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Denoising of Medical Images Using Multiwavelet Transforms and Various Thresholding Techniques

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ABSTRACT

The problem of estimating an image corrupted by additive white Gaussian noise has been of interest for practical reasons. Non-linear denoising methods based on wavelets, have become popular but Multiwavelets outperform wavelets in image denoising. Multiwavelets are wavelets with several scaling and wavelet functions, offer simultaneously Orthogonality, Symmetry, Short support and Vanishing moments, which is not possible with ordinary (scalar) wavelets. These properties make Multiwavelets promising for image processing applications, such as image denoising. The aim of this paper is to apply various non-linear thresholding techniques such as hard, soft, universal, modified universal, fixed and multivariate thresholding in Multiwavelet transform domain such as Discrete Multiwavelet Transform, Symmetric Asymmetric (SA4), Chui Lian (CL), and Bi-Hermite (Bih52S) for different Multiwavelets at different levels, to denoise an image and determine the best one out of it. The performance of denoising algorithms and various thresholding are measured using quantitative performance measures such as, Mean Square Error (MSE), and Root Mean Square Error (RMSE), Signal-to-Noise Ratio (SNR), Peak Signal-to-Noise Ratio (PSNR). It is found that CL Multiwavelet transform in combination with modified universal thresholding has given best results

Keywords: Multiwavelets; Noise; Thresholding; Additive White Gaussian Noise; Signal-to-Noise Ratio; Discrete Multiwavelet Transforms; Chui Lian; Symmetric Asymmetric Multiwavelet Transform; Bi-Hermite Multiwavelet Transform; Modified Universal Thresholding

1. Introduction

Digital images play an important role both in daily life applications such as satellite television, Magnetic Resonance Imaging (MRI), computer tomography as well as in areas of research and technology such as geographical information systems and astronomy. Data sets collected by image sensors are generally contaminated by noise. Imperfect instruments, problems with the data acquisition process, and interfering natural phenomena can all degrade the data of interest. Furthermore, noise can be introduced by transmission errors and compression. Thus, denoising is often a necessary and the first step to be taken before the image data is analysed. It is necessary to apply an efficient denoising technique to compensate for such data corruption. Image denoising still remains a challenge for researchers because noise removal introduces artifacts and causes blurring of the images. This paper describes different methodologies for noise reduction giving an insight as to which algorithm should be used to find the most reliable estimate of the original image data, given its degraded version. Noise modeling in images is greatly affected by capturing instruments, data transmission media, image quantization and discrete sources of radiation. Different algorithms are used depending on the noise model. Most of the natural images are assumed to have additive random noise which is modeled as a Gaussian.

The developments in wavelet theory have given rise to the wavelet thresholding method, for extracting an image from noisy data. Multiwavelets, wavelets with several scaling functions, have recently been introduced and they offer simultaneous orthogonality, symmetry and short support, which is not possible with ordinary wavelets, also called scalar wavelets. This property makes Multiwavelets more suitable for various image processing applications, especially denoising.

Donoho and Johnstone pioneered the theoretical formalization of filtering additive white Gaussian noise (of zero mean and standard deviation) via thresholding the decomposed coefficients. A decomposed coefficient is subjected to a given threshold and is set to zero if its magnitude is less than the threshold; otherwise, it is kept or modified, depending upon the thresholding rule. The choice of threshold determines to a great extent the efficiency of denoising algorithm.

Denoising procedure is depicted in **Figure 1**.

Step 1: Original image is a medical tomographic image (head phantom) to which noise components like additive white Gaussian noise is added to create a noisy image.

Step 2: Pre-filtering is done on the noisy image to convert the scalar coefficients of an image into vector coefficients for Multiwavelet decomposition.

Step 3: It deals with decomposition of pre-filtered image using various Multiwavelets like GHM, CL, SA4, BiHermite52S, and their respective Multiwavelet transforms.

Step 4: Thresholding methods are used to remove noise from decomposed image. Here we apply Multiwavelet thresholds such as soft, hard, universal, modified, fixed and multivariate thresholds.

Step 5: By applying inverse Multiwavelet transforms (IMWT), to thresholded coefficients, we get the denoised vector output image.

Step 6: Post filtering is done on reconstructed image to get back the denoised image coefficients in scalar form.

