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The Ricci Operator and Shape Operator of Real Hypersurfaces in a Non-Flat 2-Dimensional Complex Space Form

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ABSTRACT

In this paper, we study a real hypersurface M in a non-at 2-dimensional complex space form $M_2(c)$ with η-parallel Ricci and shape operators. The characterizations of these real hypersurfaces are obtained.

Keywords: Real Hypersurface; η-Parallel Shape Operator; η-Parallel Ricci Operator; Hopf Hypersurface; Ruled Real Hypersurfaces

1. Introduction

A complex n-dimensional Kaeherian manifold of constant holomorphic sectional curvature c is called a *complex space form*, which is denoted by $M_n(c)$. As is well-known, a complete and simply connected complex space form is complex analytically isometric to a complex projective space $P_n(\mathbb{C})$, a complex Euclidean space $\mathbb{C}^n$ or a complex hyperbolic space $H_n(\mathbb{C})$, according to $c>0, c=0$ or $c<0$.

In this paper we consider a real hypersurface M in a complex space form $M_2(c), c\neq 0$. Then M has an almost contact metric structure (ϕ, g, ξ, η) induced from the Kaehler metric and complex structure J on $M_n(c)$. The structure vector field ξ is said to be *principal* if $A\xi=\alpha\xi$ is satisfied, where A is the shape operator of M and $\alpha=\eta(A\xi)$. In this case, it is known that α is locally constant ([1]) and that M is called a *Hopf hypersurface*.

Typical examples of Hopf hypersurfaces in $P_n(\mathbb{C})$ are homogeneous ones, R. Takagi [2] and M. Kimura [3] completely classified such hypersurfaces as six model spaces which are said to be A_1, A_2, B, C, D and E. On the other hand, real hypersurfaces in $H_n(\mathbb{C})$ have been investigated by J. Berndt [4], S. Montiel and A. Romero [5] and so on. J. Berndt [4] classified all homogeneous Hopf hyersurfaces in $H_n(\mathbb{C})$ as four model spaces which are said to be A_0, A_1, A_2 and B. Further, Hopf hypersurfaces with constant principal curvatures in a complex space form have been completely classified as follows:

Theorem 1.1. ([2]) *Let M be a homogeneous real hypersurface of $P_n(\mathbb{C})$. Then M is tube of radius r over one of the following Kaeherian submanifolds*:

(A_1) a hyperplane $P_{n-1}(\mathbb{C})$, where $0<r<\frac{\pi}{\sqrt{c}}$;

(A_2) a totally geodesic $P_k(\mathbb{C})(1\le k\le n-2)$, where $0<r<\frac{\pi}{\sqrt{c}}$;

(B) a complex quadric $\mathbb{Q}_{n-1}$, where $0<r<\frac{\pi}{2\sqrt{c}}$;

(C) $P_1(C)\times P_{\frac{n-1}{2}}(C)$, where $0<r<\frac{\pi}{2\sqrt{c}}$ and $n\ge 5$ is odd;

(D) a complex Grassmann $G_{2,5}C$, where $0<r<\frac{\pi}{2\sqrt{c}}$ and $n=9$;

(E) a Hermitian symmetric space $SO(10)/U(5)$, where $0<r<\frac{\pi}{2\sqrt{c}}$ and $n=15$.

Theorem 1.2. ([4]) *Let M be a real hypersurface in $H_n(\mathbb{C})$. Then M has constant principal curvatures and ξ is principal if and only if M is locally congruent to one of the followings*:

(A_0) a self-tube, that is, a horosphere;

(A_1) a geodesic hypersphere;

(A_2) a tube over a totally geodesic $H_k(\mathbb{C})(1\le k\le n-1)$;

(B) a tube over a totally real hyperbolic space $H_n(\mathbb{R})$.

