A Journal of Software Engineering and Applications, 2012, 5, 134-139
doi:10.4236/jsea.2012.512b026 Published Online December 2012 (http://www.scirp.org/journal/jsea)

A Distributed Compressed Sensing for Images Based on Block Measurements Data Fusion

Huaixin Chen, Jie Liu

Key Lab. of Information Processing, Southwest China Institute of Electronics Technology, Chengdu, P.R. China.
Email: chenhuaixin@sina.com

Received 2012

ABSTRACT

Compressed sensing (CS) is a new technique for simultaneous data sampling and compression. In this paper, we propose a novel method called distributed compressed sensing for image using block measurements data fusion. Firstly, original image is divided into small blocks and each block is sampled independently using the same measurement operator, to obtain the smaller encoded sparser coefficients and stored measurements matrix and its vectors. Secondly, original image is reconstructed using the block measurements fusion and recovery transform. Finally, several numerical experiments demonstrate that our method has a much lower data storage and calculation cost as well as high quality of reconstruction when compared with other existing schemes. We believe it is of great practical potentials in the network communication as well as pattern recognition domain.

Keywords: Distributed CS for Image; Information Fusion; Pattern recognition; Network Communication

1. Introduction

With the development of the optical technology, the image we got from the digital camera often has above 10 million pixels, the real world becomes clearer in the electronic world, simultaneously, it becomes harder to store and transmit these images. In conventional imaging systems, natural images are often first sampled into the digital format at a higher rate and then compressed through the JPEG or the JPEG 2000 [1] code for efficient storage purpose. However, this approach is not applicable for low-power, low resolution imaging devices such as a sensor network with limited computation capabilities. Fortunately, the compressed sensing theory (CS) [2-5] which is proposed by Donoho and Candes, who shows that under certain conditions, a signal can be precisely reconstructed from only a small set of measurements. Recently, CS has attracted considerable attentions in areas of applied mathematics, computer science, and electrical engineering due to its excellent performance.

Compressed sensing acquisition of data might have an important impact for the design of imaging devices where data acquisition is expensive. Duarte et al. [6] detail a single pixel camera that acquires random projections from the visual scene through a digital micro-mirror array. The Block diagram of a single pixel camera has showed in **Figure 1**. A similar acquisition strategy can be used in MRI imaging to reduce the acquisition time and increase the spatial resolution.

In addition, Block-based CS for image is proposed [7], block measurement is more advantageous for real-time applications as the encoder does not need to send the sampled data until the whole image is measured, and the possibility of exploiting block CS is motivated by the great success of block DCT coding systems which are widely used in the JPEG and the MPEG standards.

In this paper, we present a novel distributed CS method which uses block measurements data fusion to reconstruct original images. The main advantage is to decrease the storage of encoder and huge bytes of data traffic, and to need smaller calculate cost than that Block-based

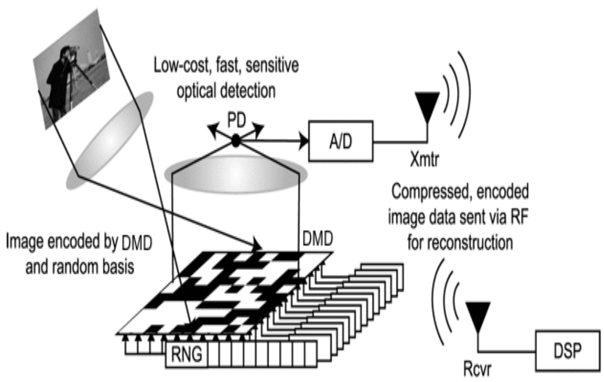


Figure 1. Block diagram of a single-pixel camera

CS restored from a set of sub-measurement recovery and joint sub-image. Therefore, our proposed method is more advantageous in many important and emerging applications, e.g., the sensor network system or network communication.