2. Theoretical Aspects of Multiwavelets

The idea of Multiwavelet originates from the generalization of scalar wavelets [1,2]. Instead of one scaling and one wavelet function, multiple scaling and multiple wavelet functions are used. This leads to more degree of freedom in constructing Multiwavelets. Therefore, opposed to scalar wavelets, properties such as orthogonality, symmetry, higher order of vanishing moments, compact support can be gathered simultaneously in Multiwavelets. Multiwavelets are of mainly two types: 1) Orthogonal type such as Geronimo-Hardin-Massopust (GHM), Symmetric Asymmetric (SA4), Chui-Lian (CL); and 2) Bi-Orthogonal type such as Bi-Orthogonal Hermite (Bih52S).

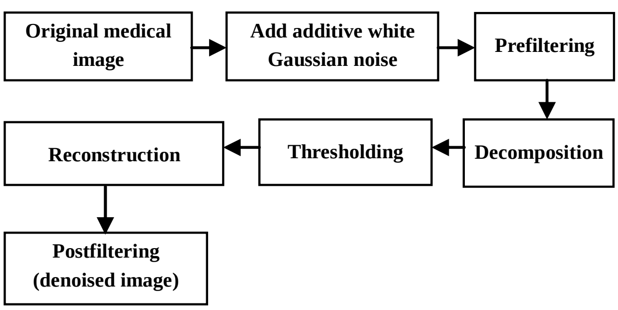

Figure 1. Block diagram of denoising using Multiwavelet transformation.

[3,4]. The scaling functions ϕ_1 and ϕ_2 are symmetric (linear phase) and they have short support (two intervals or less). The coefficients of Multiwavelets are 2 × 2 matrix. It retains the orthogonality of the Multiwavelets. The incoming signal is scalar type and is converted to vector type by using prefilter. The vector image is applied in discrete time to discrete Multiwavelet transform for low pass filtering, using low pass filter coefficients and down sampled (decimated) by 2, to get c_k coefficients and high pass filter coefficients are used for high pass filtering and down sampled by two to get d_k coefficients. This is the two band analysis bank. A perfect reconstruction synthesis bank recovers the image from the two down sampled outputs as shown in **Figure 2** [5-7].

The sub-bands of a single level Multiwavelet decomposition is shown in **Figure 3**. It has 16 sub-bands of an image [8,9].

Multiwavelets are characterized with several scaling functions and associated wavelet functions as given in Equations (1) and (2) respectively [10-12].

$$\phi(t) = \sum c_k (2t - k) \tag{1}$$

$$W(t) = \sum d_k (2t - k) \tag{2}$$

where $\phi(t)$ is a multiscaling function, $W(t)$ is a Multiwavelet function.

A multi filter has two or more low pass filters. The purpose of this multiplicity is to achieve the following properties [1].

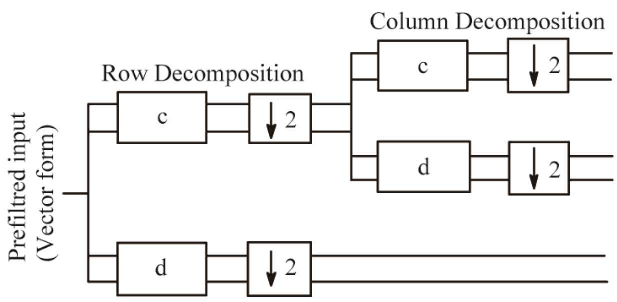

Figure 2. A multifilter bank with low pass filter iterated once.

L_1L_1	L_1L_2	L_1H_1	L_1H_2
L_2L_1	L_2L_2	L_2H_1	L_2H_2
H_1L_1	H_1L_2	H_1H_1	H_1H_2
H_2L_1	H_2L_2	H_2H_1	H_2H_2

Figure 3. Image sub-bands after one level of Multiwavelet decomposition.

Properties of ϕ_1 and ϕ_2: [13-15]

1) ***Symmetry*:** ϕ_1 and ϕ_2 have linear phase, *i.e.*, they are even functions (after a shift of the origin).

2) ***Short Support*:** Φ_i vanishes outside the interval [0, *i*]. Short support do not have much boundary problem. Long support must modify the function near boundaries. Wavelets have long support to achieve orthogonality. But Multiwavelets have short support and simultaneous orthogonality.

3) ***Second Order Accuracy*:** The scaling functions have second order accuracy, which establishes two vanishing moments.