A real hypersurface of type A_1 or A_2 in $P_n(\mathbb{C})$ or type A_0, A_1 or A_2 in $H_n(\mathbb{C})$, then M is said to be *of type* A for simplicity. As a typical characterization of real hypersurfaces of type A, in a complex space form $M_n(c)$ was given under the condition

$$g\left(\left(A\phi - A\phi\right)X, Y\right) = 0, \tag{1.1}$$

for any tangent vector fields X and Y on M by M. Okumura [5] for $c > 0$ and S. Montiel and A. Romero [6] for $c < 0$. Namely

Theorem 1.3. ([5,6]) *Let M be a real hypersurface in $M_n(c), c \neq 0, n \geq 2$. It satisfies* (1.1) *on M if and only if M is locally congruent to one of the model spaces of type A.*

The holomorphic distribution T_0 of a real hypersurface M in $M_n(c)$ is defined by

$$T_0(p) = \left\{X \in T_p(M) \middle| g(X, \xi)_p = 0\right\}. \tag{1.2}$$

The following theorem characterizes ruled real hypersurfaces in $M_n(c)$.

Theorem 1.4. ([7]) *Let M be a real hypersurface in $M_n(c), c \neq 0, n \geq 2$. Then M is a ruled real hypersurfaces if and only if $\phi A \phi = 0$, or equivalently $g(AX, Y) = 0$, for any $X, Y \in T_0$.*

A (1,1) type tensor field T of M is said to be η-parallel if

$$g\left(\left(\nabla_X T\right)Y, Z\right) = 0 \tag{1.3}$$

for any vector fields X, Y and Z in T_0. Real hypersurfaces with η-parallel shape operator or Ricci operator have been studied by many authors (see [13]). Nevertheless, the classification of real hypersurfaces with η-parallel shape operator or Ricci operator in $M_n(c)$ remains open up to this point. Recently, S.H. Kon and T.H. Loo ([9]) investigated the conditions η-parallel shape operator.

Theorem 1.5. ([9]) *Let M be a real hypersurface of $M_n(c), c \neq 0, n \geq 3$. Then the shape operator A is η-parallel if and only if M is locally congruent to a ruled real hypersurface, or a real hypersurface of type A or B.*

Also, M. Kimura and S. Maeda ([10]) and Y.J. Suh ([11]) investigated the conditions η-parallel Ricci operator.

Theorem 1.6. ([10,11]) *Let M be a real hypersurface in a complex space form $M_n(c), c \neq 0$. Then the Ricci operator of M is η-parallel and the structure vector field ξ is princial if and only if M is locally congruent to one of the model spaces of type A or type B.*

As for the structure tensor field ϕ, shape operator A and η-parallel, I.-B. Kim, K. H. Kim and one of the present authors ([12]) have proved the following.

Theorem 1.7. ([12]) *Let M be a real hypersurface in a complex space form $M_n(c), c \neq 0, n \geq 3$. If M has the cyclic η-parallel shape operator* (*resp. Ricci operator*) *and satisfies*

$$g\left(\left(A\phi - \phi A\right)X, Y\right) = 0 \tag{1.4}$$

for any X and Y in T_0, then M is locally congruent to either a real hypersurface of type A or a ruled real hypersurface (resp. M is locally congruent to a real hypersurface of type A).

The purpose of this paper is to give some characterizations of real hypersurface satisfying (1.4) and having the η-parallel shape operator or Ricci operator in $M_2(c)$. We shall prove the following.

Theorem 1.8. *Let M be a real hypersurface in a complex space form $M_2(c)$, $c \neq 0$. If M has the η-parallel shape operator and satisfies* (1.4), *then M is locally congruent a ruled real hypersurface.*

Theorem 1.9. *Let M be a real hypersurface in a complex space form $M_2(c)$, $c \neq 0$. If M has the η-parallel Ricci operator and satisfies* (1.4), *then M is locally congruent to a real hypersurface of type A.*

All manifolds in the present paper are assumed to be connected and of class C^∞ and the real hypersurfaces are supposed to be orientable.

