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Baryon wave functions and free neutron decay in the scalar strong interaction hadron theory (SSI)

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ABSTRACT

From the equations of motion for baryons in the scalar strong interaction hadron theory (SSI), two coupled third order radial wave equations for baryon doublets have been derived and published in 1994. These equations are solved numerically here, using quark masses obtained from meson spectra and the masses of the neutron, Σ^0 and Ξ^0 as input. Confined wave functions dependent upon the quark-diquark distance as well as the values of the four integration constants entering the quark-diquark interaction potential are found approximately. These approximative, zeroth order results are employed in a first order perturbational treatment of the equations of motion for baryons in SSI for free neutron decay. The predicted magnitude of neutron's half life agrees with data. If the only free parameter is adjusted to produce the known *A* asymmetry coefficient, the predicted *B* asymmetry agrees well with data and vice versa. It is pointed out that angular momentum is not conserved in free neutron decay and that the weak coupling constant is detached from the much stronger fine structure constant of electromagnetic coupling.

Keywords: Scalar Strong Interaction; Baryon Radial Wave Functions; Free Neutron Decay

1. INTRODUCTION

This paper consists of two parts.

In Part 2, the equations of motion of ground state doublet baryons in the scalar strong interaction hadron theory (SSI) derived earlier are solved numerically to yield approximate baryon wave functions and quark-diquark interaction potential.

In Part 3, these results are employed in free neutron decay to obtain decay time and the *A* and *B* asymmetry coefficients. Nonconservation of angular moemntum and detachment of weak and electromagnetic couplings are pointed out.

2. BARYON WAVE FUNCTIONS

2.1. Conversion to Six First Order Equations

Although the equations of motion for mesons [1] and for baryons [2] in the scalar strong interaction hadron theory (SSI) were both published in the early 1990's, subsequent work took place wholly in the meson sector. This due to that the meson equations can be decomposed into second order differential equations Eqs.6.7-8 of [1] or Eqs.3.2.3-4 of [3] which have analytical solutions in the rest frame providing the zeroth order background upon which higher order perturbational problems can be built and treated. Much success and new insights about mesons have been achieved, as are seen in Chapters 4-8 in [3]. Of current interest is that no Higgs boson is nee- ded to generate the mass of the gauge boson [4]. The linear Dalitz plot slope parameters in kaon decaying into three pions [5] and electromagnetic and strong decays of some vector mesons [6] have been treated. The epistemological foundation of SSI is given in the recent [7].

On the other hand, the two coupled third order radial equations for baryon doublets **Eq.A20** cannot be decoupled and reduced to lower order equations. They are eventually reduced to the six first order equations **Eq.1** below which have to be solved numerically. Further, the interquark potential **Eq.A15** contains four unknown integration constants that enter **Eq.1**. Very lengthy work has been spent in finding these constants and solving **Eq.1** below by computer. This is the reason that the baryon results to be presented below come so many years later. Such numerical computations have been carried out and the four integration constants in **Eq.A15** and **Eq.1** are obtained together with the radial wave functions.

The coupled third order equations **Eq.A20** cannot be solved analytically and have to be treated numerically.

The standard procedure is to convert them into a first system according to Eq.10.7.5 of [3] after putting the orbital angular momentum $L = 0$ there (see **Eq.1**),

In arriving at **Eq.1**, **Eq.A15** has been used. E_0 is the neutron mass, M_b^3 is the quark mass term defined in **Eq.A4** and the four d_b constants are integration constants in the solution to the homogenized **Eq.A14** $\Delta\Delta\Delta\Phi_b(r) = 0$ and are independent of flavor, *i.e.*, baryon species, as was pointed out below Eq.10.1.6 of [3].

These four d_b constants can therefore not be fixed by using four baryon masses as input. The procedure adopted is as follows. The quarks masses in **Eq.A4** are $m_u = 0.6592$, $m_d = m_u + 0.00215$ and $m_s = 0.7431$ in units of Gev taken from Table 1 of [8] or Table 5.2 of [3] obtainned from meson spectra. Three known baryon masses are $E_0 = 0.9397$, 1.1926 and 1.3148 Gev for neutron, Σ^0 and Ξ^0, respectively [9]. These are inserted into **Eq.1** and the four d_b's are varied over suitable ranges such that the solutions satisfy the boundary condition at $r \to \infty$ or converge there.

Near the origin $r \to 0$, the baryon radial wave functions can be expanded in power series in r according to Eq.10.7.6 of [3],

$$f_o(r) = -\sum_{\nu=0}^{\infty} a_\nu r^{\lambda+\nu}, \quad g_o(r) = \sum_{\nu=0}^{\infty} b_\nu r^{\lambda+\nu} \tag{2}$$

where λ satisfy the indicial equation Eq.10.7.4 of [3] and is related to λ_+ in **Eq.4** below. a_ν and b_ν are constants satisfying the recursion formula Eq.10.7.7 of [3]. At large r, Eq.10.2.7a of [3] gives

$$\begin{aligned} g_0(r \to \infty) &= -f_0(r \to \infty) \\ &= \text{constant} \times \exp\left(-\frac{3}{5}\left|d_{b2}\right|^{1/3} r^{5/3}\right) \end{aligned} \tag{3}$$

The presence of $r^{1/3}$ is incompatible with the power series **Eq.2** and **Eq.1** has to be solved numerically.

Equivalent to **Eq.2** is the expansion around the regular singular point $r = 0$ in **Eq.1**,

$$w_\alpha(r) \to w_{(\lambda+)\alpha}(r) = \sum_{\beta=0}^{\infty} r^{\lambda+} w_{(\lambda+)\alpha\beta} r^\beta, \quad \alpha = 1,2,....6 \tag{4}$$

where λ_+ are given by Eq.10.2.8a of [3]. The three cases with possible $\lambda_+ < 0$ are excluded because they lead to diverging w_α ($r = 0$). The remaining cases yield

$$\lambda_+ = 0, \quad w_{(0)40} = 1, \quad w_{(0)10} = 0 \tag{5}$$

$$\lambda_+ = 1, \quad w_{(1)40} = 0, \quad w_{(1)10} = w_{(1)} \tag{6}$$

$$\lambda_+ = 2, \quad w_{(2)40} = w_{(2)}, \quad w_{(2)10} = 0 \tag{7}$$

where the amplitude in **Eq.5** has been normalized to unity. $w_{(1)}$ and $w_{(2)}$ are amplitudes for the remaining two solutions. The general solution near $r = 0$ reads

$$w_\alpha(r) = w_{(0)\alpha}(r) + w_{(1)\alpha}(r) + w_{(2)\alpha}(r) \tag{8}$$

Substituting this into **Eq.1**, which is now solved as an initial value problem with initial conditions **Eqs.5-7**.

2.2. Numerical Solution and Results

The six unknown parameters d_b, d_{b0}, d_{b1}, d_{b2}, $w_{(1)}$, and $w_{(2)}$ are adjusted so that the six boundary conditions w_α $(r \to \infty) \to 0$ for a given M_b^3 and the associated baryon mass E_0. The calculations are done via a Fortran program employing Runge-Kutta integration subroutine on computers of the Department of Information Technology at Uppsala University. In the paramater range of interest, it was found that integration of **Eq.1** and summation of the power series **Eq.2** give the nearly the same results for $r \le$ 7-8 Gev^{-1}. Beyond this value, **Eq.2** become unreliable due to accumulated computational errors. Similarly, **Eq.1** leads often to diverging solutions at $r >$ 7-12. This is because that **Eq.3** without the minus sign in the exponent is also a solution which tends to overshadow **Eq.3** due to accumulated errors and takes over at large r.

Near the origin $r = 0$ and at large r, the potential in **Eq.1** is dominated by the d_b/r and $d_{b2}r^2$, respectively, and the flavor dependent quark mass term M_b^3 there can be dropped. The solutions $f_0(r)$ and $g_0(r)$ in these r regions are independent of the baryon flavor. Thus, d_b and d_{b2} are flavor independent constants on par with the corresponding meson sector's d_m and d_{m0} which are independent of flavor according to Subsection 4.4 of [3]. The small r region is very small and the large r region determines the asymtotic behavior of $f_0(r)$ and $g_0(r)$.

$$\begin{gathered}
w_1(r) = -f_0(r), \qquad w_4(r) = g_0(r) \\
\frac{\partial w_1}{\partial r} = \frac{w_2}{r}, \qquad \frac{\partial w_2}{\partial r} = \frac{w_3}{r}, \qquad \frac{\partial w_4}{\partial r} = \frac{w_5}{r}, \qquad \frac{\partial w_5}{\partial r} = \frac{w_6}{r} \\
\frac{\partial w_3}{\partial r} = w_1\left[-\frac{E_0^2}{4}2r\right] + w_2\left[\frac{2}{r} - \frac{E_0^2}{4}r\right] - w_3\frac{1}{r} + w_4\left[d_b r + \left(\frac{E_0^3}{8} + M_b^3 + d_{b0}\right)r^2 + d_{b1}r^3 + d_{b2}r^4\right] + (w_5 + w_6)\frac{E_0}{2} \\
\frac{\partial w_6}{\partial r} = w_1\left[d_b r + \left(-\frac{E_0^3}{8} + M_b^3 + d_{b0}\right)r^2 + d_{b1}r^3 + d_{b2}r^4 + E_0\right] - (w_2 + w_3)\frac{E_0}{2} w_5\left[\frac{2}{r} - \frac{E_0^2}{4}r\right] + w_6\frac{1}{r}
\end{gathered} \tag{1}$$

Therefore, d_{b2}, which determines the "confinement strength" via **Eq.3**, is used to lable a set of the six unknown parameters and chosen first. Extensive computer calculations showed that the remaining five constants are uniquely fixed if the solutions g_0 and f_0 are to converge at $r \to \infty$. Among a huge numbers of combinations of the six parameters, only a very narrow range of values for these parameters leads to converging $w_\alpha(r)$. Some examples of the results are given in **Table 1**.

The wave functions for the $d_{b2} = -0.4462$ case are plotted in **Figure 1** below.

Convergent solutions have been found for continuous ranges of d_{b2} values largely in the region –0.1 to –1.4, although some values outside this region seem also to lead to convergence and some values inside this region, for instance from –0.23 to –0.27, do not. It may note that it is to some degree arbitrary to regard a set of solutions to be convergent or not. Due to accumulation of computer errors at large r, all solutions eventually diverge for sufficiently large r. Convergence is regarded as good if $f_{00}(r)$ and $g_{00}(r)$ are nearly zero over a "sufficiently" large range of r when r is large.

The mean spread σ_s in **Table 1** is defined as follows. Let

$$\sigma_{db} = \frac{1}{\bar{d}_b}\sqrt{\sum\nolimits_{\text{baroyn species}}\left(d_b - \bar{d}_b\right)^2 / 3}, \qquad \bar{d}_b = \text{average value of } d_b \tag{9}$$

Repeat this step for d_{b0} and d_{b1} and define

$$\sigma_s = \sigma_{db} + \sigma_{db0} + \sigma_{db1} \tag{10}$$

which is a measure of how much the d_b, d_{b0} and d_{b1} values deviate from the averages $\bar{d}_b$, $\bar{d}_{b0}$ and $\bar{d}_{b1}$.

Near the origin $r = 0$, the potential **Eq.A15** is dominated by the d_b/r terms and the solutions $f_0(r)$ and $g_0(r)$ are independent of the baryon flavor. Thus, d_b is a flavor independent constant on par with the corresponding meson sector's d_m and d_{m0} which are independent of flavor according to Subsection 4.4 of [3]. Conversely, one expects calculations produce a common d_b value which is confirmed in **Table 1**.

If a set of d_{b2}, d_{b1}, d_{b0}, and d_b values that lead to convergent solutions for all three baryons in **Table 1**, a solution to the present baryon spectra problem has been found. **Table 1** shows that this is not the case but some sets possess values that are rather close to each other for the three baryons and may therefore be regarded as approximate solutions to the baryon spectra problem here. Possible nature of these approximations are consisdered in the next section.

The set that yields d_{b2}, d_{b1}, d_{b0}, and d_b values which are closest to each other for the three baryons in **Table 1** is the $d_{b2} = -0.4462$ case with a minimum spread $\sigma_{db} =$ 1.4% for d_b. The mean spread $\sigma_s = 6.2\%$ is also a minimum. This set and may therefore presently be regarded as representing an approximate solution to the baryon spectra problem here.

