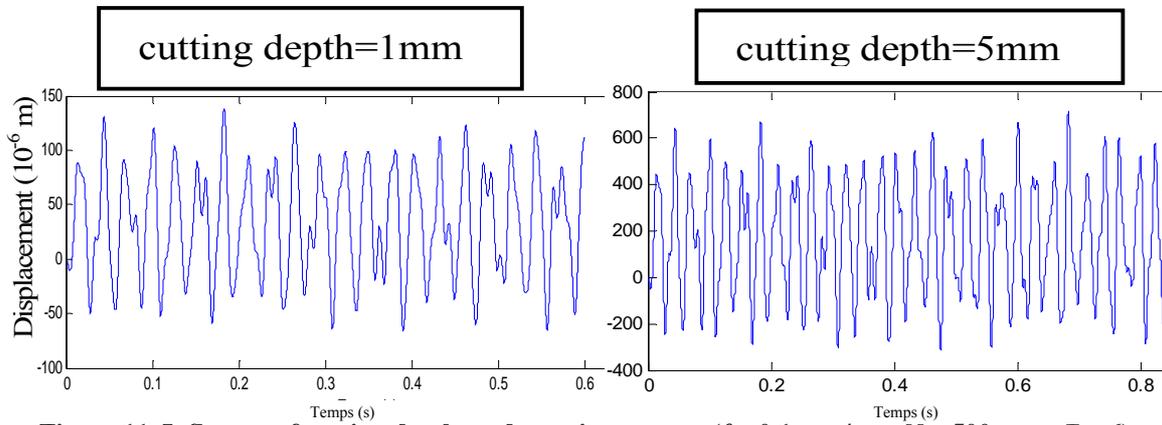
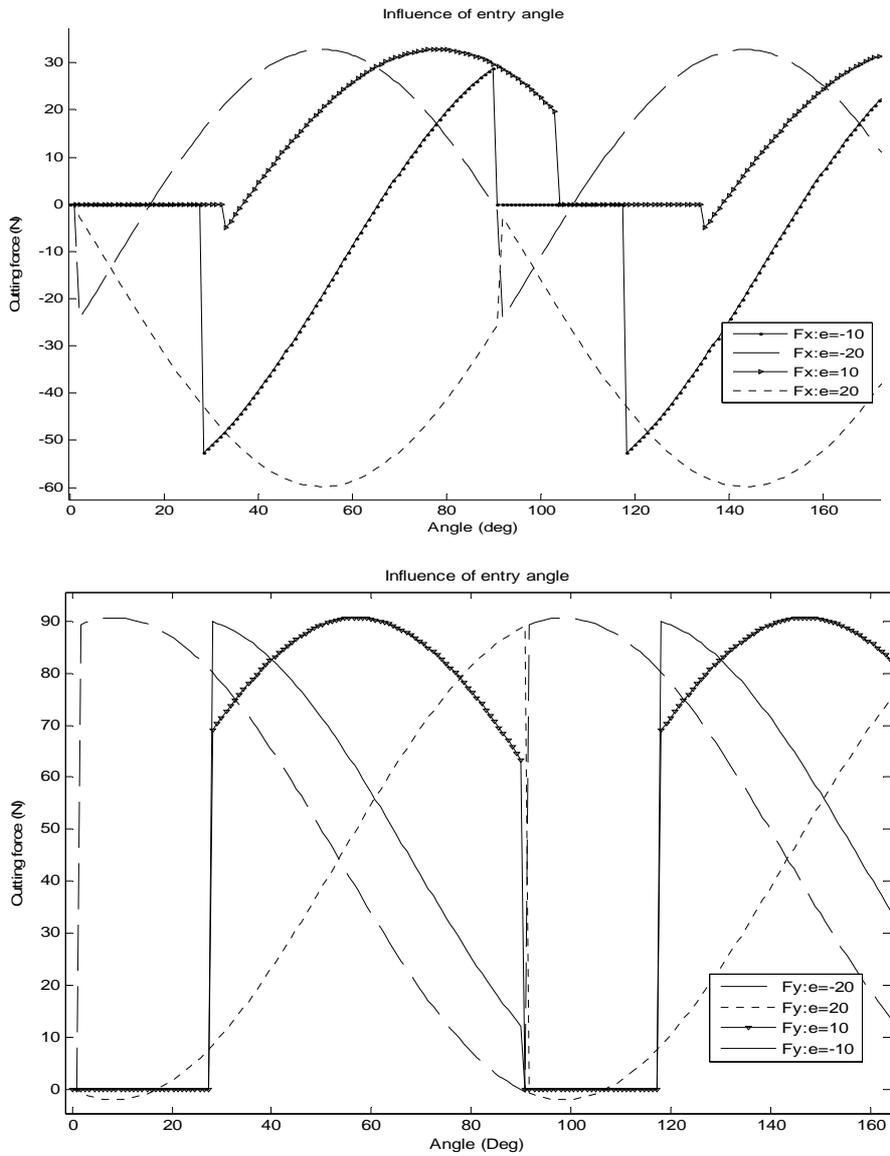