2. Preliminaries

Let M be a real hypersurface immersed in a complex space form $M_2(c)$, and N be a unit normal vector field of M. By $\tilde{\nabla}$ we denote the Levi-Civita connection with respect to the Fubini-Study metric tensor $\tilde{g}$ of $M_2(c)$. Then the Gauss and Weingarten formulas are given respectively by

$$\tilde{\nabla}_X Y = \nabla_X Y + g(AX, Y)N, \tilde{\nabla}_X N = -AX$$

for any vector fields X and Y tangent to M, where g denotes the Riemannian metric tensor of M induced from $\tilde{g}$, and A is the shape operator of M in $M_2(c)$. For any vector field X on M we put

$$JX = \phi X + \eta(X)N, JN = -\xi,$$

where J is the almost complex structure of $M_2(c)$. Then we see that M induces an almost contact metric structure (ϕ, g, ξ, η), that is,

$$\begin{gathered}\phi^2 X = -X + \eta(X)\xi, \phi\xi = 0, \eta(\xi) = 1, \\ g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \eta(X) = g(X, \xi)\end{gathered} \tag{2.1}$$

for any vector fields X and Y on M. Since the almost complex structure J is parallel, we can verify from the Gauss and Weingarten formulas the followings:

$$\nabla_X \xi = \phi AX, \tag{2.2}$$

$$(\nabla_X \phi)Y = \eta(Y)AX - g(AX,Y)\xi. \tag{2.3}$$

Since the ambient manifold is of constant holomorphic sectional curvature c, we have the following Gauss and Codazzi equations respectively:

$$\begin{aligned} R(X,Y)Z &= \frac{c}{4}\{g(Y,Z)X - g(X,Z)Y + g(\phi Y,Z)\phi X \\ &\quad - g(\phi X,Z)\phi Y - 2g(\phi X,Y)\phi Z\} \\ &\quad + g(AY,Z)AX - g(AX,Z)AY, \end{aligned} \tag{2.4}$$

$$\begin{aligned} &(\nabla_X A)Y - (\nabla_Y A)X \\ &= \frac{c}{4}\{\eta(X)\phi Y - \eta(Y)\phi X - 2g(\phi X,Y)\xi\} \end{aligned} \tag{2.5}$$

for any vector fields X, Y and Z on M, where R denotes the Riemannian curvature tensor of M. From (1.3), the Ricci operator S of M is expressed by

$$SX = \frac{c}{4}\{(2n+1)X - 3\eta(X)\xi\} + mAX - A^2X, \tag{2.6}$$

where $m = \mathrm{trace}A$ is the mean curvature of M, and the covariant derivative of (2.5) is given by

$$\begin{aligned} &(\nabla_X S)Y \\ &= -\frac{3c}{4}\{g(\phi AX,Y)\xi + \eta(Y)\phi AX\} + (Xm)AY \\ &\quad + m(\nabla_X A)Y - (\nabla_X A)AY - A(\nabla_X A)Y. \end{aligned} \tag{2.7}$$

Let U be a unit vector field on M with the same direction of the vector field $-\phi\nabla_\xi \xi$, and let μ be the length of the vector field $-\phi\nabla_\xi \xi$ if it does not vanish, and zero (constant function) if it vanishes. Then it is easily seen from (1.1) that

$$A\xi = \alpha\xi + \beta U, \tag{2.8}$$

where $\alpha = \eta(A\xi)$. We notice here that U is orthogonal to ξ. We put

$$\Omega = \{p \in M \mid \beta(p) \neq 0\}. \tag{2.9}$$

Then Ω is an open subset of M.

3. Some Lemmas

In this section, we assume that Ω is not empty, then there are sclar fields γ, ε and δ and a unit vector field U and ϕU orthogonal to ξ such that

$$AU = \beta\xi + \gamma U + \varepsilon\phi U, A\phi U = \varepsilon U + \delta\phi U \tag{3.1}$$

and

$$m = \mathrm{trace}A = \alpha + \gamma + \delta \tag{3.2}$$

in $M_2(c)$. We shall prove the following Lemmas.