Table 1. Values of the four d_b constants in **Eq.1** with Gev as basic unit, the spread σ_{db1}, σ_{db0} and σ_{db} from the mean values of the four d_b constants and the sum spread σ_s according to **Eq.10** below and $w_{(1)}$ and $w_{(2)}$ in **Eqs.5-7** for some converging solutions in **Eq.1**.

	d_{b2}	d_{b1}	d_{b0}	d_b	σ_s	$w_{(1)}$	$w_{(2)}$
Neutron	–0.140	1.0439	–3.0	0.695		0.016	0.0288
Σ^0	–0.1411	1.336	–4.06	1.085		0.0415	0.0691
Ξ^0	–0.141	1.111	–3.568	0.924		0.0019	0.0851
σ_d		10.7%	12.2%	17.7%	40.6%		
Neutron	–0.1641	1.334	–3.719	1.013		0.0668	0.045
Σ^0	–0.1641	1.279	–3.706	0.990		0.0353	0.0756
Ξ^0	–0.1641	1.220	–3.691	0.9177		0.0523	0.0721
σ_d		3.6%	0.3%	4.2%	8.1%		
Neutron	–0.3202	2.272	–4.922	1.024		0.1827	0.0674
Σ^0	–0.3202	2.167	–4.783	1.032		0.0586	0.0662
Ξ^0	–0.3202	2.142	–4.899	1.083		0.1191	0.1061
σ_d		2.6%	1.2%	2.5%	6.3%		
Neutron	–0.4462	2.968	–5.749	1.035		0.247	0.0859
Σ^0	–0.4462	2.831	–5.541	1.066		0.1624	0.090
Ξ^0	–0.4462	2.768	–5.516	1.033		0.121	0.0974
σ_d		2.9%	1.9%	1.4%	6.2%		
Neutron	–0.575	3.658	–6.55	1.064		0.2943	0.102
Σ^0	–0.575	3.471	–6.224	1.073		0.203	0.997
Ξ^0	–0.575	3.383	–6.147	1.029		0.195	0.1164
σ_d		2.2%	2.8%	1.8%	6.8%		
Neutron	–0.975	5.572	–8.328	0.8173		0.4278	0.1576
Σ^0	–0.975	5.319	–7.895	0.9048		0.316	0.1298
Ξ^0	–0.975	5.090	–7.441	0.6859		0.2902	0.1391
σ_d		3.7%	4.6%	11.2%	19.5%		

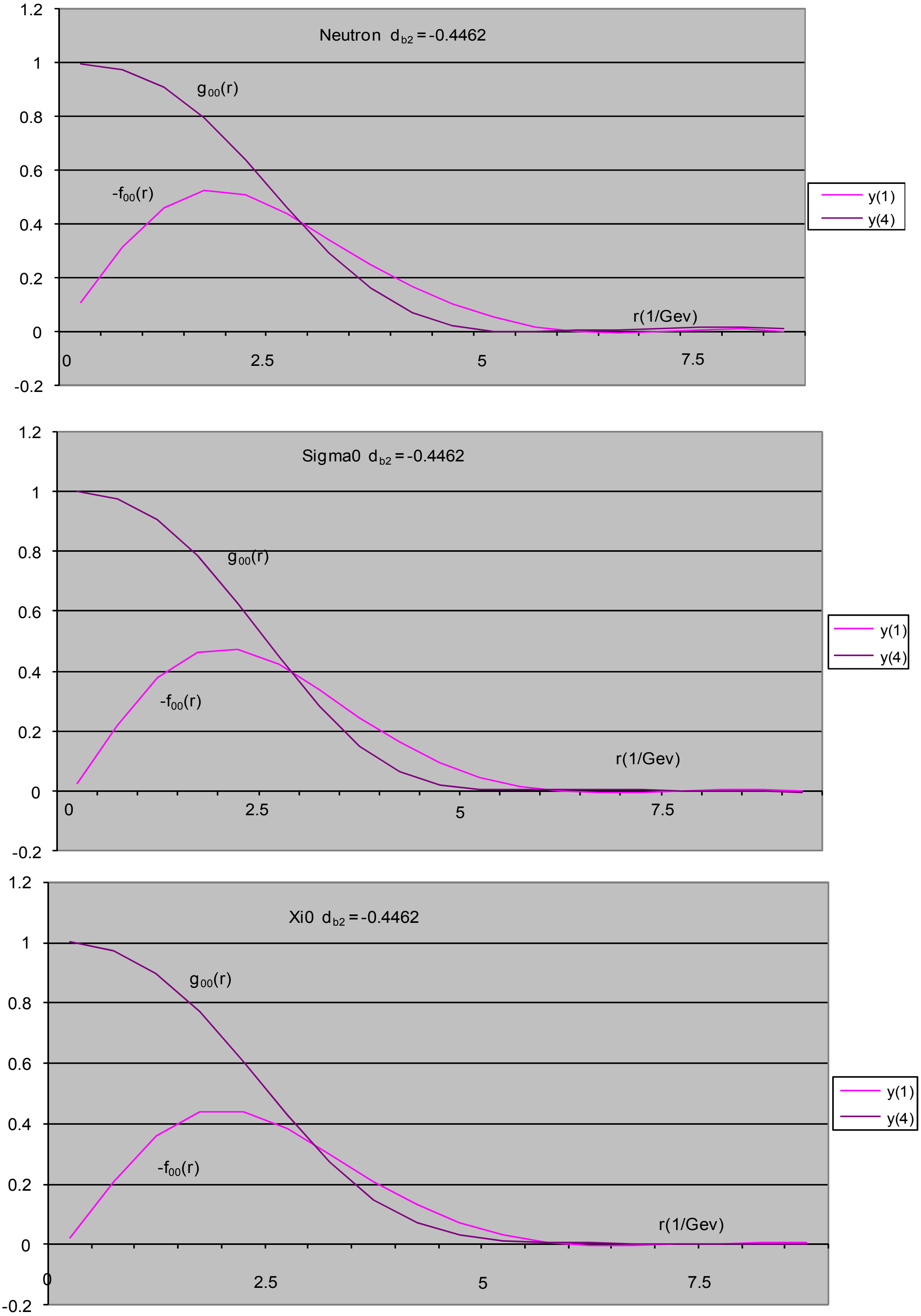

Figure 1. Baryon radial wave functions $f_0(r)$ and $g_0(r)$ in **Eq.1** normalized according to **Eq.A23** for the $d_{b2} = -0.4462$ case in **Table 1**. r is the quark-diquark distance.

However, **Table 1** also shows that this minimum is very shallow one and sets of d_{b2}, d_{b1}, d_{b0}, and d_b values with d_{b2} values in the range around –0.32 to –0.45 may also be qualified to yield approximate solutions. This range is supported by the approximative agreement of the calculated and experimental values of the half life and A and B asymmetry coefficients of free neutron decay for the d_{b2} = –0.3202 case in Part 3 below.

The value of the constant d_b corresponding to meson sector's d_m in Eq.5.2.3 of [3] and d_{m0} in Table 5.2 of [3] is close to $\overline{d}_b$ for d_{b2} values in the range around d_{b2} = –0.32 to –0.45 in **Table 1** and is

$$d_b \approx 1.04 \text{ Gev}^2 \tag{11}$$

The above results have been obtained using Eq.10.2.3a of [3] or for $j = l + 1/2$. If Eq.10.2.3b of [3] or $j = l - 1/2$ were chosen, the d_{b0} values for the three baryons in **Table 1** would deviate so much from each other that d_{b0} can no longer be considered as an approximately flavor independent constant.

The "reduced order" baryon spectra Eq.10.4.7 of [3] was obtained assuming non-relativistic qaurks, *i.e.*, $E_{0d}^2/4 >> \Delta_0, \Delta_1$ mentioned above Eq.10.4.1 of [3]. However, it can be estimated from **Figure 1** that the opposite holds. Therefore, Eq.10.4.7 of [3], hence also Eq.10.5.20 of [3] for the quartet, cannot be used. The quark and diquark in the baryons are highly relativistic just like the quarks in the mesons are at the end of Subsecction 5.3 of [3].

2.3. On Functions Nonseparable in Relative Space x and Internal Space z

Consider a two electron system. When the both electrons are far apart, it is described by two independent Dirac wave functions $\psi_\alpha(x_I)$ and $\psi_\beta(x_{II})$ totaling eight wave function components. When they are closer to each other the product wave fucntions $\psi_\alpha(x_I)\psi_\beta(x_{II})$ must be generalized to the nonseparable $\psi_{\alpha\beta}(x_I,x_{II})$ having 16 components governed by the Bethe-Salpeter equation which includes the interaction between the both electrons. The eight extra wave function components are associated with this interaction and are small perturbations when the both electrons are not too close to each other. This example illustrates the conjecture below.

The approximate solutions in Subsection 1.2 are based upon the construction that the total baryon wave functions are separable in the relative space wave function $\psi_0^a(\underline{x})$ and internal space functions $\eta_r^p(z_I,z_{II})$ in Eq.9.3.7b of [3]. According to the epistemological considerations in Subsection 5.4 of [7] or Appendix G of [3], these both spaces are "hidden", on par with each other, and at the so-called "level logic2" and can be combined to form a larger manifold (x,z_I,z_{II}). In this case the product form $\psi_0^a(\underline{x})\eta_r^p(z_I,z_{II})$ in Eq.9.3.7b of [3] needs be generalized to the nonseparable form $\Psi_0{}^a{}_r{}^p(x,z_I,z_{II})$. A *conjecture* is now made that the mass operator $m_{3op}(z_I,z_{II})$ of Eq.9.3.14 of [3] is also generalized to a nonseparable, as yet unknown form $m_{3op}(z_I,z_{II},x)$, analogous to the generalization of the masses operator $m_{2op}(z_I,z_{II})$ to $m_{2op}(z_I,z_{II},x)$ in Subsection 5.4 of [7] or Appendix G of [3]. This generalization may for instance be such that when $r = |\underline{x}|$ falls outside some region of values, $m_{3op}(z_I,z_{II},x)$ degenerates back to $m_{3op}(z_I,z_{II})$.

The product wave fucntion $\psi_0^a(\underline{x})\eta_r^p(z_I,z_{II})$ in Eq.9.3.7b of [3] has 10 components, two from a = 1, 2 and eight from $p = r$ = 1, 2, 3(less the singlet). The generalized, nonseparable $\Psi_0{}^a{}_r{}^p(x,z_I,z_{II})$ has 2 × 8 = 16 wave fucntion components, six more than 10 components. These six extra components are associated with this presently unknown dependence of $m_{3op}(z_I,z_{II},x)$ upon x. They may be the cause of the approximative nature of the results in **Table 1**. Actually, the generalization from separable to nonseparable forms can already be formally introduced at the quark level, as was pointed out at the end of Subsection 11.1.2 of [3].

3. FREE NEUTRON DECAY AND POSSIBLE NONCONSERVATION OF ANGULAR MOEMNTUM

3.1. Background

The present theory of nuclear β-decay is based upon the electroweak part of the standard model [9]. The origin of this part is the four fermion point interaction Lagrangian density

$$L_F = -C_V\left(\overline{\psi}_p\gamma_\mu\psi_n\right)\left(\overline{\psi}_e\gamma_\mu\psi_\nu\right) \tag{12}$$

for neutron decay

$$n \to p + e^- + \overline{\nu} \tag{13}$$

first proposed by Fermi in 1934. Here C_v is a constant. Subsequently, **Eq.12** has been generalized to the β-interaction Hamiltonian currently in use Eq.13.9 of [10],

$$H_W = \frac{1}{\sqrt{2}}\sum_i\int d^3\underline{X}\left(\overline{\psi}_pO_i\psi_n\right)\left(C_i\overline{\psi}_eO_i\psi_\nu + C_i'\overline{\psi}_eO_i\gamma_5\psi_\nu\right) + h.c. \tag{14}$$

Here, i refers to the scalar, vector, tensor, axial vector, and pseudoscalar interactions between the nucleon and lepton currents. The O_i's contain γ_μ and γ_5 and the C's are generally complex.

Based upon **Eq.14**, lepton kinematics and conventionnal conservation laws, including angular momentum conservation, Jackson *et al.* [11] derived in 1957 a number of decay rate formulae. The most frequently used one is

$$dW = \text{constant} \times d^3\underline{K}_e d^3\underline{K}_\nu \delta\left(E_e + E_\nu + E_p - E_n\right) \times \xi\left[1 + a\frac{\underline{K}_e\underline{K}_\nu}{E_e E_\nu} + \frac{\langle \underline{J}_n\rangle}{J_n}\left(A\frac{\underline{K}_e}{E_e} + B\frac{\underline{K}_\nu}{E_\nu} + D\frac{\underline{K}_e \times \underline{K}_\nu}{E_e E_\nu} + R\frac{\underline{K}_e \times \underline{\sigma}_e}{E_e}\right) + \ldots\right] \quad (15)$$

in which the final spins have been summed over for a given initial neutron polarization $\langle \underline{J}_n\rangle$. Here, the $\underline{K}$'s denote momenta, E energy and $\underline{\sigma}_e$ the electron polarization. The constants a, A, B, D, R, and ξ depend upon the C's and the nucleon current consisting of a vector and an axial vector part in the so-called V-A theory. Experiments on neutron decay have since then largely been devoted to determine these constants in this nearly 50 year old **Eq.15** and related formulae [12]. These have, however, yielded little physical insight into the decay mechanism.

The standard Hamiltonian **Eq.14** is a phenomenological model, not derivable from any first principles theory. It treats nucleon as a point particle and hence ignores its quark structure. There are in principle 20 real C constants in **Eq.14**, leaving the theory with little predictive power. This hints at a superfluousness of **Eq.14**, which is not invariant under SU(2) gauge transformations. Such an invariance would give rise to an intermediate vector boson W which couples to a left-handed $\bar{\nu} e$ pair.

Therefore, despite its noble origin and general acceptance, this model **Eq.14** does not differ in principle from the large number of recent phenomenological models, constructed for different and narrow application angles, present in a vast body of literature.

Angular momentum conservation has not been established experimentally in free neutron decay. Most of the experiments make use of nuclei and the conclusion is that angular momentum is conserved. But a nucleus poses a highly complex, unsolved many body problem and experiments with it cannot lead to any firm conclusion on angular momentum conservation in free neutron decay.

In this part, free neutron decay will be treated using the equations of motion for doublet baryons **Eqs.A7-A8** and its development in Chapters 9-11 of [3], incorporating vector gauge fields of Eqs.12-13 of [4] or Subsection 7.1.2 of [3] and new tensor gauge fields. The results obtained, not reachable from **Eq.15**, include a prediction of the half life of the neutron in approximate agreement with data and a relatively good prediction of the B asymmetry coefficient.

