

Application of Mathematical Model in Evaluating Undergraduate's Degree Paper*

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ABSTRACT

The level of undergraduate's degree paper is one of the important indicators of teaching quality. In this paper, mathematical modeling of FAHP (Fuzzy Analytic Hierarchy Process) is given, and then undergraduate's degree paper of the college with an example is elaborated a comprehensive evaluation of quantitative science, in order to fully mobilize the enthusiasm of teachers and students, and constantly promote the improvement of the quality of college teaching.

Keywords: Factor Set; Fuzzy Analytic Hierarchy Process; Weight Coefficient

1. Introduction

The undergraduate paper is academic thesis which is written independently by undergraduate student with the requirement of the teaching programs under the guidance of the experienced teachers before student graduate. It is an important part of the training scheme and comprehensive examination of knowledge ability and quality. With the development of the social economy and scientific technology, it put forward more higher requirement of quality for colleges and universities, which improve correspondingly the quality and requirements of the paper. But it is a complicated work to evaluate it, because the process and results are restricted in many aspects, for example, the analysis and judgment have fuzziness and uncertainty. It is very difficult to guarantee the thinking coherence in dealing many indexes for traditional Analytical Hierarchy Process(AHP). In this circumstances, it can resolve the problem to combine Fuzzy Mathematics with AHP(AHP). By this means, many factors reflected paper level can be calculated according to relativity and subjection relation from top to bottom, changing qualitative problems to quantitative problems, which make the result more correct and scientific mathematical model.

2. Mathematical Model

Here are main steps of the Fuzzy Analytical Hierarchy Process(FAHP). First, hierarchical structure of the systems is establish. Second, weights of the every factor are

calculated. Third, the degree of membership is decided by fuzzy comprehensive evaluation. Finally, its final value is computed.

2.1. Factor Set

It is crucial to establish the evaluating indexes system for undergraduate paper. The evaluation indexes system is concretization of evaluation standard and core of evaluation scheme. There are many factors that influence the scientificity and reliability, and every factor contains a number elements, so that the rational indexes system can outstand the characteristic and innovation.

The factor set is divided in to many layers, the first one is

$$U = \bigcup_{i=1}^n U_i, (U_i \cap U_k = \phi, i \neq k),$$

the second one is

$$U_i = \bigcup_{j=1}^{n_i} U_{ij}, (U_{ij} \cap U_{ik} = \phi, j \neq k),$$

the last one is

$$U_{ij} = \bigcup_{k=1}^{n_{ij}} U_{ijk}, (U_{ijk} \cap U_{ijl} = \phi, l \neq k).$$

2.2. Weight Coefficient

The important degree of every index in the index system is different, the difference can be represented by different weight coefficient, which is equal to a mapping $w: U_i \rightarrow (0,1)$, i.e.

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$$U_i \mapsto w(U_i) = a_i, (i = 1, 2, \dots, n).$$

Let the first weight coefficient be

$$A = (a_1, a_2, \dots, a_n)$$

the second one be

$$A_i = (a_{i1}, a_{i2}, \dots, a_{in_i})$$

and the last one be

$$A_{ij} = \{a_{ij1}, a_{ij2}, \dots, a_{ijn_j}\}$$

There are many method to establish weight coefficient, for example binomial coefficient, neighboring comparative gathering statistics iterative, analytical hierarchy process and so on.

2.3. Evaluation Set

The evaluation set is divided into many indexes. It is represented by membership degree, and it can be reflected correctly the result. Let evaluation set be

$$V = \{v_1, v_2, \dots, v_m\},$$

which is applicable for every layer and factor.

2.4. FAHP Model

The ordinary model is principal-factor-decision $M(\wedge, \vee)$, principal-factor-outstanding $M(\bullet, \vee)$, (or $M(\wedge, \oplus)$), and the weighted mean $M(\bullet, \oplus)$. The weighted mean is adopted because it is suitable for the factor with weight. By experiment the algorithm is effective and simple.

Step one considering the third layer comprehensive evaluation $U_{ij} = \{u_{ij1}, u_{ij2}, \dots, u_{ijn_j}\}$, let fuzzy mapping

$$\tilde{f}_{ij} : U_{ij} \rightarrow F(V), \text{ i.e.}$$

$$\tilde{f}_{ij}(u_{ijk}) = (r_{ijk1}, r_{ijk2}, \dots, r_{ijkm}) \in F(V),$$

then the third fuzzy relation matrix is built

$$R_{ij} = \begin{pmatrix} r_{ij11} & r_{ij12} & \dots & r_{ij1m} \\ r_{ij21} & r_{ij22} & \dots & r_{ij2m} \\ \dots & \dots & \dots & \dots \\ r_{ijn_j1} & r_{ijn_j2} & \dots & r_{ijn_jm} \end{pmatrix},$$

where

$$0 \leq r_{ijkq} \leq 1, i = 1, 2, \dots, n, j = 1, 2, \dots, n_i, k = 1, 2, \dots, n_{ij}, q = 1, 2, \dots, m$$

$$r_{ijkq} = \frac{\alpha}{\beta},$$

α stands for the number of the expert that u_{ijk} belong to V_q , and β stands for the number of the all ex-

pert.