2. Background

2.1. Compressed Sensing

CS builds upon the fundamental fact that we can represent many signals using only a few non-zero coefficients in a suitable basis or dictionary. Based on CS theoretic requirement that signal is assumed to be approximately sparse, we suppose that the transform coefficients in the orthogonal basis $\boldsymbol{\Psi}$ of an N dimensional vector $\boldsymbol{X}$ are sparse, the signal $\boldsymbol{X}$ can be represented as [8]:

$$\boldsymbol{X} = \boldsymbol{\Psi a} \tag{1}$$

where $\boldsymbol{a}$ is an N dimensional sparse coefficients vector, it has only K non-zero coefficients. In order to take compressed sensing measurements, we let $\boldsymbol{\Phi}$ denote an M by N matrix with M×N. The measurement matrix $\boldsymbol{\Phi}$ should be uncorrelated with the transform matrix $\boldsymbol{\Psi}$. M measurements are obtained by a linear system:

$$\boldsymbol{Y} = \boldsymbol{\Phi X} = \boldsymbol{\Phi \Psi a} = \overline{\boldsymbol{\Phi}}\boldsymbol{a} \tag{2}$$

Where, sensing matrix $\overline{\boldsymbol{\Phi}} = \boldsymbol{\Phi\Psi} \in \mathrm{R}^{M \times N}$,an n-dimensional Euclidean signal vector space is denoted by R_n . If the sensing matrix satisfies the Restricted Isometry Property (RIP) [9,10], the sparse coefficients vector can be reconstructed as solving the following minimal l_0 norm optimization problem:

$$\bar{\boldsymbol{a}} = \arg \min \| \boldsymbol{a} \|_0 , \quad s.t.\ \overline{\boldsymbol{\Phi}}\boldsymbol{a} = \boldsymbol{Y} \tag{3}$$

After obtained the sparse coefficients vector $\bar{\boldsymbol{a}}$, we can exactly reconstruct the original signal $\overline{\boldsymbol{X}}$ as follow:

$$\overline{\boldsymbol{X}} = \boldsymbol{\Psi}\bar{\boldsymbol{a}} \tag{4}$$

Essentially, the optimization problem (3) is an NP-hard problem; usually we must convert the minimal l_0 norm optimization problem into the l_1 norm or l_2 norm to solve the sub optimization problem.

The methods we often used to solve the l_1 norm optimization problem are orthogonal matching pursuit [11], iterative hard thresholding [12], gradient pursuits [13], convex optimization [14], non-convex minimization [15] and so on.

2.2. The block-based CS with united sub-images

The CS theoretic breaks the limit of Nyquist sampling rate, it can compress the data at the same of sampling, but the quantity of the CS measurement matrix's column and row equal to the dimension of the signal and measurements vector, respectively. In processing the high-resolution optical image, it still need to store the enormous measurement matrix and measurements vector, which takes a lot of time to reconstruct original image through solving the optimization problem.

The block-based CS is proposed [16] to break the bottleneck of the enormous data quantity image transmitted by band-limited communication network, which divides original image with huge gigabytes of pixels into many sub-images, which is projected and quantized in the encoder, block-by-block measurements decoded and reconstruction of subspace, finally, it make a recovery the original image from joint sub-images, as showed in **Figure 2**.

In **Figure 2**, firstly the block-based CS divides the original big image into many small sub-image, next it gets every sub-image's measurements of compressed coding. Every sub-images has been acquisition from block measurements of CS processing. Finally, original image is restored using joint reconstructed sub-images in a block-by-block manner. Since each block is processed independently in the block-based CS, Block-based measurement is more advantageous for realtime applications because the encoder does not need to send the sampled data until the whole image is measured, the initial solution can be easily obtained and the reconstruction process can be substantially speeded up. However, the approach still need to store a large of data for measurements and sub-images, and there are complex calculate cost for union sub-images to recover original image.

3. Distributed CS Based on Block Measurments Fusion

In the block CS, it ignores the strong correlation of the sub-images, which employed to reconstruct original image by union them. For the sake of mining the correlation among the sub-images and reducing the data storage as well as calculation cost, we present a novel method of distributed CS for image based on block measurements fusion, whose processing is shown in **Figure 3**.