3.2. Introduction of Vector and Tensor Gauge Fields

3.2.1. Action-Like Integrals for Doublet Baryons

The starting point is the equation of motion for doublet baryons **Eqs.A7-A8**. Multiply **Eq.A7** from the left by χ_{0a}^* and **Eq.A8** by $-\psi_0^{*\dot{b}}$, subtract the resulting expressions from their complex conjugates and integrate over x_I and x_{II} to obtain

$$S'_\chi = \frac{1}{2i}\int dx_I^4 dx_{II}^4 \Big(\chi_{0a}^* \partial_{II}^{f\dot{e}} \partial_I^{e\dot{h}} \partial_I^{a\dot{b}}\, \tfrac{1}{2}\, \chi_{\{\dot{b}\dot{h}\}f} + i\left(M_b^3 + \Phi_b\right)\chi_{0a}^* \psi_0^a - c.c.\Big) \quad (16)$$

$$S'_\psi = \frac{1}{2i}\int dx_I^4 dx_{II}^4 \Big(-\psi_0^{*\dot{b}} \partial_{II\,\dot{e}h} \partial_{I\,\dot{h}k} \partial_{I\,\dot{b}c}\, \tfrac{1}{2}\, \psi^{\{ck\}\dot{e}} + i\left(M_b^3 + \Phi_b\right)\psi_0^{*\dot{b}} \chi_{0\dot{b}} - c.c.\Big) \quad (17)$$

where * is an extra sign denoting complex conjugate, $\partial_I = \partial/\partial x_I$ and $\partial_{II} = \partial/\partial x_{II}$ and their positions have been changed so that the summations over the e and h indices conform to matrix multiplication convention and that the both ∂_I's appear next to each other to reflect that they operate on the diquark part indicated by the braces.

These integrals will not be varied with respect to χ_{0a}^* and $-\psi_0^{*\dot{b}}$ in an attempt to reproduce **Eqs.A7-A8**; handling of the last $c.c.$ terms and the necessary boundary conditions will require efforts beyond the scope of this chapter. Nor is such a variation necessary for the present purposes. If the solutions to **Eqs.A7-A8** is inserted into **Eqs.16-17**, we obtain

$$S'_\chi \to S'_{\chi 0} = 0,\; S'_\psi \to S'_{\psi 0} = 0 \quad (18)$$

3.2.2. Non-Minimal Substitution and Tensor Gauge Field

The minimal substitution of Eq.12 of [4] or Eq.7.1.4 of [3] led to the introduction of the vector gauge boson field W which is naturally associated with the vector part V of the V-A theory mentioned beneath **Eq.15**. The axial vector part A is an asymmetrical part of a tensor which can be introduced by the following non-minimal substitution (see **Eqs.19-20**),

$$\partial_I^{e\dot{h}}\partial_I^{a\dot{b}}\chi_{\{\dot{h}\dot{b}\}f} \to \partial_I^{e\dot{h}}\partial_I^{a\dot{b}}\chi_{\{\dot{h}\dot{b}\}f} + \frac{i}{4}g\left[W^{e\dot{h}}(X)\partial_I^{a\dot{b}} + W^{a\dot{b}}(X)\partial_I^{e\dot{h}} + \frac{1}{2}T^{e\dot{h}a\dot{b}}(X) + \frac{i}{4}gW^{e\dot{h}}(X)W^{a\dot{b}}(X)\right]\chi_{\{\dot{b}\dot{h}\}f} \quad (19)$$

$$\partial_{I\dot{h}k}\partial_{I\dot{b}c}\psi^{\{kc\}\dot{e}} \to \partial_{I\dot{h}k}\partial_{I\dot{b}c}\psi^{\{kc\}\dot{e}} + \frac{i}{4}g\left[W_{\dot{h}k}(X)\partial_{I\dot{b}c} + W_{\dot{b}c}(X)\partial_{I\dot{h}k} + \frac{1}{2}T_{\dot{h}k\dot{b}c}(X) + \frac{i}{4}gW_{\dot{h}k}(X)W_{\dot{b}c}(X)\right]\psi^{\{ck\}\dot{e}} \quad (20)$$

where **Eq.A10** has been noted. The right side of **Eq.19** can readily be shown to be invariant under the U(1) gauge transformations,

$$\begin{aligned} &W^{a\dot{b}}(X) \rightarrow W^{a\dot{b}}(X) - \partial_X^{a\dot{b}}\phi_s(X), \\ &T^{e\dot{h}a\dot{b}}(X) \rightarrow T^{e\dot{h}a\dot{b}}(X) - \partial_X^{e\dot{h}}\partial_X^{a\dot{b}}\phi_s(X) \\ &\chi_{\{\dot{h}\dot{b}\}f} \rightarrow \chi_{\{\dot{h}\dot{b}\}f}\exp\left(\tfrac{i}{2}g\phi_s(X)\right) \end{aligned} \tag{21}$$

where $\phi_s(X)$ is a local phase and Eq.12 of [4] or Eq.7.1.4 of [3] has been consulted. The right side of **Eq.20** transform analogously.

3.2.3. SU(3) Tensor Gauge Fields and Gauge Invariance

These expressions are now generalized to include SU(3) gauge fields analogous to Eq.7.1.4 of [3]. Limiting ourselves to baryon doublets in Eq.9.3.7b of [3], **Eqs.16-17** with **Eqs.19-20** are generalized, with the sign of the tensor term changed in **Eq.20**, to

$$S_\chi = \frac{1}{2i}\int dx_I^4 dx_{II}^4 \chi^*_{tp\ 0a} \times \left[\begin{array}{l} \left\{\partial_{II}^{f\dot{e}}\delta_{ps} + \frac{i}{4}g(\lambda_l)_{ps}W_l^{f\dot{e}}\right\}\times \\ \times\left\{\partial_I^{e\dot{h}}\partial_I^{a\dot{b}}\delta_{sq}\delta_{qr} + \frac{i}{4}g\left[\begin{array}{l}\delta_{sq}(\lambda_l)_{qr}W_l^{e\dot{h}}\partial_I^{a\dot{b}} + (\lambda_l)_{sq}W_l^{a\dot{b}}\delta_{qr}\partial_I^{e\dot{h}} + \frac{1}{2}(\lambda_l)_{sq}(\lambda_{l'})_{qr}T_{l\,l'}^{e\dot{h}a\dot{b}} \\ + \frac{i}{4}g\left\{(\lambda_l)_{sq}W_l^{e\dot{h}}\right\}\left\{(\lambda_{l'})_{qr}W_{l'}^{a\dot{b}}\right\}\end{array}\right]\right\}\chi_{rt\{\dot{b}\dot{h}\}f} + \\ + i\left(M_b^3 + \Phi_b\right)(-2)\psi^a_{pt} - c.c. \end{array}\right] \tag{22}$$

$$S_\psi = \frac{1}{2i}\int dx_I^4 dx_{II}^4 (-)\psi^{*\dot{b}}_{tp\ 0} \times \left[\begin{array}{l} \left\{\partial_{II\ \dot{e}h}\delta_{ps} + \frac{i}{4}g(\lambda_l)_{ps}W_{l\ \dot{e}h}\right\}\times \\ \times\left\{\partial_{I\ \dot{h}k}\partial_{I\ \dot{b}c}\delta_{sq}\delta_{qr} + \frac{i}{4}g\left[\begin{array}{l}\delta_{sq}(\lambda_l)_{qr}W_{l\ \dot{h}k}\partial_{I\ \dot{b}c} + (\lambda_l)_{sq}W_{l\ \dot{b}c}\delta_{qr}\partial_{I\ \dot{h}k} - \frac{1}{2}(\lambda_l)_{sq}(\lambda_{l'})_{qr}T_{l\,l'\ \dot{h}k\dot{b}c} \\ + \frac{i}{4}g\left\{(\lambda_l)_{sq}W_{l\ \dot{h}k}\right\}\left\{(\lambda_{l'})_{qr}W_{l'\ \dot{b}c}\right\}\end{array}\right]\right\}\psi_{rt}^{\{ck\}\dot{e}} + \\ + i\left(M_b^3 + \Phi_b\right)2\chi_{pt\,0\dot{b}} - c.c. \end{array}\right] \tag{23}$$

Equation **Eq.22** is invariant with respect to the SU(3) gauge transformations Eq.7.1.7 of [3], with the obvious replacement of the two meson indices by the three baryon indices associated with χ in **Eq.22**, together with a generalization of the second of **Eq.21**,

$$(\lambda_l)_{sq}(\lambda_{l'})_{qr}T_{l\,l'}^{e\dot{h}a\dot{b}}(X)\chi_{rt\{\dot{b}\dot{h}\}f} \rightarrow \left((\lambda_l)_{sq}(\lambda_{l'})_{qr}T_{l\,l'}^{e\dot{h}a\dot{b}}(X) + \frac{2i}{g}\left(\partial_X^{e\dot{h}}\partial_X^{a\dot{b}}U_{3sq}(X)\right)U_{3qr}^{-1}(X)\right)\chi_{rt\{\dot{b}\dot{h}\}f} \tag{24}$$

Analogously, the same invariance also holds for Eq.2.3 with Eq.2.4 replaced by

$$(\lambda_l)_{sq}(\lambda_{l'})_{qr}T_{l\,l'\ \dot{h}k\dot{b}c}(X)\psi_{rt}^{\{ck\}\dot{e}} \rightarrow \left((\lambda_l)_{sq}(\lambda_{l'})_{qr}T_{l\,l'\ \dot{h}k\dot{b}c}(X) - \frac{2i}{g}\left(\partial_{X\dot{h}k}\partial_{X\dot{b}c}U_{3sq}(X)\right)U_{3qr}^{-1}(X)\right)\psi_{rt}^{\{ck\}\dot{e}} \tag{25}$$

For application to neutron decay, only the SU(2) part of Eqs.7.1.4-5 of [3] is needed and l and l' run from 1 to 3. Apart from the flavor indices l and l', the tensor $T_{l\,l'}^{e\dot{h}a\dot{b}}$ has 16 components, 10 symmetrical and 6 asymmetrical, which in its turn is grouped into a vector E(electric field in electromagnetism) and an axial vector H(magnetic field), which is assigned to the axial vector A mentioned above **Eq.19**. Identification of the tensor components corresponding to H has been given in Subsection 4-5 of [13] which are found by means of the invariant aymmetrical operator ε_{kl} of Eq.B15 of [3]. These are

$$\begin{aligned} &T_{l\,l'}^{\dot{h}\dot{b}} = T_{l\,l'}^{e\dot{h}a\dot{b}}\varepsilon_{ea} = \frac{1}{2}\left(T_{l\,l'}^{\dot{h}\dot{b}21} - T_{l\,l'}^{\dot{h}\dot{b}12}\right) \\ &T_{l\,l'}^{\{\dot{1}\dot{2}\}} = T_{l\,l'}^{\{\dot{2}\dot{1}\}} = 2iH_3,\ T_{l\,l'}^{\dot{1}\dot{1}} = -2i\left(H_1 + iH_2\right), \\ &T_{l\,l'}^{\dot{2}\dot{2}} = 2i\left(H_1 - iH_2\right) \end{aligned} \tag{26}$$

The remaining 13 components of $T_{ll'}^{\dot{e}ha\dot{b}}$ do not enter here and are put to zero.

3.3. First Order Relations

The gauge boson and tensor field in **Eqs.22-23** are decay products of the neutron whose wave functions now acquire a weak time dependence. Follow Eq.6.4.1 or Eq.7.3.1 of [3], noting **Eq.A10**, and let the nucleon wave functions in **Eqs.22-23** take the form

$$
\begin{aligned}
&\chi_{\{\dot{b}\dot{h}\}f}(X,x) \\
&= \left(a_{op} + a_{op}^{(1)}(X^0)\right)\chi_{\{\dot{b}\dot{h}\}f}(\underline{x})\exp\left(-iEX^0 + i\underline{K}\,\underline{X}\right) \\
&= \left(1 + \frac{a_{op}^{(1)}(X^0)}{a_{op}}\right)\chi_{0\{\dot{b}\dot{h}\}f}(X,x) \\
&= \chi_{0\{\dot{b}\dot{h}\}f}(\underline{x})\exp\left(-iEX^0 + i\underline{K}\,\underline{X}\right) + \chi_{1\{\dot{b}\dot{h}\}f}(X,x),\ \chi \to \psi
\end{aligned}
\tag{27}
$$

where E is the energy, $\underline{K}$ the momentum, $a_{op} = 1$ and $a_{op}^{(1)}(X^0)$ is a first order quantity varying slowly with time. Both can be elevated to operators in quantized case, as are described beneath Eq.6.4.2 of [3]. Ordering of these small quantities $a_{op}^{(1)}(X^0)$, g, W etc is the same as that in Eqs.18-19 of [4] or Eq.7.3.2 of [3]. The subscripts 0, 1 denote zeroth order and first order quantities, respectively.

Following the rudimentary quantization procedure of Eqs.23-25 of [4] or Eqs.6.4.12-15 of [3], let the initial and final states be denoted by

$$
\begin{aligned}
&\left|i\right\rangle = \left|n(\underline{K}_n = 0)\right\rangle, \\
&\left\langle f\right| = \left\langle p(\underline{K}_p), W(E_W, \underline{K}_W), T(E_W, \underline{K}_W)\right|
\end{aligned}
\tag{28}
$$

respectively. n, p, W, T denote the neutron, proton, gauge boson, and tensor gauge field, respectively. Further,

$$
\begin{aligned}
&\left\langle 0 \middle| i\right\rangle = \left\langle f \middle| 0\right\rangle = \left\langle f \middle| i\right\rangle = 0, \\
&\left\langle 0 \middle| 0\right\rangle = \left\langle i \middle| i\right\rangle = \left\langle f \middle| f\right\rangle = 1
\end{aligned}
\tag{29}
$$

Let a_{op} in **Eq.27** and its hermetian conjugate a_{op}^{+} be elevated to annihilation and creation operators. Insert **Eq.27** into **Eq.22** and sandwich the resulting expression between $<f|$ and $|i>$. There are two types of first order terms: 1) those containing $a_{op}^{(1)}(X^0 \to \infty)$ and 2) those linear in gW and gT.