The third layer fuzzy comprehensive evaluation is calculated according to weight distribution

$$A_j = (a_{j1}, a_{j2}, \dots, a_{jn_j}),$$

$$B_{ij} = A_j \circ R_{ij} = (a_{ij1}, a_{ij2}, \dots, a_{ijn_j}) \begin{pmatrix} r_{ij11} & r_{ij12} & \dots & r_{ij1m} \\ r_{ij21} & r_{ij22} & \dots & r_{ij2m} \\ \dots & \dots & \dots & \dots \\ r_{ijn_j1} & r_{ijn_j2} & \dots & r_{ijn_jm} \end{pmatrix} = (b_{ij1}, b_{ij2}, \dots, b_{ijm}),$$

where

$$b_{ijq} = \sum_{p=1}^{n_j} a_{ijp} \times r_{ijpq} \quad (i = 1, 2, \dots, n, j = 1, 2, \dots, n_i, q = 1, 2, \dots, m)$$

Step two considering the second layer comprehensive evaluation $U_i = \{U_{i1}, U_{i2}, \dots, U_{in_i}\}$, the second fuzzy relation matrix is obtained, i.e.

$$R = \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & \dots & \dots & \dots \\ b_{n1} & b_{n2} & \dots & b_{nm} \end{pmatrix}$$

If weight distribution is

$$A = (a_1, a_2, \dots, a_n)$$

then the first fuzzy comprehensive evaluation is obtained

$$B = A \circ R = (b_1, b_2, \dots, b_m) \in F(V)$$

where $b_q = \sum_{p=1}^n a_p \times b_{pq}, (q = 1, 2, \dots, m)$.

3. Application Example

The evaluation index system is different for various university. Here is an example of undergraduate's degree thesis in Tonghua Normal university.

3.1. Evaluation Index System

There are three indexes in the first layer of evaluation index system, thirteen indexes in the second layer, and sixteen indexes in the third layer, which is in the paper quality because of its importance.

3.2. Weight Coefficient

A number of experienced experts who is invited score according to their importance, and then calculate their weight coefficient by superiority chart.

3.3. Evaluation Result

Ten experienced examiners give the grade according to evaluation index system.

Table 1. Weight coefficient.

The First Index	The Second Index	The Third Index
Attitude (0.15)	Discipline(0.15)	
	Activeness(0.15)	
	Consulting(0.15)	
	Investigation(0.40)	
Quality (0.70)	Writing (0.15)	Innovation level(0.3)
	Ability for selecting topic (0.25)	Academic value(0.36)
		Application value(0.18)
		Difficult degree (0.16)
	Choice of data (0.15)	Richness(0.27)
		Reality(0.27)
		Timeliness(0.27)
		Correlation(0.19)
	Verifying ability (0.35)	Argument establishment(0.34)
		Verification method (0.20)
Material structure(0.17)		
Logic structure(0.29)		
Expression ability (0.25)	Title(0.16)	
	Abstract , keywords(0.36)	
	Language, punctuation, signal(0.32)	
Answering statement (0.15)	Notes(0.16)	
	Content introduce (0.40)	
	Answering problem (0.35)	
	Language (0.15)	
	Politeness, apparatus (0.10)	

Table 2. Evaluation grade.

No.	Index	Evaluation Set			
		A	B	C	D
1	Discipline	8	2	0	0
2	Activeness	9	1	0	0
3	Consulting	8	1	1	0
4	Investigation	9	1	0	0
5	Writing	9	1	0	0
6	Innovation level	7	2	1	0
7	Academic value	8	1	1	0
8	Application value	7	1	1	1
9	Difficult degree	8	1	1	0
10	Richness(7	2	1	0
11	Reality	8	1	1	0
12	Timeliness	7	1	1	0
13	Correlation	8	0	1	1
14	Argument establishment	8	2	0	0
15	Verification method	8	1	1	0
16	Material structure	7	2	1	0
17	Logic structure	9	1	0	0
18	Title	9	1	0	0
19	Abstract , keywords	9	0	1	0
20	Language, punctuation, signal	8	2	0	0
21	Notes	7	1	1	1
22	Content introduce	9	1	0	0
23	Answering problem	8	1	1	0
24	Language	9	1	0	0
25	Politeness, apprance	10	0	0	0