Being different from Block-based CS, the proposed method in this paper doesn't reconstruct original image from a set of the recovery sub-images separately, but restored original sub-image's block measurement matrix and its measurement vectors at firstly, and reconstruction

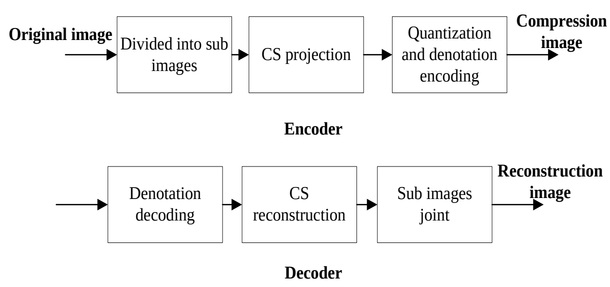
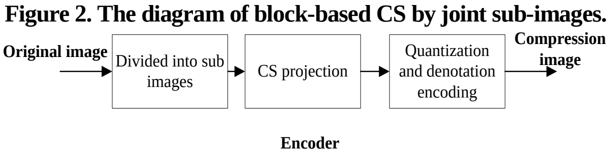


Figure 2. The diagram of block-based CS by joint sub-images.



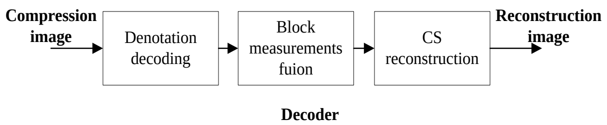


Figure 3: The diagram of distribution CS based on block measurements fusion.

original image though to synthesis measurement matrix and its measurement vectors using data fusion, so as to reduces the storage in the decoder and reduce the calculations from united sub-images. The associated algorithm is given in the next section detailedly.

3.1. Synthese's measurements matrix and its vectors

In the decompressed of distribution CS, we get every compressed measurements blocks of n column by n column or m row by m row. Let n dimensional vector $\boldsymbol{x}$ denote one column of the original image. We can obtain the measurements of $\boldsymbol{x}$ by $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$, $\boldsymbol{y} \in \mathrm{R}^m$, where m is a number of measurement vector, $\boldsymbol{A} \in \mathrm{R}^{m\times n}$ is the measurement matrix. This process shows in figure 4.

In [17], the authors indicate that if the measurement matrix is random gauss matrix, the sensing matrix can satisfy the RIP with high probability. The theoretic analysis and experimental results show that all of the different measurement matrixes perform excellently and we can't find which one is better than others. So we employ the random Gauss matrix with the normal distribution as the measurement matrix of CS.

Suppose the column of image divided into u segments(or blocks), denoted by $\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_u$, the number of every segment are $n_1, n_2, ..., n_u$,respectively. In a cast of overlap segment with the neighbors, therefore, $n_1 + n_2 + ... + n_u > n$. In other case of no overlap segment， $n_1 + n_2 + ... + n_u = n$. Every segment's compressed measurements $\boldsymbol{y}_i$ can obtain as follow:

$$\boldsymbol{y}_i = \boldsymbol{A}_i \boldsymbol{x}_i ;\ \ i = 1, 2, ..., u \tag{5}$$

Where, $\boldsymbol{A}_i \in \mathrm{R}^{m_i \times n_i}$ is the i-th segment's measurement matrix, that is a block measurement matrix, m_i is the number of its compressed measurements.

3.2. Fusion algorithm

The reconstruction optimization problem in CS is: $\min \| \boldsymbol{f} \|_1,\ \ s.t.\ \ \boldsymbol{y} = \boldsymbol{A}\boldsymbol{\Psi} \boldsymbol{f}$. which indicates that to reconstruct the sparse coefficients $\boldsymbol{f}$ of vector $\boldsymbol{x}$, we must know the measurement matrix $\boldsymbol{A}$, measurements vector $\boldsymbol{y}$, and the sparse matrix $\boldsymbol{\Psi}$. Because the matrix $\boldsymbol{\Psi}$ doesn't be used in the encoder, so we just need to synthesis the measurement matrix A and measurements of vector y of original image $\boldsymbol{x}$ from every sub-block measurements.

The reconstructed $\boldsymbol{A}$ and $\boldsymbol{y}$ satisfy the equation as follow:

$$\begin{aligned} \boldsymbol{y} &= \boldsymbol{A}\boldsymbol{x} \\ &= \boldsymbol{A}.[\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_u] \\ &= [\boldsymbol{A}(:, 1:n_1),\ \ \boldsymbol{A}(:, n_1+1:n_1+n_2), ..., \\ &\quad \boldsymbol{A}(:, 1:n-n_u+1:n)].[\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_u] \\ &= [\boldsymbol{A}_1^{'},\ \boldsymbol{A}_2^{'}, ..., \boldsymbol{A}_u^{'}].[\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_u] \end{aligned} \tag{6}$$

Then we obtain:

$$\boldsymbol{y}(j) = \boldsymbol{A}_1^{'}(j,:)\boldsymbol{x}_1 + \boldsymbol{A}_2^{'}(j,:)\boldsymbol{x}_2 +, ..., + \boldsymbol{A}_u^{'}(j,:)\boldsymbol{x}_u \tag{7}$$

Where, $j = 1, 2, ..., m$.