Carrrying out integration over the time X^0, the type 1) terms read

$$
\begin{aligned}
&S_{fi}\int d^3\underline{X}\int d^4x\left\{-i\chi_{0a}^{*}(\underline{x})\left(\frac{E_n^2}{4} + \Delta\right)\chi_{0\dot{a}}(\underline{x})\right\}, \\
&S_{fi} = \left\langle f\left|a_{op}^{+}a_{op}^{(1)}(X^0 \to \infty)\right|i\right\rangle
\end{aligned}
\tag{30}
$$

in which the $c.c.$ term in **Eq.22** contributes equally. S_{fi} is the decay amplitude and **Eq.A6** has been used. For the evaluation of the type 2) terms, the final state $< f|$ in **Eq.28** will contain the gauge fields,

$$
\begin{aligned}
&\left(W_1^{a\dot{b}}(X) - iW_2^{a\dot{b}}(X)\right)/\sqrt{2} \\
&= W^{-a\dot{b}}(X) \\
&= W^{-0}\delta^{a\dot{b}} - \underline{\sigma}^{a\dot{b}}\,\underline{W}^{-} \\
&= w^{a\dot{b}}\exp\left(iE_W X^0 - i\underline{K}_W\,\underline{X}\right) \\
&= \left(w_0\delta^{a\dot{b}} - \underline{\sigma}^{a\dot{b}}\,\underline{w}\right)\exp\left(iE_W X^0 - i\underline{K}_W\,\underline{X}\right)
\end{aligned}
\tag{31}
$$

$$
\begin{aligned}
&\left(T_{13}^{\dot{e}ha\dot{b}} - T_{31}^{\dot{e}ha\dot{b}} + iT_{32}^{\dot{e}ha\dot{b}} - iT_{23}^{\dot{e}ha\dot{b}}\right)/\sqrt{2} \\
&= T^{\dot{e}ha\dot{b}}(X) = t^{\dot{e}ha\dot{b}}\exp\left(iE_W X^0 - i\underline{K}_W\,\underline{X}\right)
\end{aligned}
\tag{32}
$$

where Eq.12-13 of [4] or Eq.7.1.4-5 of [3] limited to its SU(2) part has been consulted. The initial and final nucleon states in **Eq.28** will have a laboratory space time dependence of the form given in **Eq.27** with subscripts p for proton and n for neutron attached to the variables there. Here, use has been made of Eq.9.3.7b and Eq.9.3.18a of [3] which gives $tp = 31$ for proton and $rt = 23$ for neutron in **Eq.22**. After summing over the flavor indices t, p, s, q, and r and carrying out the integration over X, the type 2) terms become

$$
\begin{aligned}
&\frac{g}{4\sqrt{2}}(2\pi)^4\delta\left(\underline{K}_p + \underline{K}_W\right)\delta\left(E_p + E_W - E_n\right) \\
&\times\int d^4x\left\{w_0\left[\chi_{p0a}^{*}(\underline{x})\left(-2\left(\frac{E_n^2}{4} + \Delta\right) - E_n^2\right)\chi_{n0\dot{a}}(\underline{x}) + c.c.\right]\right. \\
&\left.- i\left[\chi_{p0a}^{*}(\underline{x})\partial_{II}^{f\dot{e}}t^{\dot{h}bea}\chi_{n0\{\dot{b}\dot{h}\}f}(\underline{x}) + c.c.\right]\right\}
\end{aligned}
\tag{33}
$$

where **Eq.A6** has been used and E_W and K_W terms have been neglected because they are small relative to $|\partial_{I,II}\chi|/|\chi|$. With Eq.26, Eq.32 and Eq.B5 of [3], the 16 t's in **Eq.33** reduce similarly to three for an axial vetor:

$$
\begin{aligned}
&t^{\dot{h}\dot{b}} = t^{\dot{e}ha\dot{b}}\varepsilon_{ea} = \frac{1}{2}\left(t^{\dot{h}\dot{b}21} - t^{\dot{h}\dot{b}12}\right), \\
&t^{ea} = t^{\dot{e}ha\dot{b}}\varepsilon_{\dot{h}\dot{b}} = \frac{1}{2}\left(t^{ea\dot{2}\dot{1}} - t^{ea\dot{1}\dot{2}}\right) \\
&t^{\dot{1}\dot{1}} = t^{22} = t_{\dot{2}\dot{2}} = t_{11},\ t^{\dot{2}\dot{2}} = t^{11} = t_{\dot{1}\dot{1}} = t_{22}, \\
&t^{12} = t^{21} = -t^{\{\dot{1}\dot{2}\}} = t_{\{\dot{1}\dot{2}\}} = -t_{\{12\}} \\
&T^{\dot{h}\dot{b}}(X) = t^{\dot{h}\dot{b}}\exp\left(iE_W X^0 - i\underline{K}_W\,\underline{X}\right), \\
&T^{ea}(X) = t^{ea}\exp\left(iE_W X^0 - i\underline{K}_W\,\underline{X}\right)
\end{aligned}
\tag{34}
$$

3.4. Decay Amplitude

The decay amplitude $S_{fi\chi}$ for the χ function is found by putting **Eq.30** to the negative of **Eq.33**. Letting $\Omega = \int d^3\underline{X}$, the result is

$$S_{fi\chi} =-\frac{ig}{4\sqrt{2}}(2\pi)^4\delta\left(\underline{K}_p+\underline{K}_W\right)\delta\left(E_p+E_W-E_n\right)\times$$
$$\frac{\int d^3\underline{x}\left\{w_0\left[\chi^*_{p0a}(\underline{x})\left(-2\left(\frac{E_n^2}{4}+\Delta\right)-E_n^2\right)\chi_{n0\dot{a}}(\underline{x})+c.c.\right]-i\left[\chi^*_{p0a}(\underline{x})\partial_{II}^{f\dot{e}}t^{\dot{h}bea}\chi_{n0\{\dot{b}\dot{h}\}f}(\underline{x})+c.c.\right]\right\}}{\Omega\int d^3\underline{x}\left\{\chi^*_{0a}(\underline{x})\left(\frac{E_n^2}{4}+\Delta\right)\chi_{0\dot{a}}(\underline{x})\right\}} \tag{35}$$

In an analogous fashion, The decay amplitude $S_{fi\psi}$ for the ψ function is obtained from **Eq.23** and reads

$$S_{fi\psi} =-\frac{ig}{4\sqrt{2}}(2\pi)^4\delta\left(\underline{K}_p+\underline{K}_W\right)\delta\left(E_p+E_W-E_n\right)\times$$
$$\times\frac{\int d^3\underline{x}\left\{w_0\left[\psi^{*\dot{a}}_{p0}(\underline{x})\left(-2\left(\frac{E_n^2}{4}+\Delta\right)-E_n^2\right)\psi^{a}_{n0}(\underline{x})+c.c.\right]+i\left[\psi^{*\dot{a}}_{p0}(\underline{x})\partial_{II\dot{e}d}t_{\dot{d}\dot{a}hb}\psi^{\{bh\}\dot{e}}_{n0}(\underline{x})+c.c.\right]\right\}}{\Omega\int d^3\underline{x}\left\{\psi^{*\dot{a}}_{0}(\underline{x})\left(\frac{E_n^2}{4}+\Delta\right)\psi^{a}_{0}(\underline{x})\right\}} \tag{36}$$

The starred wave functions in nominators of **Eqs.35-36** represent final states or proton, irrespective the nucleon lables; the *c.c.* terms will turn out to contribute equally and can be dropped together with an overall factor 2 multiplying the right of **Eqs.35-36**. The wave functions in denoiminators of **Eqs.35-36** are those of the initial neutron, as $<f|$ has been included in S_{fi} of **Eq.30**.

The proton may have a different m or spin value in **Eqs.A16-A19** relative to that pertaining to the initial neutron in **Eqs.35-36**. There are four combinations which are denoted by

$$\begin{array}{lcccc} \text{neutron} & m=\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \text{proton} & m=\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ \text{notation} & F+ & GT+ & GT- & F \end{array} \tag{37}$$

As there are two equally correct solutions **Eqs.A16-A17** and **Eqs.A18-A19**, **Eq.30** and **Eq.33** are to be averaged over these two equally probable solutions. The averaging will turn out to not affect **Eq.30** so that it can be carried out for $S_{fi\chi}$ and $S_{fi\psi}$ of **Eqs.35-36** directly to obtain $S_{fi\chi av}$ and $S_{fi\psi av}$. It removes terms of containing $f_0(r)g_0(r)$ or their derivatives that will appear in **Eqs.35-36** after application of **Eqs.A16-A17** or **Eqs.A18-A19** and **Eq.37**. Such terms will for instance differentiate between neutron spin up and spin down decay rates contray to the measured A asymmetry coefficient [9]. Inserting **Eqs.A16-A17** and **Eqs.A18-A19** into **Eq.35** and **Eq.36** for the four combinations of **Eq.37**, making use of **Eq.A10**, summing over the spinor indices, employing **Eq.34**, and integrating over the angles ϑ and ϕ in the relative space, one finds that the both averaged decay amplitudes $S_{fi\chi av}$ and $S_{fi\psi av}$ are the same, as may be expected from the symmetry between the χ part **Eq.22** and the ψ part **Eq.23**;

$$S_{fi\chi av}=S_{fi\psi av}=\begin{pmatrix} S_{fiF+} \\ S_{fiGT+} \\ S_{fiGT-} \\ S_{fiF-}\end{pmatrix}=-i\frac{g}{2\sqrt{2}}\frac{(2\pi)^4}{\Omega}\delta\left(E_p+E_W-E_n\right)\delta\left(\underline{K}_p+\underline{K}_W\right)\frac{1}{N_{dr}}$$
$$\left[-2w_0\left(N_{dr}+E_n^2\left(I_{g0}+I_{f0}\right)\right)\begin{pmatrix}1\\0\\0\\1\end{pmatrix}-i\frac{2}{3}E_n\left(I_{g0}-\frac{I_{f0}}{3}\right)\begin{pmatrix}t^{\{12\}}\\-t^{22}\\t^{11}\\-t^{\{12\}}\end{pmatrix}\right] \tag{38}$$

$$I_{g0} = \int_0^\infty drr^2 g_0^2(r), \quad I_{f0} = \int_0^\infty drr^2 f_0^2(r), \quad I_{g00} = \int_0^\infty drr^2 g_{00}^2(r), \quad I_{f00} = \int_0^\infty drr^2 f_{00}^2(r) \tag{39}$$

where N_{dr} is given by **Eq.A22** and I_{g00} and I_{f00}. are defined **Eq.A23**.

3.5. Expressions for Vector and Tensor Gauge Fields

3.5.1. Mass Generation of W Boson Via Virtual π^0

In the analogous meson case, the pion beta decay $\pi^+ \to \pi^0 e^+ \nu_e$ of [4] or Subsection 7.4.5 of [3], the gauge boson W decay into a pair of leptons. The mass of the W boson comes from the $(gW)^2$ terms in the meson action integral S_m in Eq.11 of [4] via Eq.32 and Eq.35 of [4] or S_{m3} in Eq.7.1.8 of [3] via Eq.7.4.4 and Eq.7.4.6b of [3] and is generated by the integral of the |pion wave functions|^2 over the relative space x. It can be seen from Eq.7.4.3a and Eq.7.4.4 of [3] that the energy of pion does not enter and can be zero. In this case, the pion is virtual.

In the corresponding action-like integrals $S\chi$ and $S\psi$ of **Eqs.22-23**, there is no such $(gW)^2$ mass term, only terms of the form $(gW)(g\partial_{I,II}W)$. Therefore, the gauge boson W from neutron decay cannot decay into a pair of leptons via the integrals **Eqs.22-23** for neutrons.

The interpretation of this situation is that gauge boson$W$ in the neutron case here decays into a pair of leptons via a virtual π^0 action as has been considered in Subsection 7.6.2 3 of [3] and employed earlier for muon decay in Subsection 7.6.3 4 of [3]. In the pion beta decay, the W is positively charged and decays into a lepton pair via Eqs.44-45 of [4] or Eq.7.4.10 of [3]. In neutron decay, the W is negatively charged and W_I^- in Eq.7.4.5 of [3] is replaced by W_I^+ so that Eq.7.4.5 and Eq.7.4.6a [3], neglecting the first three terms there and noting Eq.7.4.10 of [3], are modified to

$$\begin{aligned} \frac{M_W^2}{2} W_I^{-b\dot{a}} &= \frac{M_W^2}{2} \begin{pmatrix} W_0 - W_3 & -W_1 + iW_2 \\ -W_1 - iW_2 & W_0 + W_3 \end{pmatrix} \\ &= \delta S_L / \delta W^{+a\dot{b}} = \frac{g}{2\sqrt{2}} \psi_{L\dot{b}}^{(+)} \psi_{\nu La}^{(-)} \\ &= \frac{g}{2\sqrt{2}} \begin{pmatrix} \psi_{L\dot{1}}^{(+)} \psi_{\nu L1}^{(-)} & \psi_{L\dot{1}}^{(+)} \psi_{\nu L2}^{(-)} \\ \psi_{L\dot{2}}^{(+)} \psi_{\nu L1}^{(-)} & \psi_{L\dot{2}}^{(+)} \psi_{\nu L2}^{(-)} \end{pmatrix}, \\ M_W^2 &= g^2 \frac{\int dx^0 \to \infty}{\Omega \to \infty} = (80.42 \text{ Gev})^2 \end{aligned} \tag{40}$$

where the gauge boson mass M_W is given by Eq.35 and Eq.41 of [4] or Eq.7.4.6b and Eq.7.4.9 of [3], x^0 is the relative time between the quarks in π^0 and Ω a large normalization volume of the π^0.