4) ***Orthogonality*:** The translates $\phi_1(t-k) \& \phi_2(t-k)$ are all mutually orthogonal.

5) ***Matrix Multiscaling* (*Dilation*) *and Multiwavelet Equations*:** The coefficient "c_k" and "d_k" are 2 × 2 matrices multiplying vectors of scaling and wavelet functions as given in Equations (3) and (4) respectively.

$$\Phi(t) = \begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix} = \sum_{k=0}^{3} c_k \begin{bmatrix} \phi_1(2t-k) \\ \phi_2(2t-k) \end{bmatrix} \tag{3}$$

$$\Psi(t) = \begin{bmatrix} \psi_1(t) \\ \psi_2(t) \end{bmatrix} = \sum_{k=0}^{3} c_k \begin{bmatrix} \psi_1(2t-k) \\ \psi_2(2t-k) \end{bmatrix} \tag{4}$$

3. Thresholding Techniques

3.1. Hard Thresholding

It chooses coefficients greater than the given threshold λ and sets others to zero. It is unsuccessful in removing large noise coefficients.

The hard thresholding, operator is defined in Equation (5) as

$$F_h(x) = \begin{cases} x & \text{if } |x| \geq \lambda \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

The transfer function of the same is shown in **Figure 4**.

If coefficient > λ, value attributes to original pixel, else its noise and we discard it.

3.2. Soft Thresholding

Soft thresholding yields smaller error than hard and is generally preferred over hard thresholding. Soft thresholding shrinks coefficients by the threshold λ towards zero. It is also called as shrinkage function. The soft thresholding operator is defined in Equation (6) as,

$$F_s(x) = \begin{cases} x-\lambda & \text{if } x \geq \lambda \\ 0 & \text{if } |x| < \lambda \\ x+\lambda & \text{if } x \leq -\lambda \end{cases} \tag{6}$$

The transfer function of the same is shown in **Figure 5**.

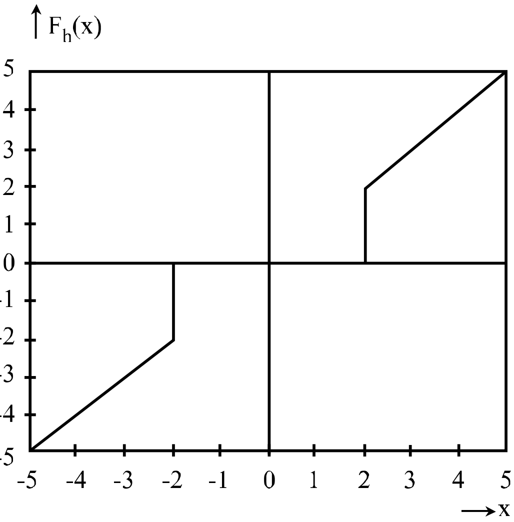

Figure 4. Hard thresholding.

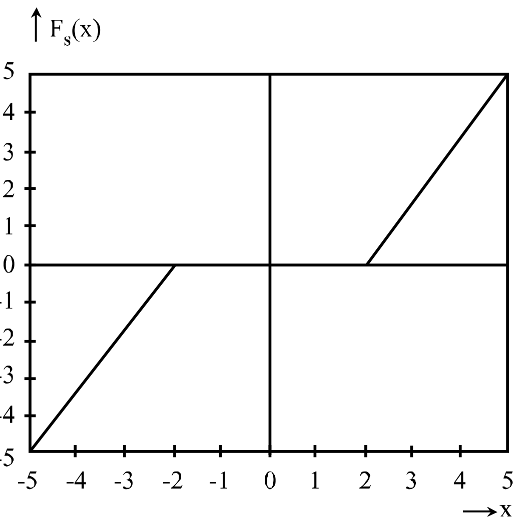

Figure 5. Soft thresholding.

3.3. Wavelet Thresholding [16,17]

Noise is generally present in the higher order frequency components obtained after wavelet and Multiwavelet decomposition. wavelet thresholding is the estimation technique which exploits the capabilities of wavelet transforms. Here small coefficients dominated by noise are set to zero, when they are below a certain threshold. In this way, noise can be removed.

3.3.1. Universal Threshold

The Universal threshold was introduced by Donoho and Johnston for scalar wavelets. It is given in Equation (7) as,

$$\lambda_{\text{universal}} = \sigma\sqrt{2}\log N, \tag{7}$$

where σ is the standard deviation of the noise.