Lemma 3.1. *Let M be a real hypersurface in a complex space form $M_2(c), c \neq 0$. If M satisfies* (1.4), *then we have $AU = \beta\xi + \gamma U$, $A\phi U = \delta\phi U$ and $\delta = \gamma$.*

Proof. If we put $X = Y = U$, or $X = U$ and $Y = \phi U$ into (1.4) and make use of (3.1), then we have

$$\varepsilon = 0 \text{ and } \delta = \gamma. \tag{3.3}$$

Therefore, it follows that AU is expressed in terms of ξ and U only and $A\phi U$ given by ϕU. □

It follows from (2.6), (2.8) and Lemma 3.1 that

$$\begin{aligned} S\xi &= \left(\frac{c}{2} + 2\alpha\gamma - \beta^2\right)\xi + \beta\gamma U, \\ SU &= \beta\gamma\xi + \left(\frac{5c}{4} + \alpha\gamma - \beta^2 + \gamma^2\right)U, \\ S\phi U &= \left(\frac{5c}{4} + \alpha\gamma + \gamma^2\right)\phi U. \end{aligned} \tag{3.4}$$

Lemma 3.2. *Under the assumptions of Lemma* 3.1. *If M has the η-parallel Ricci operator S then we have $U\beta = 0$ and $(\phi U)\beta = -\gamma^2$.*

Proof. Differentiating the second of (3.4) covariantly along vector field X in T_0, we obtain

$$\begin{aligned} &(\nabla_X S)U \\ &= \left\{\left(\frac{5c}{4} + \alpha\gamma - \beta^2 + \gamma^2\right)I - S\right\}\nabla_X U + \beta\gamma\phi AX \\ &\quad + X(\beta\gamma)\xi + X\left(\frac{5c}{4} + \alpha\gamma - \beta^2 + \gamma^2\right)U. \end{aligned} \tag{3.5}$$

Taking inner product of (3.5) with U and ϕU and making use of (3.5) and Lemma 3.1, we have

$$2\beta\gamma^2 g(\phi U, X) = X\left(\frac{5c}{4} + \alpha\gamma - \beta^2 + \gamma^2\right) \tag{3.6}$$

and

$$\beta g(\nabla_X U, \phi U) = \gamma^2 g(U, X). \tag{3.7}$$

If we put $X = U$ and $Y = \phi U$ into (3.6) then we have

$$(\alpha + 2\gamma)U\gamma + \gamma U\alpha - 2\beta U\beta = 0 \tag{3.8}$$

and

$$2\beta\gamma^2 = (\alpha + 2\gamma)(\phi U)\gamma + \gamma(\phi U)\alpha - 2\beta(\phi U)\beta. \tag{3.9}$$

Putting $X = U$ and $Y = \phi U$ into (3.7), then we obtain

$$\beta g(\nabla_U U, \phi U) = \gamma^2 \text{ and } \beta g(\nabla_{\phi U} U, \phi U) = 0. \tag{3.10}$$

If we differentiate the third of (3.4) covariantly along vector field X in T_0, we obtain

$$
\begin{aligned}
\left(\nabla_X S\right)\phi U = & \left\{\left(\frac{5c}{4}+\alpha\gamma+\gamma^2\right)I - S\right\}\nabla_X \phi U \\
& + \left\{X\left(\frac{5c}{4}+\alpha\gamma+\gamma^2\right)\right\}\phi U.
\end{aligned} \tag{3.11}
$$