3.5.2. Variation of the Total Action for Neutron Decay

Having found an expression for w_0 in **Eq.38** from **Eq.40** via **Eq.31**, expressions for the other unknowns t^{ab} there will be obtained in this and the next paragraph. The vector gauge boson fields **Eq.40** was found by varying the total action Eq.7.1.1 of [3], after removing its last term, or Eq.5 of [4] with respect to W^- in Subsection 7.4.2 of [3] or Eqs.30-32 of [4]. For neutron decay, the meson action S_{m3} in Eq.7.1.1 of [3] is to be replaced by the corresponding baryons actions **Eqs.22-23**, $S_\chi \pm S_\psi$, and the vector gauge boson action S_{GB} be replaced by a gauge boson-tensor field action S_{GBT}. Equation Eq.5 of [4] or Eq.7.1.1 of [3] is here replaced by

$$S_{Tn} = S_{GBT} + \kappa_b \left(S_{\chi av} - S_{\psi av}\right) + S_L + S_{Lm} \tag{41}$$

Here, the lepton actions $S_L + S_{Lm}$ are the same as those in Eqs.8-10 of [4] or Eqs.7.1.16-17 of [3]. κ_b is a dimensionless proportional constant that, for instance, signifies that the action-like baryon integrals S_χ and S_ψ differ basically from the meson action S_m in Eq.11 of [4] or S_{m3} in Eq.7.1.8 of [3] in that the normalization of the wave function amplitudes are different. $S_{\chi av}(S_{\psi av})$ denotes that $S_\chi(S_\psi)$ has been averaged over the both equally probable solutions **Eqs.A16-A17** and **Eqs.A18-A19** for the baryon wave functions that enter it, just like that $S_{fi\chi}$ and $S_{fi\psi}$ of **Eqs.35-36** have been averaged to $S_{fi\chi av}$ and $S_{fi\psi av}$ in **Eq.38**.

The – sign in **Eq.41** is chosen because **Eq.41** will lead to an expression for t^{ab} in **Eq.38**; if this sign is replaced by +, the needed **Eq.42** below will vanish. Further, this – sign will remove terms linear in gW, as is implied by $S_{fi\chi av} - S_{fi\psi av} = 0$ from **Eq.38**. This will cause $S_{\chi av} - S_{\psi av}$ in **Eq.41** to possess $(gW)^2$ terms, apart from the t^{ab} terms, to that order and render it to have an extremum when solutions near the correct ones are inserted into $S_{\chi av} - S_{\psi av}$.

Unlike S_{GB} in Eqs.6-7 of [4] or Eq.7.1.2 of [3], S_{GBT} in **Eq.41** is unknown. If the tensor gauge field is to have an equation of motion like that for the gauge boson Eqs.34-35 of [4] or Eq.7.4.6 of [3], S_{GBT} is expected to contain terms of the form $(\partial_X W)(\partial_X T)$. Physical existence and interpretation of the tensor gauge field are not known and will be left to eventual future work. For the present purpose, it is sufficient to obtain an expression for t^{ab} from **Eq.41** for use in **Eq.38**.

Follow the steps of Eqs.30-33 of [4] or Subsection 7.4.2 of [3] and vary **Eq.41** with respect to $W^{+a\dot{b}}$. The unknown S_{GBT} is expected to give rise to energy-momentum terms of the form $(\partial_X^2 T)$ corresponding to the first terms on the left of Eq.34 of [4] or Eq.7.4.6a of [3], which have been neglected because they are much smaller than the gauge boson mass term that follows them. Similarly, the obtained $(\partial_X^2 T)$ terms can also be dropped on the same ground so the exact but unknown form of S_{GBT} is of no concern here. Variation of the $S_L + S_{Lm}$ terms in **Eq.41** with respect to $W^{+a\dot{b}}$ is analogous to that given by Eq. 33 of [4] or Eq.7.4.5 of [3] and leads to **Eq.40**.

3.5.3. Expressions for Tensor and Vector Gauge Fields

Variation of $S_{\chi av}$–$S_{\psi av}$ in **Eq.41** with respect to $W^{+a\dot{b}}$ is limited to terms of order g^2. When evaluating the average $S_{\chi av}(S_{\psi av})$ using **Eqs.A16-A17** and **Eqs.A18- A19**, it is practically sufficient to use one of them, for instance **Eqs.A16-A17** for $S_\chi(S_\psi)$ of **Eqs.22-23**, and drop the $f_0(r)g_0(r)$ terms, as was mentioned beneath **Eq.37**.

In the evaluation of **Eqs.22-23**, use is made of **Eq.27**, **Eq.A6** and **Eq.A10**. When summing over the flavor indices t, p, s, q, and r, tp = 32 and rt = 23 for neutron and **Eqs.31-32** are employed. Carrying out the angular integrations, one finds

$$\frac{\delta\kappa_b\left(S_{\chi av} - S_{\psi av}\right)}{\delta W^{+a\dot{b}}} = U_{\dot{b}a} = \begin{pmatrix} U_0 + U_3 & U_1 - iU_2 \\ U_1 + iU_2 & U_0 - U_3 \end{pmatrix} \quad (42)$$

$$U_0 = \frac{\kappa_b g^2}{96} E_n \tau_0 \left(I_{g0} - \frac{1}{3} I_{f0} \right) 7W_3^-(X) \begin{pmatrix} 1 \\ -1 \end{pmatrix} = -\frac{g}{4\sqrt{2}} \left(\psi_{L1}^{(+)}\psi_{\nu L1}^{(-)} + \psi_{L2}^{(+)}\psi_{\nu L2}^{(-)} \right) \quad (43)$$

$$U_3 = -\frac{\kappa_b g^2}{96} E_n \tau_0 \left(I_{g0} - \frac{1}{3} I_{f0} \right) 3W_0^- \begin{pmatrix} 1 \\ -1 \end{pmatrix} - i\frac{\kappa_b g^2}{24} \tau_0 \left(I_{g0} + I_{f0} \right) T^{12}(X) \begin{pmatrix} 1 \\ 1 \end{pmatrix} = -\frac{g}{4\sqrt{2}} \left(\psi_{L1}^{(+)}\psi_{\nu L1}^{(-)} - \psi_{L2}^{(+)}\psi_{\nu L2}^{(-)} \right) \quad (44)$$

$$U_1 - iU_2 = -i\frac{\kappa_b g^2}{24} \tau_0 \left(-T^{11} \right)(X) \begin{pmatrix} 2I_{f0}/3 \\ I_{g0} + I_{f0}/3 \end{pmatrix} = -\frac{g}{4\sqrt{2}} \psi_{L1}^{(+)}\psi_{\nu L2}^{(-)} \quad (45)$$

$$U_1 + iU_2 = -i\frac{\kappa_b g^2}{24} \tau_0 T^{22}(X) \begin{pmatrix} I_{g0} + I_{f0}/3 \\ 2I_{f0}/3 \end{pmatrix} = -\frac{g}{4\sqrt{2}} \psi_{L2}^{(+)}\psi_{\nu L1}^{(-)}, \qquad \tau_0 = \int dx^0 \quad (46)$$

where the upper and lower rows in the U′s refer to neutron spin up m = 1/2 and spin down m = −1/2, respectively, in **Eqs.A16-A19**. Further, **Eq.42** has been equated to the negative of **Eq.40** as is prescribed in **Eq.41**. Note that U_0 in **Eq.43** does not contain W_0, which however enters **Eq.40** and stems from the virtual π^0 action in Eq.7.4.4 of [3].

Comparing the X dependence of W's and T's and the ψ's in **Eqs.43-46** via **Eq.31**, **Eq.34** and Eq.7.4.19 of [3], replacing L and νL there e and ν, we find

$$E_W = E_\nu + E_e^{(+)}, \quad \underline{K}_W = \underline{K}_e^{(+)} - \underline{K}_\nu \quad (47)$$

Removing the X dependence in **Eqs.43-46** and observing **Eq.31** and **Eq.34**, we obtain

$$0 = \frac{1}{M_W^2\Omega} \frac{g}{4\sqrt{2\Omega_\nu\Omega_e}} \left(u_{L1}^{(+)}u_{\nu L1}^{(-)} + u_{L2}^{(+)}u_{\nu L2}^{(-)} \right) + \frac{7}{96} E_n \left(I_{g0} - I_{f0}/3 \right)\kappa_b w_3 \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (48)$$

$$t^{\{12\}} = -i\frac{24}{M_W^2\Omega\left(I_{g0} + I_{f0}\right)} \frac{g}{4\sqrt{2\Omega_\nu\Omega_e}} \left(u_{L1}^{(+)}u_{\nu L1}^{(-)} - u_{L2}^{(+)}u_{\nu L2}^{(-)} \right) + i\frac{3}{4} E_n \frac{\left(I_{g0} - I_{f0}/3\right)}{\left(I_{g0} + I_{f0}\right)} \kappa_b w_0 \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (49)$$

$$-t^{11}\kappa_b \begin{pmatrix} 2I_{f0}/3 \\ I_{g0} + I_{f0}/3 \end{pmatrix} = -i\frac{24}{M_W^2\Omega} \frac{g}{4\sqrt{2\Omega_\nu\Omega_e}} u_{L1}^{(+)}u_{\nu L2}^{(-)},$$
$$t^{22}\kappa_b \begin{pmatrix} I_{g0} + I_{f0}/3 \\ 2I_{f0}/3 \end{pmatrix} = -i\frac{24}{M_W^2\Omega} \frac{g}{4\sqrt{2\Omega_\nu\Omega_e}} u_{L2}^{(+)}u_{\nu L1}^{(-)} \quad (50)$$

The w′s in **Eq.31** are similarly found from **Eq.40** and read

$$w_0 = \frac{1}{M_W^2} \frac{g}{2\sqrt{2\Omega_\nu\Omega_e}} \left(u_{L1}^{(+)}u_{\nu L1}^{(-)} + u_{L2}^{(+)}u_{\nu L2}^{(-)} \right),$$
$$w_3 = \frac{1}{M_W^2} \frac{g}{2\sqrt{2\Omega_\nu\Omega_e}} \left(u_{L1}^{(+)}u_{\nu L1}^{(-)} - u_{L2}^{(+)}u_{\nu L2}^{(-)} \right) \quad (51)$$

3.5.4. Decay Amplitude as Function of Lepton Wave Functions

Eqs.48-51 can now be inserted into **Eq.38**. Here, it is noted that the neutron spin is up or m = 1/2 for the upper two amplitudes corresponding to the upper case in **Eqs.48-50** and down or m = −1/2 for the lower two amplitudes corresponding to the lower case in **Eqs. 48-50**. Thus, the w_0 in the third component of a triplet **Eq.44** and **Eq.49** become a singlet to be combined to the w_0 in **Eq.51**. Analogously, the singlet **Eq.48**, when inserted into Eq.38 becomes the third component of a triplet to be combined with **Eq.49**. Effectively, the 7 W_3 in **Eq.43** and 3 W_0 in **Eq.44** change place after insertion into **Eq.38**. The two terms are not the dominating ones but will lead to the two c's (< 0.4 or so) in **Eq.56** below. The resulting decay amplitude, noting **Eq.47** and **Eq.39**, reads

$$S_{fi} = \begin{pmatrix} S_{fiF+} \\ S_{fiGT+} \\ S_{fiGT-} \\ S_{fiF-} \end{pmatrix} = i\frac{g^2}{8}\frac{1}{M_W^2}\frac{(2\pi)^4}{\Omega\sqrt{\Omega_\nu\Omega_e}}$$

$$\delta\left(E_p + E_\nu + E_e^{(+)} - E_n\right)\delta\left(\underline{K}_p + \underline{K}_e^{(+)} - \underline{K}_\nu\right)\frac{1}{N_{dr}}$$

$$\times\begin{bmatrix} b_{F+0} & 0 & 0 & b_{F+3} \\ 0 & b_{GT+p} & 0 & 0 \\ 0 & 0 & b_{GT-m} & 0 \\ b_{F-0} & 0 & 0 & b_{F-3} \end{bmatrix}\begin{pmatrix} \left(u_{L1}^{(+)}u_{\nu L1}^{(-)} + u_{L2}^{(+)}u_{\nu L2}^{(-)}\right) \\ u_{L2}^{(+)}u_{\nu L1}^{(-)} \\ u_{L1}^{(+)}u_{\nu L2}^{(-)} \\ \left(u_{L1}^{(+)}u_{\nu L1}^{(-)} - u_{L2}^{(+)}u_{\nu L2}^{(-)}\right) \end{pmatrix} \tag{52}$$

$$b_{F+0} = b_{F-0} = -2\left(N_{dr} + E_n^2\left(I_{g0} + I_{f0}\right)\right)\frac{1}{1 + c_{F0}} \tag{53}$$

$$b_{F+3} = -b_{F-3} = -\frac{8E_n}{\kappa_b}\frac{\left(I_{g0} - I_{f0}/3\right)}{a_g^2\left(I_{g00} + I_{f00}\right)}\left(1 + c_{F3}\right) \tag{54}$$

$$b_{GT+p} = b_{GT-m} = b_{GT} = \frac{16E_n}{\kappa_b}\frac{\left(I_{g0} - I_{f0}/3\right)}{a_g^2\left(I_{g0} + I_{f0}/3\right)} \tag{55}$$

$$c_{F0} = \frac{1}{16}E_n\kappa_b a_g^2\left(I_{g00} - I_{f00}/3\right), \quad c_{F3} = \frac{7}{48}E_n a_g^2\left(I_{g00} - I_{f00}/3\right) \tag{56}$$

Because κ_b in **Eq.41** can be incorporated into the wave functions χ and ψ, it can be absorbed into the normalization condition Eq.10.3.13 of [3] by chosing a different normalization constant or equivalently a different normalized amplitude a_g given by **Eq.A23**.