Figure 11. Influence of cutting depth on dynamic response ( $f = 0.1 \text{ mm/rev}$ ;  $N = 500 \text{ rpm}$ ;  $Z = 6$ ).



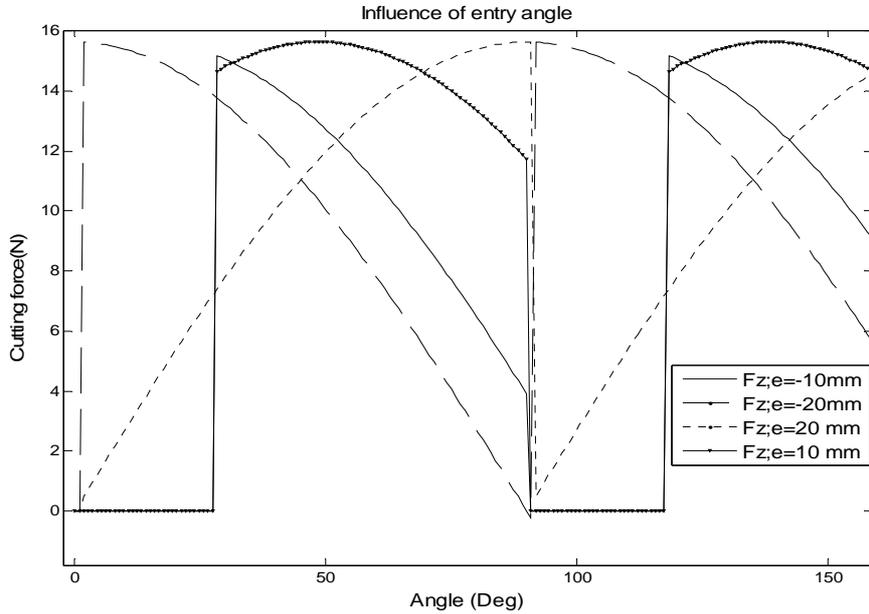


Figure 12. Influence of entry angle on cutting forces.

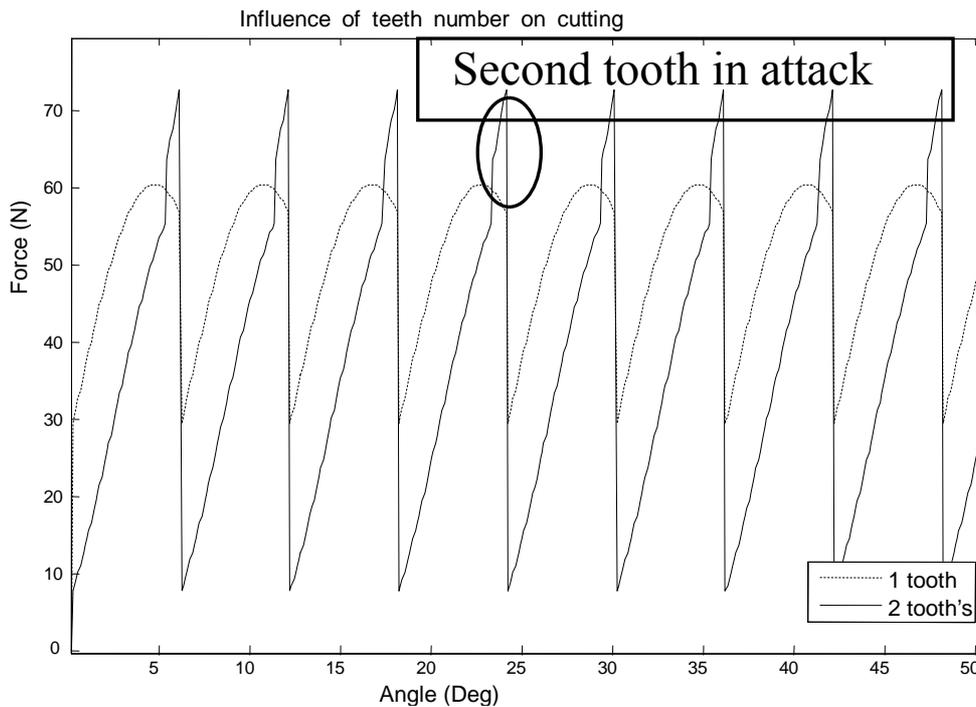


Figure 13. Influence of teeth number on cutting forces.

their value because the summation of forces of two teeth in attack.