N is the sample size.

Same threshold is applied to all coefficients regardless of the decomposition level.

For images, N is chosen to be the number of pixels in a row rather than the total number of pixels in the image. It is observed that this choice for N results in higher SNR values.

3.3.2. Modified Universal Threshold

1) The modified universal threshold is given in Equation (8) as,

$$\lambda_{\text{mod}} = \sigma\sqrt{2}\log 2N , \quad (8)$$

where σ is the standard deviation of the noise. N is the sample size.

2) Another modified universal threshold is given in Equation (9) as,

$$\lambda_N = \sigma\left(\sqrt{2}\log N + 2\log\log N\right) \quad (9)$$

where N is the sample size.

3.3.3. New Thresholding

The new threshold is given in Equation (10) as,

$$\lambda_{\text{Newthr}} = \sigma\left(\sqrt{2}\log\log N\right) \quad (10)$$

3.3.4. Multivariate Thresholding

This method is based on parent-child relationship which exists between image pixels of various sub-bands of Multiwavelet decomposition. If the parent coefficient has a small value, then the children would likely have small values. If the parent coefficient has a large value, the child might also have large values.

3.4. Measuring Parameters

3.4.1. Mean Square Error (MSE)

The MSE between the original image $I(x,y)$ and the reconstructed image $I'(x,y)$ is given in Equation (11) as,

$$\text{MSE} = \frac{1}{MN}\sum_{y=0}^{M-1}\sum_{x=0}^{N-1}\left[I(x,y) - I'(x,y)\right]^2 \quad (11)$$

where $I(x,y)$ is the original image, $I'(x,y)$ is the approximated version, which is actually the decompressed image and $M * N$ represents the size of an image.

3.4.2. Signal-to-Noise Ratio (SNR)

It is another measure often used to compare the performance of reproduced images. It is given in Equation (12) as,

$$\text{SNR} = 10\log_{10}\left(\frac{1}{MN}\sum_{y=0}^{M-1}\sum_{x=0}^{N-1}\left[I(x,y)\right]^2 \Big/ \text{MSE}\right) \quad (12)$$

3.4.3. Peak Signal-to-Noise Ratio (PSNR)

It is more subjective qualitative measurement of distortion. For 8-bit image, it is given in Equation (13) as,

$$\text{PSNR} = 10 \times \log_{10}\left(\frac{(255)^2}{(\text{MSE})}\right) \quad (13)$$

3.5. Experimental Results

The measuring parameters such as MSE, RMSE, PSNR and SNR values for various Multiwavelet transforms with repeated row prefilter and thresholding techniques are tabulated in **Table 1**.

3.5.1. Using Repeated Row Pre-Filtering Method

The measuring parameters such as MSE, RMSE, PSNR and SNR values for various Multiwavelet transforms with approximation prefilter and thresholding techniques are tabulated in **Table 2**.

3.5.2. Using Approximation Pre-Filtering Method

The measuring parameters such as MSE, RMSE, and PSNR and SNR values for GHM Multiwavelet transforms with various thresholding techniques are tabulated in **Table 3**.

The measuring parameters such as MSE, RMSE, and PSNR and SNR values for GHMAP Multiwavelet transforms with various thresholding techniques are tabulated in **Table 4**.

The measuring parameters such as MSE, RMSE, and PSNR and SNR values for CARDBAL2 Multiwavelet transforms with various thresholding techniques are tabulated in **Table 5**.

Denoising of tomographic image of phantom is shown in **Figure 6**.

Comparison of various Multiwavelet transformation with thresholding techniques with respect to parameters such as MSE, SNR, RMSE are depicted in the **Figures 7-9** respectively.

Comparison of various measuring parameters before and after thresholding is depicted in **Figure 10**.

Comparison between various Multiwavelet transforms along with various thresholding techniques is given in **Figure 11** and Comparison between various Multiwavelet transforms is given in **Figure 12**.

3.6. Conclusions

Multiwavelets became a focus of research partly because they made possible the construction of wavelet systems that are simultaneously orthogonal, symmetric and Finite Impulse Response. However, it has become clear that the implementation of the discrete Multiwavelet transform does not require prefilters. The performance of the Multiwavelets with different thresholding methods were investigated. We have as well modified a thresholding method to give better performance for the tomographic

Table 1. Measurement of denoising parameters using various Multiwavelet transformation with repeated row prefilter and thresholding techniques.