If we take inner product of ϕU and using (3.4), then we have

$$X\left(\frac{5c}{4}+\alpha\gamma+\gamma^2\right)\phi U = 0. \tag{3.12}$$

Substituting $X = U$ and ϕU into (3.12), we obtain

$$
\begin{aligned}
&(\alpha+2\gamma)U\gamma+\gamma U\alpha = 0 \text{ and} \\
&(\alpha+2\gamma)(\phi U)\gamma+\gamma(\phi U)\alpha = 0.
\end{aligned} \tag{3.13}
$$

By comparing (3.8) and (3.9) with (3.13), we have $U\beta = 0$ and $(\phi U)\beta = -\gamma^2$. □

Lemma 3.3. *Under the assumptions of Lemma* 3.2, *we have* $\nabla_X U = \gamma g(\phi U, X)\xi + \frac{\gamma^2}{\beta} g(U, X)\phi U$.

Proof. Since we have $A\phi U = \gamma\phi U$ and using (3.7), we get

$$
\begin{aligned}
&a(X) = g(\nabla_X U, \xi) = \gamma(\phi U, X) \text{ and} \\
&c(X) = g(\nabla_X U, \phi U) = \frac{\gamma^2}{\beta} g(U, X).
\end{aligned} \tag{3.14}
$$

Thus, it follows from (3.14) that

$\nabla_X U = \gamma g(\phi U, X)\xi + \frac{\gamma^2}{\beta} g(U, X)\phi U.$ □

Lemma 3.4. *Under the assumptions of Lemma 3.2*, *we have* $\xi\alpha = \xi\beta = \xi\gamma = 0$ *and* $U\alpha = U\gamma = 0$.

Proof. Differentiating the smooth function $\alpha = g(A\xi, \xi)$ along any vector field X on Ω and using (2.2) and (2.5) and Lemma 3.1, we have

$$X\alpha = g\left(\left(\nabla_\xi A\right)\xi - 2\beta\gamma\phi U, X\right). \tag{3.15}$$

Since we have $\left(\nabla_\xi A\right)\xi = \nabla_\xi\left(\alpha\xi+\beta U\right) - A\nabla_\xi\xi$, we see from this equation above that the gradient vector field $\nabla\alpha$ of α is given by

$$\nabla\alpha = \beta\nabla_\xi U + (\xi\alpha)\xi + (\xi\beta)U + (\alpha\beta - 3\beta\gamma)\phi U.$$

If we put $X = \xi$ into Lemma 3.3, then we have

$$\nabla_\xi U = 0. \tag{3.16}$$

Thus, the above equation is reduced to

$$\nabla\alpha = (\xi\alpha)\xi + (\xi\beta)U + (\alpha\beta - 3\beta\gamma)\phi U. \tag{3.17}$$

Taking inner product of this equation with U and ϕU respectively, we obtain

$$U\alpha = \xi\beta \text{ and } (\phi U)\alpha = \alpha\beta - 3\beta\gamma. \tag{3.18}$$

If we differentiate the smooth function $\beta = g(AU, \xi)$ along any vector field X on M and using (2.2), (2.5) and (2.8) and Lemma 3.2, we have

$$\nabla\beta = \beta\nabla_U U + (U\alpha)\xi + (U\beta)U + \left(\frac{c}{2} + 2\left(\alpha\gamma - \gamma^2\right)\right)\phi U. \tag{3.19}$$

Putting $X = U$ into Lemma 3.3, then we have

$$\nabla_U U = \frac{\gamma^2}{\beta}\phi U. \tag{3.20}$$

If we substitute (3.20) into (3.19), then we obtain

$$\nabla\beta = (U\alpha)\xi + (U\beta)U + \left(\frac{c}{2} + 2\alpha\gamma - \gamma^2\right)\phi U. \tag{3.21}$$

If we take inner product of this equation with ϕU and using $(\phi U)\beta = -\gamma^2$ in Lemma 3.2, then we have

$$\alpha\gamma + \frac{c}{4} = 0. \tag{3.22}$$

As a similar argument as the above, we can verify that the gradient vector fields of the smooth function $\gamma = g(AU, U) = g(A\phi U, \phi U)$ is given respectively by

$$\nabla\gamma = -(A - \gamma I)\nabla_U U + (U\beta)\xi + (U\gamma)U + 3\beta\gamma\phi U \tag{3.23}$$

and

$$\nabla\gamma = \left((\phi U)\gamma\right)\phi U \tag{3.24}$$

by virtue of (2.3) and Lemma 3.2.