3.6. Decay Rate and Asymmetry Coefficients *A* and *B*

3.6.1. Decay Rate

As in Eq.7.5.1 of [3], the decay rate is

$$\Gamma = \sum\nolimits_{\text{final states}}\left|S_{fi}\right|^2/T_d \tag{57}$$

where T_d is a long decay time. The subscript "final states" refers to four final lepton spin states and all possible momenta of the proton, electron and antineutrino, like that in Eq.7.5.2 of [3]. The decay rates are

$$\begin{pmatrix} \Gamma_{F\pm} \\ \Gamma_{GT\pm} \end{pmatrix} = \frac{\Omega\Omega_e\Omega_\nu}{(2\pi)^9}\int d^3\underline{K}_p\int d^3\underline{K}_e\int d^3\underline{K}_\nu\frac{1}{T_d}\sum\nolimits_{\text{e spins}}\sum\nolimits_{\nu\text{ spins}}\begin{pmatrix} \left|S_{fiF\pm}\right|^2 \\ \left|S_{fiGT\pm}\right|^2 \end{pmatrix} \tag{58}$$

The square of S_{fi} contains squares of the δ functions in **Eq.52**, which are "linearized" by Eq.7.5.6 of [3] type of formula. Following the common approach, integration over the recoil momenta $\underline{K}_p$ in **Eq.58** is carried out first. By **Eq.47**, this gives $\underline{K}_p = \underline{K}_\nu - \underline{K}_e$, where the superscript (+) has been dropped. Introduce

$$\underline{K}_\nu = K_\nu\left(k_{\nu1}, k_{\nu2}, k_{\nu3}\right),$$

$$k_{\nu\pm} = k_{\nu1} \pm ik_{\nu2} = \sin\vartheta_n \exp\left(\pm i\phi_\nu\right), \quad k_{\nu3} = \cos\vartheta_\nu$$
$$K_{e\pm} = K_{e1} \pm iK_{e2} = K_e\sin\vartheta_e\exp\left(\pm i\phi_e\right), \tag{59}$$
$$K_{e3} = K_e\cos\vartheta_e$$

These and **Eq.52** are inserted into the decay rate expression **Eq.58** to produce

$$\begin{pmatrix} \Gamma_{F\pm} \\ \Gamma_{GT\pm} \end{pmatrix} = \frac{g^4}{8192\pi^5M_W^4N_{dr}^2}\int dK_eK_e^2\int dK_\nu K_\nu^2\delta\left(E_p + E_e + E_\nu - E_n\right)\left(1 + \frac{m_e}{E_e}\right)\begin{pmatrix} I_{F\pm} \\ I_{GT\pm} \end{pmatrix} \tag{60}$$

$$\begin{pmatrix} I_{F\pm} \\ I_{GT\pm} \end{pmatrix} = \int d\vartheta_e\sin\vartheta_e d\phi_e\int d\vartheta_\nu\sin\vartheta_\nu d\phi_\nu\sum\nolimits_{\text{e spins}}\sum\nolimits_{\nu\text{ spins}}\begin{pmatrix} i_{F\pm} \\ i_{GT\pm} \end{pmatrix} \tag{61}$$

$$\sum\nolimits_{\text{e spins}}\sum\nolimits_{\nu\text{ spins}}\left(i_{F\pm}\right) = i_{F\pm\uparrow\uparrow} + i_{F\pm\uparrow\downarrow} + i_{F\pm\downarrow\uparrow} + i_{F\pm\downarrow\downarrow} \tag{62}$$

$$\sum\nolimits_{\text{e spins}}\sum\nolimits_{\nu\text{ spins}}\left(i_{GT\pm}\right) = i_{GT\pm\uparrow\uparrow} + i_{GT\pm\uparrow\downarrow} + i_{GT\pm\downarrow\uparrow} + i_{GT\pm\downarrow\downarrow} \tag{63}$$

where the both arrows denote the spin directions, separated by the commas in Eq.7.4.19 of [3], of the electron and the antineutrino and

$$i_{F\pm\uparrow\uparrow} = \left|\left(-b_{F\pm0} + b_{F\pm3}\right)\left(1 - \frac{K_{e3}}{E_e + m_e}\right)k_{\nu-} + \left(b_{F\pm0} + b_{F\pm3}\right)\frac{K_{e-}}{E_e + m_e}\left(1 - k_{\nu3}\right)\right|^2$$

$$\begin{aligned} i_{F\pm\uparrow\downarrow} &= \left| (b_{F\pm0} - b_{F\pm3})\left(1 - \frac{K_{e3}}{E_e + m_e}\right)(1 + k_{\nu3}) + (-b_{F\pm0} - b_{F\pm3})\frac{K_{e-}}{E_e + m_e} k_{\nu+} \right|^2 \\ i_{F\pm\downarrow\uparrow} &= \left| (-b_{F\pm0} - b_{F\pm3})\left(1 + \frac{K_{e3}}{E_e + m_e}\right)(1 - k_{\nu3}) + (b_{F\pm0} - b_{F\pm3})\frac{K_{e+}}{E_e + m_e} k_{\nu-} \right|^2 \\ i_{F\pm\downarrow\downarrow} &= \left| (b_{F\pm0} + b_{F\pm3})\left(1 + \frac{K_{e3}}{E_e + m_e}\right)k_{\nu+} + (-b_{F\pm0} + b_{F\pm3})\frac{K_{e+}}{E_e + m_e}(1 + k_{\nu3}) \right|^2 \end{aligned} \tag{64}$$

$$\begin{aligned} i_{GT+\uparrow\uparrow} &= \left| (b_{GT+p})\left(1 - \frac{K_{e3}}{E_e + m_e}\right)(1 - k_{\nu3}) \right|^2, \quad i_{GT-\uparrow\uparrow} = \left| (-b_{GT-m})\frac{K_{e-}}{E_e + m_e} k_{\nu-} \right|^2 \\ i_{GT+\uparrow\downarrow} &= \left| (-b_{GT+p})\left(1 - \frac{K_{e3}}{E_e + m_e}\right)k_{\nu+} \right|^2, \quad i_{GT-\uparrow\downarrow} = \left| (b_{GT-m})\frac{K_{e-}}{E_e + m_e}(1 + k_{\nu3}) \right|^2 \\ i_{GT+\downarrow\uparrow} &= \left| (-b_{GT+p})\frac{K_{e+}}{E_e + m_e}(1 - k_{\nu3}) \right|^2, \quad i_{GT-\downarrow\uparrow} = \left| (b_{GT-m})\left(1 + \frac{K_{e3}}{E_e + m_e}\right)k_{\nu-} \right|^2 \\ i_{GT+\downarrow\downarrow} &= \left| (b_{GT+p})\frac{K_{e+}}{E_e + m_e} k_{\nu+} \right|^2, \quad i_{GT-\downarrow\downarrow} = \left| (-b_{GT-m})\left(1 + \frac{K_{e3}}{E_e + m_e}\right)(1 + k_{\nu3}) \right|^2 \end{aligned} \tag{65}$$

Let $\Delta_m = E_n - E_p \approx m_n - m_p = 1.2933 \text{ Mev}$ (66)

Eq.61 can now be evaluated using **Eqs.64-65** and **Eq.59**. Carrying out the angular integrations, it is found that the cross terms in **Eq.64** drop out and one finds

$$\begin{pmatrix} I_{F\pm} \\ I_{GT\pm} \end{pmatrix} = 4\pi^2 \begin{pmatrix} 2b_{F\pm0}^2 + 2b_{F\pm3}^2 \\ b_{GT}^2 \end{pmatrix} \frac{16E_e}{E_e + m_e} \tag{67}$$

which is independent of the antineutrino energy $E_\nu = |K_\nu|$. Carrying out the K_ν integration in **Eq.60** using **Eq.66**, one gets

$$\int dK_\nu K_\nu^2 \delta(E_p + E_e + E_\nu - E_n)\begin{pmatrix} I_{F\pm} \\ I_{GT\pm} \end{pmatrix} = (\Delta_m - E_e)^2 \begin{pmatrix} I_{F\pm} \\ I_{GT\pm} \end{pmatrix} \tag{68}$$

where $0 \le K_\nu \le \Delta - E_e$. Inserting **Eqs.67-68** into **Eq.60**, changing the variable dK_eK_e to dE_eE_e and noting **Eqs. 53-55** leads to

$$\begin{pmatrix} \Gamma_{F\pm} \\ \Gamma_{GT\pm} \end{pmatrix} = \begin{pmatrix} \Gamma_F \\ \Gamma_{GT} \end{pmatrix} = \frac{1}{128\pi^3}\frac{g^4}{M_W^4}\frac{1}{N_{dr}^2}\int dE_e P_F(E_e)\begin{pmatrix} 2b_{F\pm0}^2 + 2b_{F\pm3}^2 \\ b_{GT}^2 \end{pmatrix} \tag{69}$$

$$P_F(E_e) = \sqrt{E_e^2 - m_e^2}\,E_e(\Delta_m - E_e)^2 \tag{70}$$

where $m_e \le E_e \le \Delta + m_e$ and $P_F(E_e)$ is the conventional Fermi electron energy spectrum.

The half life of the neutron to be compared to the known $\tau_{exp} = 885.7$ sec. is

$$\tau_{th}(\log 2) = \frac{h}{2\pi}\frac{1}{(\Gamma_F + \Gamma_{GT})}(\log 2) \tag{71}$$

where h is the Planck constant.

3.6.2. Asymmetry Coefficients *A* and *B*

The asymmetry coefficients A and B are obtained from **Eq.61**, just like **Eq.67**, but without carrying out integration over θ_ν and θ_e, respectively. One finds,

$$\begin{aligned} &\begin{pmatrix} I_{F+} + I_{GT+} \\ I_{F-} + I_{GT-} \end{pmatrix} = \int d\vartheta_e \sin\vartheta_e 4\pi^2 \begin{pmatrix} b_{FGT+} \\ b_{FGT-} \end{pmatrix} \frac{8E_e}{E_e + m_e}\left(1 + A_\pm \frac{K_e}{E_e}\cos\vartheta_e\right) \\ &b_{FGT} = b_{FGT+} = 2b_{F+0}^2 + 2b_{F+3}^2 + b_{GT+p}^2 = b_{FGT-} = 2b_{F-0}^2 + 2b_{F-3}^2 + b_{GT-m}^2 \\ &A_+ = \left(4b_{F+0}b_{F+3} - b_{GT+p}^2\right)/b_{FGT+} = -A_- = -\left(4b_{F-0}b_{F-3} + b_{GT-m}^2\right)/b_{FGT-} \end{aligned} \tag{72}$$

The antineutrino moves in the opposite direction relative to that of the neutrino so that $\underline{K}_\nu$ is to be replaced by $-\underline{K}_\nu$ in **Eq.47**, hence also **Eq.52**. With this replacement, both $\underline{K}_e$ and $\underline{K}_\nu$ are now on equal footing in these both expressions and **Eq.61** can be written in the form

$$\begin{aligned} &\begin{pmatrix} I_{F+} + I_{GT+} \\ I_{F-} + I_{GT-} \end{pmatrix} = \int d\vartheta_\nu \sin\vartheta_\nu 4\pi^2 \begin{pmatrix} b_{FGT+} \\ b_{FGT-} \end{pmatrix} \frac{8E_e}{E_e + m_e}(1 + B_\pm \cos\vartheta_\nu) \\ &B_+ = \left(4b_{F+0}b_{F+3} + b_{GT}^2\right)/b_{FGT} = -B_- = -\left(4b_{F-0}b_{F-3} - b_{GT}^2\right)/b_{FGT} \end{aligned} \tag{73}$$

3.6.3. Comparison with Data

The expressions in **Eqs.71-73** have been evaluated using **Eqs.53-56**, **Eq.39** and the normalized radial wave functions $g_{00}(r)$ and $f_{00}(r)$ for the neutron associated with some of the confinement cases in **Table 1**. The results are summarized in **Table 2** below.

These results have been derived starting from **Eqs.A7-A8**, as was mentioned in Subsection 2.2.1, and using **Eq.A6** which stems from putting the quartet wave functions Eq.9.2.8 of [3] entering **Eqs.A7-A8** to zero.

If the symmetric quark postulate [H15] mentioned beneath Eq.9.3.7 of [3] is used, **Eq.A9** can be inserted into **Eqs.A7-A8** and **Eq.A6** is no longer needed. The expressions **Eqs.71-73** remain valid if c_{F0} and c_{F3} in **Eq.56** are replaced by c'_{F0} and c'_{F3},

$$c'_{F0} = c_{F0} / 2, \; c'_{F3} = 9c_{F3} / 14 \tag{74}$$

The corresponding results are similarly summarized inside parentheses in **Table 2**.

Table 2. Values of the calculated decay rate $\tau_{th}(log2)$, A and B asymmetry coefficients given by **Eqs.71-73** and Γ_{GT}/Γ_F by **Eq.69** are presented for a number of confinement strengths d_{b2} given in **Table 1**. The normalization constant κ_b in Eqs.53-56 are chosen such that in one case the A coefficient agrees with data [9] and in the other case the B coefficient agrees with data. The integrals appearing in **Eqs.53-56**, given by **Eq.39**, depend upon the approximate, normalized radial wave functions for the neutron $g_{00}(r)$ and $f_{00}(r)$ obtained in Subsection 2.2 and shown in **Figure 2** for the $d_{b2} = -0.3202$ case. The corresponding results stemming from symmetric quark postulate using **Eq.A69** and **Eq.74** are given inside parentheses. $\Gamma_A/\Gamma_V = \left(2b_{F+3}^2 + b_{GT+p}^2\right)/2b_{F+0}^2 \approx 4.8$–$5.5$ for all these cases.