Parameters	GHM		CL		SA4		BiH52S	
	BT	AT	BT	AT	BT	AT	BT	AT
New Threshold								
MSE	0.0025	0.0004	0.0025	0.0011	0.0025	0.0012	0.0025	0.0155
RMSE	0.0498	0.0269	0.0498	0.0335	0.05	0.0349	0.0502	0.1246
PSNR (dB)	74.1916	82.1668	74.1866	77.6343	74.1541	77.273	74.1102	66.2236
SNR (dB)	6.324	10.3117	6.3215	8.0453	6.3052	7.8647	6.2833	2.34
Universal Threshold								
MSE	0.0025	0.0007	0.0025	0.0003	0.0025	0.0008	0.0025	0.0166
RMSE	0.0499	0.0263	0.0501	0.0186	0.05	0.028	0.0499	0.1288
PSNR (dB)	74.11648	79.7172	74.1375	82.7634	74.1442	7912769	74.1866	65.9309
SNR (dB)	6.3106	9.0868	6.297	10.6101	6.3003	8.8167	6.3215	2.1936
Modified Universal Threshold								
MSE	0.0025	0.0006	0.0025	0.0003	0.0025	0.0005	0.0025	0.0166
RMSE	0.0503	0.0251	0.0499	0.0167	0.0502	0.023	0.05	0.129
PSNR (dB)	74.1604	80.1536	74.1706	83.6994	74.1248	80.8961	74.1467	65.9207
SNR (dB)	6.2816	9.3049	6.3135	11.0779	6.2908	9.6747	6.3016	2.1886
Multivariate Threshold								
MSE	0.0025	0.0008	0.0025	0.0004	0.0025	0.0009	0.0025	0.0166
RMSE	0.0499	0.0275	0.0498	0.0195	0.05	0.0298	0.0501	0.1289
PSNR (dB)	74.1616	79.3328	74.1816	82.2447	74.1556	78.6606	74.1863	65.9273
SNR (dB)	6.309	8.8947	6.319	10.3887	6.306	8.5585	6.297	2.1896
Fixed Threshold								
MSE	0.0025	0.0003	0.0025	0.0004	0.0025	0.0019	0.0025	0.0008
RMSE	0.0503	0.0186	0.0497	0.0257	0.0498	0.0439	0.0497	0.1278
PSNR (dB)	74.1036	82.7341	74.1957	79.9169	74.1816	75.2828	74.1957	79.2436
SNR (dB)	6.28	10.5956	6.3262	9.1871	6.319	6.8696	6.3262	8.8504

Table 2. Measurement of denoising parameters using various Multiwavelet transformation with approximation prefilter and thresholding techniques.

Parameters	GHM		CL		SA4		BiH52S	
	BT	AT	BT	AT	BT	AT	BT	AT
New Threshold								
MSE	0.0025	0.0008	0.0025	0.0025	0.0025	0.0005	0.0025	0.0038
RMSE	0.0499	0.0282	0.0499	0.0499	0.0502	0.0228	0.05	0.062
PSNR (dB)	74.1765	79.14	74.1644	74.1633	74.112	80.9821	74.156	72.2764
SNR (dB)	6.3165	8.7986	6.3104	6.3224	6.3165	8.7986	6.3062	5.3664
Universal Threshold								
MSE	0.0025	0.0038	0.0025	0.0023	0.0025	0.0021	0.0025	0.0131
RMSE	0.0499	0.0615	0.0499	0.0481	0.0499	0.0455	0.0501	0.1143
PSNR (dB)	74.1655	72.3592	74.203	74.4897	74.161	74.973	74.1257	66.967
SNR (dB)	6.3109	5.4078	6.3297	6.4731	6.3109	5.4078	6.2912	2.7117
Modified Universal Threshold								
MSE	0.0025	0.0034	0.0025	0.0025	0.0025	0.0014	0.0025	0.0087
RMSE	0.05	0.0581	0.0499	0.0499	0.0501	0.0378	0.0498	0.0933
PSNR (dB)	74.1518	72.8456	74.1467	74.1612	74.1351	76.5872	74.1943	68.7368
SNR (dB)	6.3109	5.4078	6.3083	6.3088	6.3109	5.4078	6.3254	2.9201
Multivariate Threshold								
MSE	0.0025	0.0043	0.0025	0.0023	0.0025	0.0023	0.0025	0.0135
RMSE	0.05	0.0653	0.05	0.0478	0.0502	0.048	0.0499	0.1163
PSNR (dB)	74.1437	71.8352	74.1937	74.5359	74.1199	74.5136	74.1766	66.8158
SNR (dB)	6.3001	5.1458	6.3016	6.4962	6.3001	5.1458	6.3166	2.3276
Fixed Threshold								
MSE	0.0025	0.0005	0.0025	0.0006	0.0025	0.0008	0.0025	0.0011
RMSE	0.0501	0.023	0.05	0.0251	0.05	0.028	0.0502	0.0329
PSNR (dB)	74.1417	80.895	74.1442	80.1489	74.1455	79.173	74.1201	77.7814
SNR (dB)	6.299	9.6758	6.3003	9.3027	6.299	9.6758	6.2883	8.119