If we substitute (3.24) into (3.23) and make use of (3.20) and Lemma 3.1, then we obtain

$$(U\beta)\xi + (U\gamma)U - \left((\phi U)\gamma - 3\beta\gamma\right)\phi U = 0. \tag{3.25}$$

If we take inner product of this equation with U and ϕU respectively, then we have

$$U\gamma = 0 \text{ and } (\phi U)\gamma = 3\beta\gamma. \tag{3.26}$$

If we substitute (3.26) into (3.14) and take account of (3.21), then we have $U\alpha = 0$. Also, if we differentiate (3.21) along any vector field ξ, then we have

$$\alpha\xi\gamma + \gamma\xi\alpha = 0. \tag{3.27}$$

Taking inner product of (3.23) with ξ and using (3.18), we get $\xi\gamma = U\beta$. Since $U\alpha = 0$, we see from (3.27) and the first of (3.18) that $\xi\gamma = 0, \xi\alpha = 0$ and $\xi\beta = 0$. □

4. Proofs of Theorems

Proof Theorem 1.8. If (1.4) is given in M, then we see that Lemma 3.1 holds on M. If we differentiate (1.3) along any vector field X in T_0 and using (2.3) and (2.8), then we have

$$\begin{aligned} & g\left(\left(A\phi-\phi A\right)Z,\nabla_X Y\right)+g\left(\left(A\phi-\phi A\right)Y,\nabla_X Z\right) \\ & =\beta\left(g\left(U,Z\right)g\left(AX,Y\right)+g\left(U,Y\right)g\left(AX,Z\right)\right) \end{aligned} \tag{4.1}$$

for any vector fields X, Y and Z on T_0. Putting $X=Y=Z=U$ into (4.1) and using Lemma 3.1 and 3.3, then we have

$$\beta\gamma=0. \tag{4.2}$$

Since Ω is not empty, we have $\gamma=0$ hold on Ω. It follows from (2.8) and Lemma 3.1 that

$$A\xi=\alpha\xi+\beta U, AU=\beta\xi \text{ and } A\phi U=0.$$

Thus M is locally congruent to ruled real hypersurface (see [7]). □

Proof Theorem 1.9. Assume that the open set $\Omega=\left\{p\in M \middle| \beta(p)\neq 0\right\}$ is not empty. Then we consider from Lemma 3.2 and 3.3 that $(\phi U)\beta=-\gamma^2$ and $c(U)=\dfrac{\gamma^2}{\beta}$. If we differentiate the smooth function $\beta=g(A\xi,U)$ along vector field X on M and (2.2), (2.5) and (2.8), we have

$$X\beta=g\left(\left(\nabla_\xi A\right)U+\left(\frac{c}{4\alpha}+\alpha\gamma-\gamma^2\right)\phi U, X\right). \tag{4.3}$$

Since we have $\left(\nabla_\xi A\right)U=\nabla_\xi\left(\beta\xi+\gamma U\right)-A\nabla_\xi U$, we see from this equation above that gradient vector field $\nabla\beta$ of β is given by

$$\begin{aligned} \nabla\beta= & -\left(A-\gamma I\right)\nabla_\xi U+\left(\xi\beta\right)\xi+\left(\xi\gamma\right)U \\ & +\left(\beta^2+\frac{c}{4}+\alpha\gamma-\gamma^2\right)\phi U, \end{aligned} \tag{4.4}$$

where I indicates the identity transformation on M. If we substitute (3.16) into (4.4) and using Lemma 3.4, then we obtain

$$\nabla\beta=\left(\beta^2+\frac{c}{4}+\alpha\gamma-\gamma^2\right)\phi U. \tag{4.5}$$