PDF data [9]		$A_+ = -0.1173$	$B_+ = 0.9807$		$\tau_{exp} = 885.7$ sec
d_{12}	κ_b	A	B	Γ_{GT}/Γ_F	$\tau_{th}(log2)$(sec)
−0.1621	3.436	0.0595	0.9807	0.854	73.5
	2.245	−0.1174	0.9984	1.26	38.0
	(2.928	0.0130	0.9808	0.938	56.1)
	(2.203	−0.1172	1.0000	1.27	36.6)
−0.1641	3.472	0.0000	0.9806	0.962	91.1
	2.639	−0.1172	0.9996	1.26	59.9
	(3.049	−0.0381	0.9807	1.04	73.0)
	(2.571	−0.1171	0.9965	1.26	56.7)
−0.1670	3.413	−0.0580	0.9807	1.08	109.2
	2.982	−0.1172	0.9937	1.25	89.16
	(3.081	−0.0883	0.9807	1.15	91.60)
	(2.897	−0.1172	0.9879	1.24	83.73)
−0.3202	9.585	−0.1173	0.9781	1.21	1036
	9.374	−0.1265	0.9807	1.24	1001
	(8.815	−0.1173	0.9732	1.20	872)
	(8.347	−0.1422	0.9807	1.28	805)
−0.3622	12.41	−0.1173	0.9679	1.19	1956
	11.25	−0.1578	0.9807	1.32	1684
	(11.30	−0.1173	0.9631	1.18	1614)
	(10.05	−0.1708	0.9807	1.36	1359)
−0.4042	17.40	−0.1173	0.9571	1.16	4345
	14.66	−0.1861	0.9807	1.40	3349
	(15.55	−0.1174	0.9526	1.15	3455)
	(13.03	−0.1969	0.9807	1.43	2671)

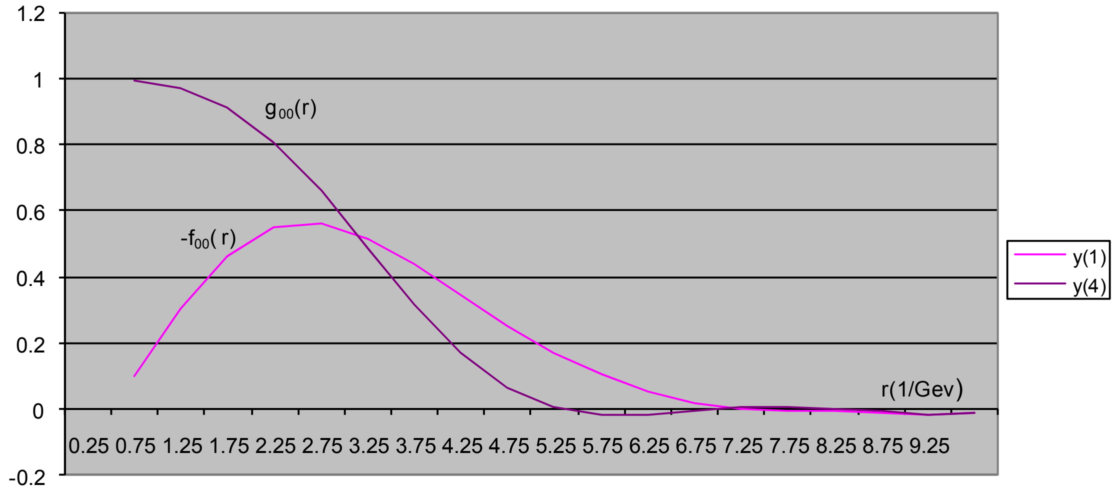

Figure 2. Normalized neutron radial wave functions $f_{00}(r)$ and $g_{00}(r)$ in (A19) for the $d_{b2} = -0.3202$ case in **Table 1**. r is the quark-diquark distance.

Subsection 2.2 shows that the approximate solutions to the baryon spectra problem are those having confinement strength constant d_{b2} values in the range around –0.32 to –0.45. **Table 2** shows that neutron wave functions associated with the $d_{b2} = -0.3202$ case leads to half life $\tau_{th}(log2)$ which are about 15%(12%) off from data [9]. The associated A and B asymmetry coefficients for this $d_{b2} = -0.3202$ case are also rather close to data. These are obtained by adjusting the only free parameter or chosen normalization constant κ_b such that the calculated A coefficient coincides with data. The so predicted B coefficients deviate only 0.3% (0.8%) from data [9], close to experimental error. If κ_b is adjusted such that the calculated B coefficient coincides with data, the so predicted A coefficient deviates from data [9] by 8% (21%).

For $d_{b2} \le -0.3622$, the predicted half life time are too long and the both the predicted A and B coefficients are futher way from data. For $d_{b2} > -0.3202$, it was pointed out beneath **Figure 1** that no satisfactory convergent solutions were found in the range $-0.23 > d_{b2} > -0.27$. For $d_{b2} \ge -0.1670$, the predicted half life time are too short and the both the predicted A and B coefficients are also futher way from data, particularly for the A coefficient. Thus, agreement of predictions of the half life $\tau_{th}(log2)$ and A or B asymmetry coefficients with data [9] is best for $d_{b2} = -0.3202$ and deteriorates considerably for larger or smaller d_{b2} values. This supports the results of Subsection 2.2 that confinement strength constant d_{b2} lies in the range –0.32 to –0.45.

In conclusion, the present treatment leads to two approximate predictions, bearing in mind that they are based upon the approximate results of Subsection 2.2. Possible source of the approximations there has been conjectured in Subsection 2.3. Firstly, the predicted half life time for the chosen confinement strength $d_{b2} = -0.3202$ is consistent with the approximate solution to the baryon spectra problem given in Subsection 2.2. Secondly, this appro- ximate solution also leads to B coefficient in agreement with data [9].

3.6.4. Detachment of Weak and Electromagnetic Couplings

Γ_{GT}/Γ_F and Γ_A/Γ_V in **Table 2** have not been measured. Γ_A/Γ_V is the ratio between the decay rates stemming from the axial vector or tensor part and the vector part in the decay amplitude **Eq.38**. Both these ratios are > 1 and shows that the axial vector or tensor part of the amplitude is greater than that of the vector part. They behave qualitatively in a similar way as does the conventional $(g_A/g_V)^2 = 1.611$ [9], which lies between Γ_{GT}/Γ_F and Γ_A/Γ_V but cannot be related to them.

In the present theory, there is only one weak coupling constant g in **Eqs.19-21** identified or associated with g_V in the literature [9]. Gauge transformations in **Eqs.19-21** does not allow that the axial vector or tensor field is associated with a different coupling constant g_A. That $\Gamma_A > \Gamma_V$ is due to that the normalization type of constant κ_b in **Eq.41**, the only free parameter in this article, is connected to Γ_A but not to Γ_V and κ_b is rather large in **Table 2**.

Even this g or g_V will drop out in the decay rate. Equation **Eq.69** shows that the magnitude of neutron decay rate is proportional to $(g/M_W)^4$. Now, M_W^2 itself is proportional to g^2 according to **Eq.40** so that

$$\frac{g^4}{M_W^4} = 32G_F^2 = \left(\frac{\Omega \to \infty}{\int dx^0 \to \infty}\right)^2 \tag{75}$$

is independent of g. Here, G_F is the Fermi constant of Eq.7.4.29 of [3]. Since $g=e/\sin\vartheta_W$, where ϑ_W is the Weinberg angle and $-e$ the electron charge, **Eq.75** *is independent of* e.

Because ϑ_W is not a basic constant in the present theory but can be derived as in Eq.7.2.3 and Eq.7.2.12 of [3] or Eq.3.3 and Eq.3.12 of [5], the more genuine weak coupling **Eq.75** is detached from the much stronger electromagnetic coupling characterized by e, just like such a detachment found in the meson case in Subsection 7.5.3 of [3]. *Nature is too economical* to deal out two fundamental constnts g and e that are so close to each other. Instead, the strength of weak interactions is characterized by the dimensionless constant $F_W{}^2 = 1.737 \times 10^{-13}$ in Eq.7.5. 22a of [3] which is much smaller than the corresponding $\alpha = 1/137$ for electromagnetic interactions.

On the contrary, based upon an anlysis of some vector meson decay rates, the strong coupling α_s and electromagnetic coupling $\alpha = 1/137$ are unified into one single "electrostrong" coupling via the hypothesis Eq.9.2 of [6] or Eq.8.3.11 of [3], $\alpha_s{}^4 = \alpha$ or $\alpha_s = 0.2923$.

That the Fermi constant G_F in **Eq.75** is a ratio between a large volume and a long relative time indicates that the weak interaction is related to the large scale, long time or low energy aspects of physics.

3.7. Possible Nonconservation of Angular Momentum

3.7.1. Theoretical Background of Possible $\underline{J}$ Nonconservation

The angular momentum $\underline{J}$ of an observable, spin ½ point particle in conventional form

$$\underline{J} = -i\underline{X} \otimes \frac{\partial}{\partial \underline{X}} + i\begin{pmatrix} \underline{\sigma} & \\ & \underline{\sigma} \end{pmatrix} \tag{76}$$

is a constant of motion, hence a conserved quantity. Therefore, the total angular momentum of an ensemble of such particles is also conserved.

However, a baryon is neither a point particle nor a detachable ensemble of such particles. Therefore, conservation of $\underline{J}$ in **Eq.76** cannot be applied to a baryon without reservation. $\underline{J}$ conservation also does not apply to the quarks that constitute the baryon because a quark is not observable in the sense implied by **Eq.76**. For a slowly moving doublet baryon, however, Subsection 10.2.4 of [3] shows that Eq.10.2.1 of [3] can be reverted to a Dirac equation when the diquark coordinate x_I merges into the quark co- ordinate x_{II}. This merger reduces the present nonlocal description **Eqs.A1-A2** to a local one so that **Eq.76** is applicable and $\underline{J}$ is conserved.

By reducing a baryon with extension into a point particle, a great amount of underlying physics leading to various observable baryon phenomena is irretrievably lost. However, the point particle approximation of a baryon can be valid in certain low energy interactions. For instance, the strong charge of a baryon may be considered to be concentrated at a point source in pion-nucleon scattering. Analogously, in Rutherford scattering, the proton can be regarded as having a point charge. In these cases, **Eq.76** can be applied and $\underline{J}$ is conserved.

In weak interactions, however, the gauge boson interacts differently with the differently flavored quarks, broadly speaking. Therefore, reduction of the baryon to a point particle cannot be made. This is evident in the introduction of a gauge fields in Eqs.12-13 of [4] or Eq.7.1.4 of [3], where $\partial^{a\dot{b}}$ operating in the relative space of the diquark and quark cannot be neglected in the ensuing calculations. Thus, the angular momentum $\underline{J}$ of **Eq.76** is not applicable to the baryon wave functions in **Eqs.A1-A2**, which depend upon both the laboratory coordinates X and the relative space coordinates x. Therefore, $\underline{J}$ needs not be conserved in neutron decay. This is supported by noting the following.

In the four possile spin combinations for the nucleons in **Eq.37**, the total spin of the lepton pair is fixed, being zero for the Fermi decays and unity for the Gamow-Teller decays. However, **Eqs.62-65** show that this total spin can also be unity for the Fermi decays and zero for the Gamow-Teller decays in violation of angular momentum conservation. Thus, a *qualitative prediction* is that *angular momentum is not conserved in neutron decay* and hence in weak interactions in general.

3.7.2. Experimental Tests of $\underline{J}$ Conservation Involving Nucleons

$\underline{J}$ in **Eq.76** is conserved in muon decay which involves four spin ½ point particles.

As was mentioned in Subsection 3.1, **Eq.15** makes use of conservation of angular momentum. When combined with Coulomb correction functions, its predictions are relatively consistent with nuclear β-decay and free neutron decay data. Therefore, there is a prevalent view that angular momentum is conserved in such decays. Arguments aginst this view has been given in Subsection 3.1. The conclusion given at the end of Subsection 2.7.1 thus invalidates **Eq.15** by Jackson *et al*. [11] and hence also the interpretations of the experimental results that ensue from it.

Already in their classical paper of 1956, Lee and Yang [14] remarked that in weak interaction experiments up to that time, the baryon number, electric charge, energy and momentum are conserved. Conservation of angular momnetum $\underline{J}$ and parity P as well as invariances under charge conjugation C and time reversal T had however not been established. Nonconservation of P and C was soon discovered and P violation has been extensively measured [15]. A small violation of T has also been detected and subjected to many experimental investigations [12] and [16].

In contrast, no experiment dedicated to test conservation of $\underline{J}$ in nuclear β-decay or free neutron decay has been performed to my knowledge. In fact, no experiment exists that directly distinguishes Fermi from Gamow-Teller transitions in free neutron decay without making use of **Eq.14**.

Therefore, specific tests on $\underline{J}$ conservation in such decays seem to be called for. Such a test is however not strictly a test of the present theory which holds for free neutron decay only. As was indicated below **Eq.15**, the unknown effects of internucleon interactions intervene the theory and eventual experimental results. To this end, decays of free neutrons are needed and will give results of more fundamental importance. That this has not been done is due to the great technical difficulty of such an experiment.

In view of the wealth of raw data available on coincidental experiments with free, polarized neutrons, it may be possible to obtain some indication as to whether $\underline{J}$ is conserved in free neutron decay. No analysis along this line has been carried out to my knowledge. Important clues may be obtained by reviewing existing data.