Table 3. Measurement of denoising parameters using direct Multiwavelet transform (GHM) using various thresholding techniques.

Parameters	New Thr		Universal Thr		Modified Universal Thr		Multivariate Thr		Fixed Thr	
	BT	**AT**	**BT**	**AT**	**BT**	**AT**	**BT**	**AT**	**BT**	**AT**
MSE	0.0025	0.0011	0.0025	0.0036	0.0025	0.0025	0.0025	0.004	0.0025	0.001
RMSE	0.0502	0.0328	0.05	0.0596	0.0502	0.0503	0.0499	0.0633	0.0499	0.0321
PSNR (dB)	74.1246	77.8223	74.1529	72.6189	74.1131	74.094	74.165	69.506	74.1541	78.0126
SNR (dB)	6.2908	8.1397	6.3046	5.5377	6.2848	6.2752	6.3108	5.2826	6.3085	8.2347

Table 4. Measurement of denoising parameters using direct Multiwavelet transform (GHMAP) using various thresholding.

Parameters	New Thr		Universal Thr		Modified Universal Thr		Multivariate Thr		Fixed Thr	
	BT	**AT**	**BT**	**AT**	**BT**	**AT**	**BT**	**AT**	**BT**	**AT**
MSE	0.0025	0.0011	0.0025	0.0029	0.0025	0.0027	0.0025	0.0032	0.0025	0.001
RMSE	0.05	0.0325	0.0498	0.0538	0.05	0.0517	0.0501	0.0564	0.05	0.0321
PSNR (dB)	74.1855	77.5138	74.1406	73.4042	74.1584	73.8591	74.1346	73.1048	74.1467	78.0114
SNR (dB)	6.3109	8.1434	6.2985	5.9305	6.3075	6.1581	6.2955	5.7807	6.3015	8.2341

Table 5. Measurement of denoising parameters using Multiwavelet transform (CARDBAL2) with identity prefilter using various thresholding.

Parameters	New Thr		Universal Thr		Modified Universal Thr		Multivariate Thr		Fixed Thr	
	BT	AT	BT	AT	BT	AT	BT	AT	BT	AT
MSE	0.0025	0.0009	0.0025	0.0008	0.0025	0.0008	0.0025	0.0008	0.0025	0.001
RMSE	0.0498	0.0292	0.0499	0.0282	0.0497	0.0281	0.0501	0.0283	0.05	0.0312
PSNR (dB)	74.1816	78.8104	74.1675	79.1367	74.1957	79.1642	74.1352	79.0973	74.1504	78.2357
SNR (dB)	6.319	8.6334	6.3121	8.797	6.3262	8.8107	6.2958	8.7768	6.3034	8.346

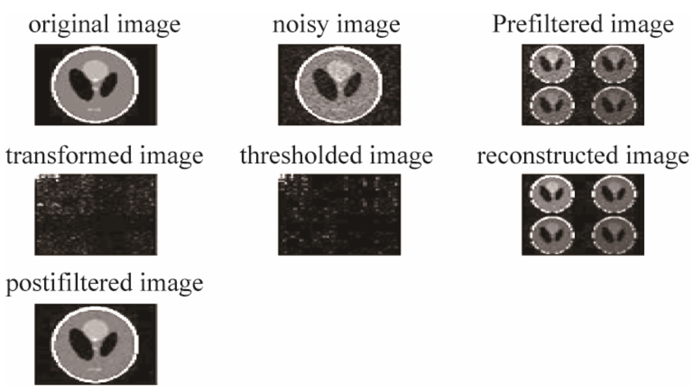

Figure 6. Denoising of a phantom using Multiwavelet transformation and thresholding techniques.