Figure 14 proposes the two modes of milling up milling and down milling. Figure 15 gives the results obtained during two simulations carried out under the same cutting conditions, except the mode of selected milling: the first is in configuration of up milling and the second

in configuration of down milling.

We remark that for down milling frequency of vibration rises. Besides, amplitude seems to be little changed.

### 5. Spectral Response

The spectral representation of displacement following

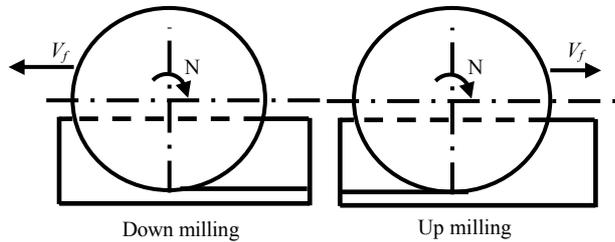


Figure 14. Two modes of milling.

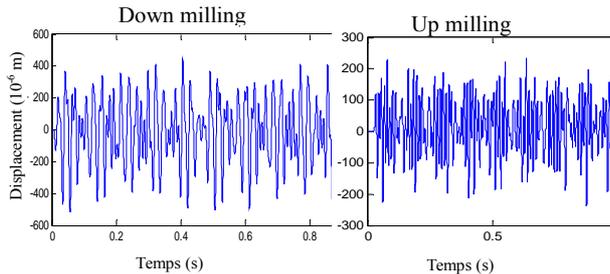


Figure 15. Numerical results for the two modes of milling.

axis X, Y in **Figure 16** proves that there are dominant vibrations due to periodical cutting pressure at the frequencies of passage of the teeth (TPF, 2TPF and 3TPF) and the maximum vibration is in the feed direction. In the feed direction (X), simulation shows the unstable frequencies for the cut with approximately 37 Hz, 60 Hz and 85 Hz, which is almost one of the natural frequencies of the system.

TPF: Frequency of teeth passage

FP: Eigen frequency

## 6. Conclusions

In this work, numerical study is conducted to predict

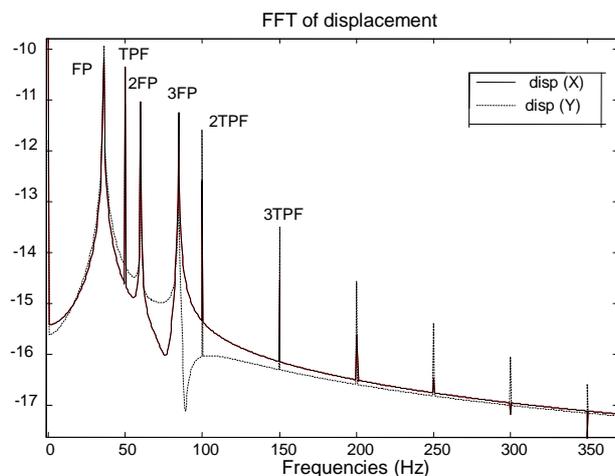


Figure 16. Spectral response.

cutting forces variation according to cutting parameters in milling operation. So, dynamic equation is established using finite element method and projected on the basis of the eigenmodes. The resolution of the obtained equation by numerical method gives dynamic response and cutting force variation. The main objective of this method is to take the influence of machine, tool and part simultaneously. Results showed that most influenced parameters are feed and cutting depth. In fact, clear influence appears on dynamic response for little variation of these parameters. Indeed, simulation reveals that the adopted cutting models have an effect on dynamic response of the system. Besides predicted spectral response of the system machine-tool-part showed that frequency vibrations are due to frequencies of passage of the teeth and eigen frequency of the system.

Finally, modelling milling operations, by finite element method represents a very useful tool for the prediction of the behaviour of the system. So, the need for establishing extensive cutting experiments is reduced.

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