3.4. Fuzzy Comprehensive Evaluation

Fuzzy comprehensive evaluation of the **third layer** factor set is $U_{21} = \{U_{211}, U_{212}, U_{213}, U_{214}\}$

$$B_{21} = A_{21} \circ R_{21} = (0.3, 0.36, 0.18, 0.16) \begin{pmatrix} 0.7 & 0.2 & 0.1 & 0 \\ 0.8 & 0.1 & 0.1 & 0 \\ 0.7 & 0.1 & 0.1 & 0.1 \\ 0.8 & 0.1 & 0.1 & 0 \end{pmatrix} = (0.752, 0.13, 0.10, 0.018),$$

likewise, $B_{22} = (0.746, 0.108, 0.10, 0.046),$

$$B_{23} = (0.812, 0.151, 0.037, 0),$$

$$B_{24} = (0.836, 0.096, 0.052, 0.016)$$

Fuzzy comprehensive evaluation of the **second layer** factor set is $U_1 = \{U_{11}, U_{12}, U_{13}, U_{14}\}$

$$B_1 = A_1 \circ R_1 = (0.15, 0.15, 0.15, 0.40, 0.15) \begin{pmatrix} 0.8 & 0.2 & 0 & 0 \\ 0.9 & 0.1 & 0 & 0 \\ 0.8 & 0.1 & 0.1 & 0 \\ 0.9 & 0.1 & 0 & 0 \\ 0.9 & 0.1 & 0 & 0 \end{pmatrix} = (0.87, 0.115, 0.015, 0).$$

$$B_3 = A_3 \circ R_3$$

$$= (0.40, 0.35, 0.15, 0.10) \begin{pmatrix} 0.9 & 0.1 & 0 & 0 \\ 0.8 & 0.1 & 0.1 & 0 \\ 0.9 & 0.1 & 0 & 0 \\ 1.0 & 0 & 0 & 0 \end{pmatrix} = (0.875, 0.09, 0.035, 0).$$

$$B_2 = A_2 \circ R_2$$

$$= (0.25, 0.15, 0.35, 0.25) \begin{pmatrix} 0.752 & 0.13 & 0.1 & 0.018 \\ 0.746 & 0.108 & 0.1 & 0.046 \\ 0.812 & 0.151 & 0.037 & 0 \\ 0.836 & 0.096 & 0.052 & 0.016 \end{pmatrix} = (0.7931, 0.1256, 0.0659, 0.0154).$$

Fuzzy comprehensive evaluation of the **first layer** factor set is $U = \{U_1, U_2, U_3\}$, then

Table 3. Grades and score segments.

Number	Grades	Score segments
1	Excellent	90-----100
2	Good	80-----90
3	Pass	60-----80
4	Fail	60 Below

$$B = A \circ R$$

$$= (0.15, 0.70, 0.15) \begin{pmatrix} 0.87 & 0.115 & 0.015 & 0 \\ 0.7931 & 0.1256 & 0.0659 & 0.0154 \\ 0.875 & 0.090 & 0.035 & 0 \end{pmatrix}$$

$$= (0.8169, 0.1186, 0.0537, 0.0108).$$

The results indicate that 81.69 % of experts think it is excellent, that 11.86 % of experts think it is good, 5.37% of experts think it is pass, and another think it is fail. According to maximum subordination principle, the result of above evaluation is excellent.

The grade theory domain is used for the sake of observing visual image. The evaluation set is described by different score segments, see **Table 3**.

Let 95, 85, 75, 35 represent the score of the different grades, then the grade matrix is $C = (95, 85, 75, 35)^T$. The final score is

$$F = B \circ C^T = (0.8169, 0.1186, 0.0537, 0.0108) \begin{pmatrix} 95 \\ 85 \\ 75 \\ 35 \end{pmatrix} = 92.09.$$

It is well known that the grade of the undergraduate's degree paper is excellent.

4. Conclusion

The undergraduate paper is one of the important indica-

tor that show practical teaching of university. It is very crucial to evaluate undergraduate paper. FAHP is not only considering the internal relationship between the various indicator fuzziness of the system, but also possessing the basis of Fuzzy Mathematics, Matrix Theory, and AHP, therefore FAHP can accurately reflect the level of the undergraduate's degree thesis, this method is scientific.

The steps is clear, the judgment is simple in entire model, and it can be calculated by using mathematical software Mathematica and Matlab when calculation amount is very complicated, so FAHP can avoid confusion that caused by inaccurate scoring and reduce the workload of teacher.

It is rigorous in theory, and convenient in application, especially, the programming effective of the FAHP is verified through some examples, there is extensive application space and wonderful development prospects in the field of life and production.

REFERENCES

- [1] F. Chiclana, F. Herrera, E. Herrera-Vidm, "Integrating three representation models in fuzzy multipurpose decision making based on fuzzy preference relations," *Fuzzy Set and Systems*. vol. 97, 1998, pp.33-48.
- [2] Yong -yue Zhu, "FAHP-based evaluation of the service quality of private-owned express companies," *Proceedings of 2010 International Colloquium on Computing, Communication, Control, and Management (CCCM2010) Volume 2*[C], 2010.
- [3] Jing Yang, "FAHP-Basic Comprehensive Economic Assessment of Wind Power Project." *Proceedings of 2009 Asia-Pacific Conference on Information Processing (Volume 2)* [C], 2009.
- [4] Deming Fan, "Research on the teaching effect evaluation of teaching website of Quality courses in universities." *Science and Technology Information*, 2007(11): pp 110 - 111.