Since we have $(\phi U)\beta=-\gamma^2$, we get

$$\beta^2+\frac{c}{4}+\alpha\gamma=0. \tag{4.6}$$

By (4.6) and (3.22), we have $\beta=0$ and hence it is a contradiction. Thus the set $\Omega=\left\{p\in M \middle| \beta(p)\neq 0\right\}$ is empty, and hence M is a Hopf hypersurface. Since M is a Hopf hypersurface, we see from (2.1) and (2.8) that $\left(A\phi-\phi A\right)\xi=0$, which together with our assumption (1.4) implies (1.1), that is $A\phi=\phi A$ on M. Thus, Theorem 1.9 shows that M is locally congruent to a real hypersurface of type A. □

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REFERENCES

[1] U.-H. Ki and Y. J. Suh, "On Real Hypersurfaces of a Complex Space Form," *Mathematical Journal of Okayama University*, Vol. 32, 1990, pp. 207-221.

[2] R. Takagi, "On Homogeneous Real Hypersurfaces in a Complex Projective Space," *Osaka Journal of Mathematics*, Vol. 10, 1973, pp. 495-506.

[3] M. Kimura, "Real Hypersurfaces and Complex Submanifolds in Complex Projective Space," *Transactions of the American Mathematical Society*, Vol. 296, 1986, pp. 137-149. doi:10.1090/S0002-9947-1986-0837803-2

[4] J. Berndt, "Real Hypersurfaces with Constant Principal Curvatures in Complex Hyperbolic Space," *Journal Für Die Reine und Angewandte Mathematik*, Vol. 1989, No. 395, 1989, pp. 132-141.

[5] M. Okumura, "On Some Real Hypersurfaces of a Complex Projective Space," *Transactions of the American Mathematical Society*, Vol. 212, 1975, pp. 355-364. doi:10.1090/S0002-9947-1975-0377787-X

[6] S. Montiel and A. Romero, "On Some Real Hypersurfaces of a Complex Hyperbolic Space," *Geometriae Dedicata*, Vol. 20, No. 2, 1986, pp. 245-261. doi:10.1007/BF00164402

[7] S. Maeda and T. Adachi, "Integral Curves of Characteristic Vector Fields of Real Hypersurfaces in Nonflat Complex Space Forms," *Geometriae Dedicata*, Vol. 123, No. 1, 2006, pp. 65-72. doi:10.1007/s10711-006-9100-1

[8] W. H. Sohn, "Characterizations of Real Hypersurfaces of Complex Space Forms in Terms of Ricci Operators," *Bulletin of the Korean Mathematical Society*, Vol. 44, No. 2007, pp. 195-202. doi:10.4134/BKMS.2007.44.1.195

[9] S. H. Kon and T. H. Loo, "Real Hypersurfaces of a Complex Space Form with η-Parallel Shape Operator," *Mathematische Zeitschrift*, Vol. 269, No. 1-2, 2011, pp. 47-58. doi:10.1007/s00209-010-0715-4

[10] M. Kimura and S. Maeda, "On Real Hypersurfaces of a Complex Projective Space," *Mathematische Zeitschrift*, Vol. 202, No. 3, 1989, pp. 299-311. doi:10.1007/BF01159962

[11] Y. J. Suh, "On Real Hypersurfaces of a Complex Space Form with η-Parallel Ricci Tensor," *Tsukuba Journal of Mathematics*, Vol. 14, 1990, pp. 27-37.

[12] I.-B. Kim, K. H. Kim and W. H. Sohn, "Characterizations of Real Hypersurfaces in a Complex Space Form," *Canadian Mathematical Bulletin*, Vol. 50, 2007, pp. 97-104. doi:10.4153/CMB-2007-009-5