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Appendix

This appendix provides the basic equations underlying the present work given in [2] and Chapters 9 and 10 of [3]. The equations of motion for the baryons of interest here are given by Eq.2.9 and Eq.8.5 of [2] or **Eq.9.3.16** and **Eq.9.3.19** of [3],

$$\begin{aligned}&\partial_I^{a\dot b}\partial_I^{gh}\partial_{II\dot e f}\chi^f_{\{\dot b\dot h\}}(x_I,x_{II})\\&=-i\left(M_b^3+\Phi_b(x_I,x_{II})\right)\psi^{\{ag\}}_{\dot e}(x_I,x_{II})\end{aligned} \quad (A1)$$

$$\begin{aligned}&\partial_{I\dot bc}\partial_{I\dot hk}\partial_{II}^{d\dot e}\psi^{\{ck\}}_{\dot e}(x_I,x_{II})\\&=-i\left(M_b^3+\Phi_b(x_I,x_{II})\right)\chi^d_{\{\dot b\dot h\}}(x_I,x_{II})\end{aligned} \quad (A2)$$

$$\Box_I\Box_I\Box_{II}\Phi_b(x_I,x_{II})=\frac{1}{4}g_s^6\left\{\chi^f_{\{\dot b\dot h\}}(x_I,x_{II})\psi_f^{\{\dot b\dot h\}*}(x_I,x_{II})+c.c.\right\} \quad (A3)$$

$$M_b^3=\frac{1}{8}\left(m_p+m_s+m_q\right)^3 \quad (A4)$$

where x_I and x_{II} are the diquark and quark coordinates, respectively. χ and ψ are baryon wave functions. The spinor are defined in Eqs.C1-C3 of [3],

$$X^{\dot b a}=\delta^{\dot b a}X^0-\underline{\sigma}^{\dot b a}\underline{X},\ \partial_X^{\dot b a}=-\delta^{\dot b a}\frac{\partial}{\partial X^0}-\underline{\sigma}^{\dot b a}\frac{\partial}{\partial\underline{X}} \quad (A5)$$

the m's are the quark masses obtained from meson spectra in Table 1 of [8] or Table 5.2 of [3] and are m_u = 0.6592, $m_d = m_u + 0.00215$ and m_s = 0.7431 in unit of Gev. Φ_b is the scalar quark-diquark interaction potential and the strong quark-quark interaction constant $g_s^6/4$ can be absorbed into the amplitudes of χ and ψ.

The six component wave functions χ and ψ can each be decomposed into a doublet part ψ^c and $\chi_{\dot b}$ and a quartet part. Since we are only concerned with the doublet or spin ½ baryons Eq.9.3.7b of [3], the quartet baryon wave function components are put to zero. From Eq.2.6 of [2] or Eq.9.2.8 of [3], one finds that

$$\begin{aligned}&\chi_{\dot b}=\tfrac{1}{2}\left(\chi_{\{\dot b\dot 1\}1}+\chi_{\{\dot b\dot 2\}2}\right),\\&\psi^a=-\tfrac{1}{2}\left(\psi^{\{a1\}\dot 1}+\psi^{\{a2\}\dot 2}\right)\\&\chi_{\{\dot 1\dot 2\}2}=\tfrac{1}{2}\chi_{\{\dot 1\dot 1\}1}=\tfrac{2}{3}\chi_{\dot 1},\\&\chi_{\{\dot 1\dot 2\}1}=\tfrac{1}{2}\chi_{\{\dot 2\dot 2\}2}=\tfrac{2}{3}\chi_{\dot 2},\\&\chi_{\{\dot 1\dot 1\}2}=\chi_{\{\dot 2\dot 2\}1}=0\\&\psi^{\{12\}\dot 2}=\tfrac{1}{2}\psi^{\{11\}\dot 1}=-\tfrac{2}{3}\psi^1,\\&\psi^{\{12\}\dot 1}=\tfrac{1}{2}\psi^{\{22\}\dot 2}=-\tfrac{2}{3}\psi^2,\\&\psi^{\{11\}\dot 2}=\psi^{\{22\}\dot 1}=0\end{aligned} \quad (A6)$$

Rules for manipulating the spinor indices are given in Appendix B of [3]. Here, an upper index 1(2) can be lowered into a index 2(1 and a − sign) and vice versa according to Eq.B5b of [3]. For doublets, the e, f and d indices in **Eqs.A1-A2** are raised and lowered. Multiply the so-modified **Eq.A1** and **Eq.A2** by $\delta_{\dot e g}/2$ and $\delta^{d\dot h}/2$, respectively, and apply **Eq.A6**. **Eqs.A1-A2** are now reduced to

$$\begin{aligned}&\partial_I^{a\dot b}\partial_{II}^{f\dot e}\partial_I^{e\dot h}\,\tfrac{1}{2}\,\chi_{\{\dot b\dot h\}f}(x_I,x_{II})\\&=-i\left(M_b^3+\Phi_b(x_I,x_{II})\right)\psi_0^a(x_I,x_{II})\end{aligned} \quad (A7)$$

$$\begin{aligned}&\partial_{I\dot bc}\partial_{II\dot eh}\partial_{I\dot hk}\,\tfrac{1}{2}\,\psi^{\{ck\}\dot e}(x_I,x_{II})\\&=i\left(M_b^3+\Phi_b(x_I,x_{II})\right)\chi_{0\dot b}(x_I,x_{II})\end{aligned} \quad (A8)$$

where a subscript 0 has been added to ψ and χ on the right.

If the symmetric quark postulate Section 4 of [2] below (9.3.7) of [3] is used, Eq.10.0.7 of [3]

$$\chi_{\{\dot b\dot h\}f}=\chi_{0\dot b}\delta_{\dot hf},\ \psi^{\{ck\}\dot e}=\psi_0^c\delta^{k\dot e} \quad (A9)$$

can be inserted into **Eqs.A7-A8** which now take a simpler form.

The quark coordinates are transformed into a laboratory coordinate X and a relative coordinate x according to Section 5 of [2] or Eq.3.1.3a, Eq.3.1.10a and Eq.3.5.6 of [3],

$$X=\frac{1}{2}\left(x_I+x_{II}\right),\ x=x_{II}-x_I \quad (A10)$$

As has been mentioned at the end of Section 3.1 of [3], the relative coordinates $x = x^\mu$ is a "hidden variable" not directly observable.

Consider solutions of the separable form Eq.5.1 of [2] or Eq.10.1.1 of [3],

$$\chi^e_{\{\dot a\dot c\}}(x_I,x_{II})=\chi^e_{\{\dot a\dot c\}}(\underline{x})\exp\left(-iK_\mu X^\mu\right),\ \chi\to\psi \quad (A11)$$

where $K_\mu = (E_0, -\underline{K})$ is the four momentum of the baryon. The rest frame rest ($\underline{K} = 0$) doublet equations in the in relative space are obtained from **Eqs.A7-A8** using **Eqs. A10-A11** and read Eq.5.4 of [2] or Eq.10.2.1 of [3],

$$\begin{aligned}&\left(i\delta^{a\dot b}E/2+\underline{\sigma}^{a\dot b}\underline{\partial}\right)\left(E_0^2/4+\Delta\right)\chi_{0\dot b}(\underline{x})\\&=i\left(M_b^3+\Phi_b(\underline{x})\right)\psi_0^a(\underline{x})\end{aligned} \quad (A12)$$

$$\begin{aligned}&\left(i\delta_{\dot bc}E/2-\underline{\sigma}_{\dot bc}\underline{\partial}\right)\left(E_0^2/4+\Delta\right)\psi_0^c(\underline{x})\\&=i\left(M_b^3+\Phi_b(\underline{x})\right)\chi_{0\dot b}(\underline{x}),\qquad \Delta=\underline{\partial}\underline{\partial}\end{aligned} \quad (A13)$$

Similarly, **Eq.A3** with **Eq.A6**, **Eq.A10** and **Eq.A11** leads to Eq.5.5 of [2],

$$\Delta\Delta\Delta\Phi_b(\underline{x})=\frac{4}{3}\operatorname{Re}\left\{\chi^*_{0\dot a}(\underline{x})\psi_0^a(\underline{x})\right\} \quad (A14)$$

The doublet wave functions χ_0 and ψ_0 include a nomalization type of factor $1/\sqrt{\Omega_{cb}}$ factor according to **Eq.A23** which vanishes for a free baryon. In this case, the right side of **Eq.A14** drops out and **Eqs.A12-13** are linear, which is necessary for wave packet building. Equation **Eq.A14** has now the solution Eq.10.2.2a [3] dropping the nonlinear terms there;

$$\Phi_b(r) = \frac{d_b}{r} + d_{b0} + d_{b1}r + d_{b2}r^2, \; r = |\underline{x}| \qquad (A15)$$

where the four d_b's are unknown integration constants.

The doublet wave functions in the relative space are entirely analogous to those of the hydrogen atom and are expanded into spherical harmonics according to Eq.6.3 (with $g_l \to -g$) of [2] or Eq.10.2.3a of [3]. These relations give for orbital quantum number $l = 0$ and azimuthal quantum number $m = \pm½$ Eq.10.3.8 of [3] which consists of two equivalent solutions,

$$\begin{aligned} & m = \frac{1}{2}, \; \psi_0^1(\underline{x}) = \frac{1}{\sqrt{4\pi}}\left(g_0(r) + if_0(r)\cos\vartheta\right), \\ & \chi_{0\dot{1}}(\underline{x}) = \left(\psi_0^1(\underline{x})\right)^* \\ & \psi_0^2(\underline{x}) = \frac{1}{\sqrt{4\pi}} if_0(r)\sin\vartheta\exp(i\phi), \\ & \chi_{0\dot{2}}(\underline{x}) = -\psi_0^2(\underline{x}) \end{aligned} \qquad (A16)$$

$$\begin{aligned} & m = -\frac{1}{2}, \; \psi_0^1(\underline{x}) = \frac{1}{\sqrt{4\pi}} if_0(r)\sin\vartheta\exp(-i\phi), \\ & \chi_{0\dot{1}}(\underline{x}) = -\psi_0^1(\underline{x}) \\ & \psi_0^2(\underline{x}) = \frac{1}{\sqrt{4\pi}}\left(-g_0(r) + if_0(r)\cos\vartheta\right), \\ & \chi_{0\dot{2}}(\underline{x}) = \left(\psi_0^2(\underline{x})\right)^* \end{aligned} \qquad (A17)$$

$$\begin{aligned} & m = \frac{1}{2}, \; \psi_0^1(\underline{x}) = \frac{1}{\sqrt{4\pi}}\left(-g_0(r) + if_0(r)\cos\vartheta\right), \\ & \chi_{0\dot{1}}(\underline{x}) = -\left(\psi_0^1(\underline{x})\right)^* \\ & \psi_0^2(\underline{x}) = \frac{1}{\sqrt{4\pi}} if_0(r)\sin\vartheta\exp(i\phi), \\ & \chi_{0\dot{2}}(\underline{x}) = \psi_0^2(\underline{x}) \end{aligned} \qquad (A18)$$

$$\begin{aligned} & m = -\frac{1}{2}, \; \psi_0^1(\underline{x}) = \frac{1}{\sqrt{4\pi}} if_0(r)\sin\vartheta\exp(-i\phi), \\ & \chi_{0\dot{1}}(\underline{x}) = \psi_0^1(\underline{x}) \\ & \psi_0^2(\underline{x}) = \frac{1}{\sqrt{4\pi}}\left(g_0(r) + if_0(r)\cos\vartheta\right), \\ & \chi_{0\dot{2}}(\underline{x}) = -\left(\psi_0^2(\underline{x})\right)^* \end{aligned} \qquad (A19)$$

Substituting **Eqs.A16-A17** and **Eqs.A18-A19** into **Eqs.A12-A13** yields, respectively, Eq.10.2.12 of [3] and Eq.6.9 of [2] or Eq.10.2.12 (with $f_0 \to -f_0$,) of [3],

$$\begin{aligned} & \left[\frac{E_0^3}{8} + M_b^3 + \Phi_{bd}(r) + \frac{E_0}{2}\Delta_0\right] g_0(r) \\ & + \left(\frac{E_0^2}{4} + \Delta_0\right)\left(\frac{\partial}{\partial r} + \frac{2}{r}\right) f_0(r) = 0 \\ & \left[\frac{E_0^3}{8} - M_b^3 - \Phi_{bd}(r) + \frac{E_0}{2}\Delta_1\right] f_0(r) \\ & - \left(\frac{E_0^2}{4} + \Delta_1\right)\frac{\partial}{\partial r} g_0(r) = 0 \end{aligned} \qquad (A20)$$

$$(A16) \;\; \text{with} \;\; f_0(r) \to -f_0(r) \qquad (A21)$$

The doublet χ and ψ wave functions in the rest frame have been normalized in Subsection 10.3 of [3]. The resulting normalization integral given by Eq.10.3.9b of [3] reads

$$\begin{aligned} N_{dr} = 2\int drr^2 \Bigg[& \frac{E_0^2}{4}\left(g_0^2(r) + f_0^2(r)\right) - \left(\underline{\partial} g_0(r)\right)^2 \\ & - \left(\underline{\partial} f_0(r)\right)^2 - \frac{2}{r^2} f_0^2(r) \Bigg] \end{aligned} \qquad (A22)$$

$$\begin{aligned} & \left(f_0(r),\; g_0(r)\right) = \frac{a_g}{\sqrt{\Omega_{cb}}}\left(f_{00}(r), g_{00}(r)\right), \\ & g_{00}(0) = 1, \; a_g^2 = \frac{E_{0d}}{2|N_{dr0}|} \\ & N_{dr0} = N_{dr} \text{ in } (A18) \text{ with } f_0(r), \\ & g_0(r) \text{ replaced by } f_{00}(r), g_{00}(r) \end{aligned} \qquad (A23)$$

according to Eq.10.3.14 of [3]. Ω_{cb} is a large normalization volume for the doublet baryon.