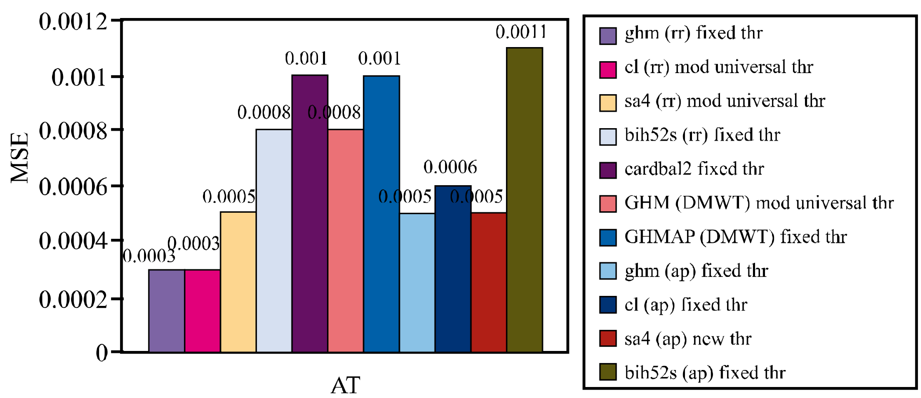

Figure 7. Comparison of various Multiwavelet transformation with thresholding techniques with respect to Mean Square Error.

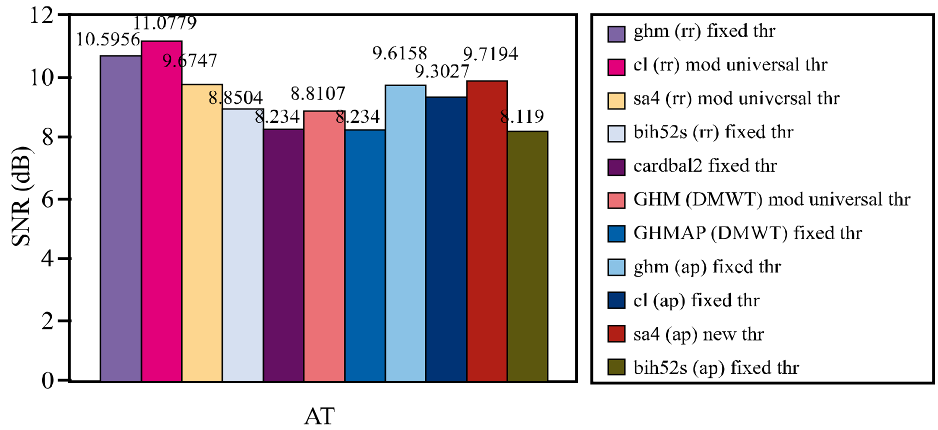

Figure 8. Comparison of various Multiwavelet transformation with thresholding techniques with respect to Signal-to-Noise Ratio.

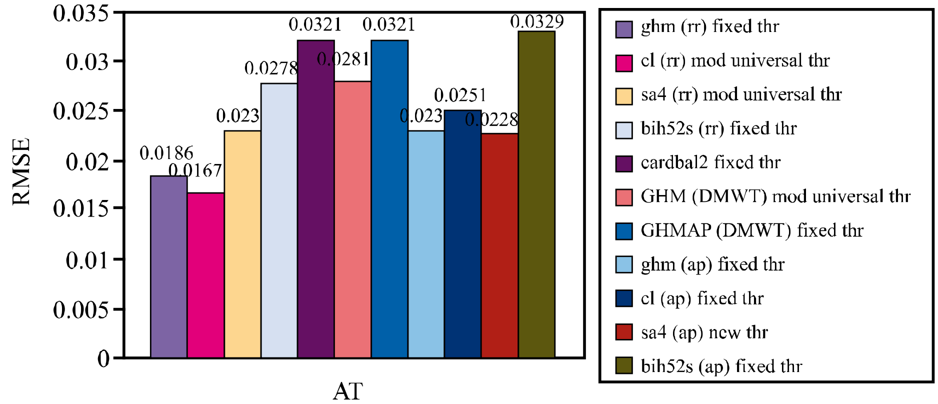

Figure 9. Comparison of various Multiwavelet transformation with thresholding techniques with respect to Root Mean Square Error.

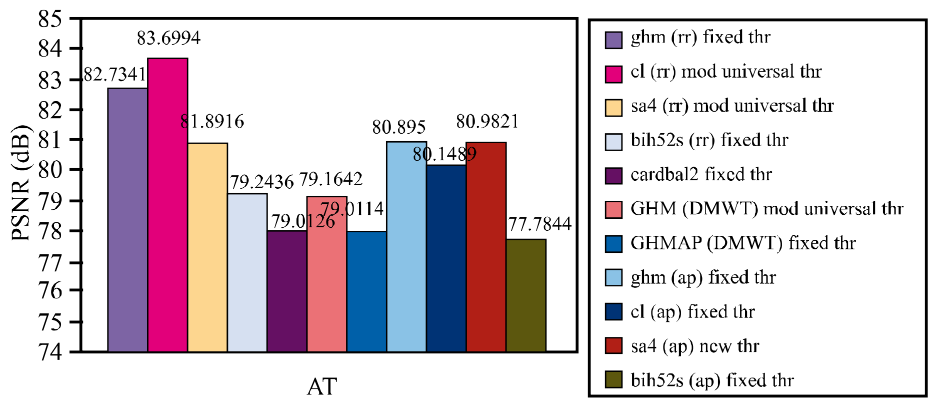

Figure 10. Comparison of various measuring parameters before and after thresholding.

image used in our algorithms and named it as new threshold.

To test the effectiveness of our algorithm, we added Gaussian noise to the 256 × 256 gray image phantom to, get noisy image and then applied Multiwavelet transforms and thresholding techniques to obtain denoised image. In this paper, different Multiwavelets like GHM, SA4, CL Bi-Hermite., and Multiwavelet transforms like Dec_2D, GHM and GHMAP with approximation and repeated row prefilters and various thresholding techniques like universal threshold, modified universal threshold, new threshold, multivariate threshold and fixed threshold is used to denoise the test image. Noise estimation parameters such as Root Mean Square Error (RMSE), Mean Square Error (MSE), Peak Signal to Noise Ratio (PSNR) and Signal to Noise Ratio (SNR) are

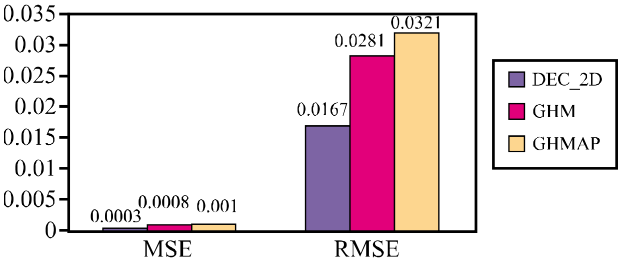

Figure 11. Comparison between various Multiwavelet transforms along with various thresholding techniques.

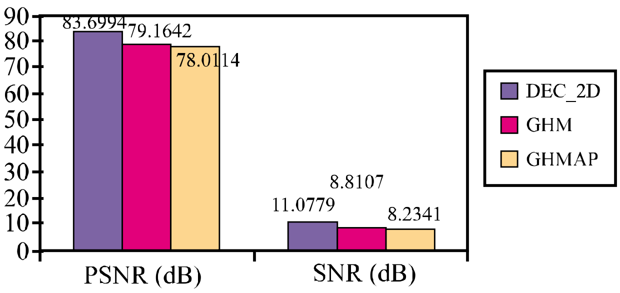

Figure 12. Comparison between various Multiwavelet transforms.

used to evaluate the performance of algorithm.

We have compared different Multiwavelets and thresholding techniques used. It is found that CL is the best Multiwavelet, when used with modified universal threshold and repeated row prefilter.

3.7. Future Scope

Nonetheless, there is always room for improvement. Since Multiwavelets are relatively a new subject of study, only a few construction methods for Multiwavelets are available. Most current filter available have two, three or fourth order of approximation. Future construction methods may add even higher order of approximation, while preserving the desirable features of current methods, would most likely result in multifilters that perform even better in image denoising and compression applications. Moreover the Multiwavelet systems available presently have the multiscaling and Multiwavelet coefficients which are 2 × 2 matrices. There is a possibility that in future many more Multiwavelet systems might be developed with matrix coefficients with higher order, which could provide even better results in the field of image denoising.

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