Using the equation (7), we know that every entry $\boldsymbol{y}(j),\ j = 1, 2, ..., m$ of $\boldsymbol{y}$, contains all entries' information of $\boldsymbol{x}$ .The measurements are obtained through multiplying measurement matrix by vector, so the every entry of $\boldsymbol{A}$ correlates with $\boldsymbol{A}_i$.

As the case of no overlap segments,

$$n_1 + n_2 + ... + n_u = n ,$$

every entry $\boldsymbol{y}_i(k), k = 1, 2, ..., m_i$ of $\boldsymbol{y}_i$ is obtained through multiplying one whole row of measurement matrix by vector $\boldsymbol{x}_i$, so the rows of sub-measurement matrix just can be operated linearly and must treat the whole row as an entry. From the equation (6), $\boldsymbol{y}(j) = \boldsymbol{A}(j,:)\boldsymbol{x}$ and $n_1 + n_2 + ... + n_u = n$, the row number of all the sub-measurement matrix should be extended to m , and then fused the block matrix to reconstruct the whole measurement matrix $\boldsymbol{A}$, which shows in figure 5.

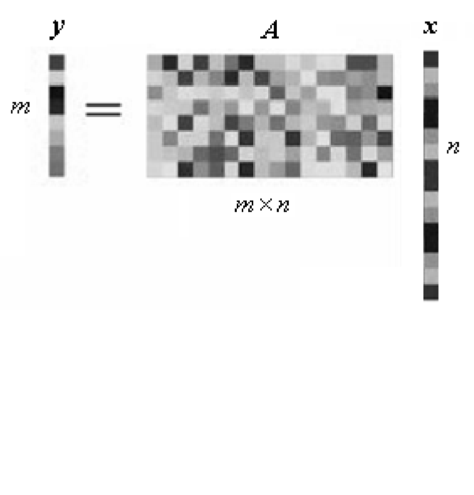


Figure 4. The projection of CS

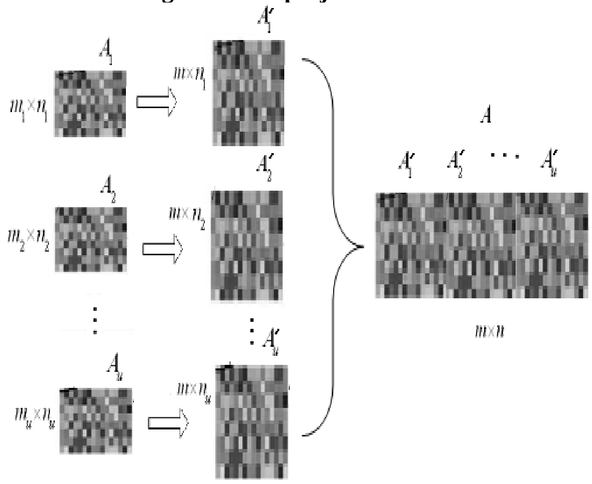


Figure 5: The fusion of block measurement's data matrix.

The next work is to expand the $m_i \times n_i$ matrix $\boldsymbol{A}_i$ to the $m \times n_i$ matrix $\boldsymbol{A}_i'$. In the section 3.1, we let the measurement matrix be random Gauss matrix, so that the reconstruction whole matrix should also be random Gauss matrix satisfied the RIP.

From the applied probability, if $\boldsymbol{A}_i(p,q) \sim \mathrm{N}(0,1)$, and $\boldsymbol{A}_k(w,t) \sim \mathrm{N}(0,1)$, let $\boldsymbol{S} = a\boldsymbol{A}_i(p,q) + b\boldsymbol{A}_k(w,t)$, where *a* and *b* are constant.

So that we can obtain: $\boldsymbol{S} \sim \mathrm{N}(0, a^2 + b^2)$.

The encoder with high confidence nearly loses data so to bring error. By contrast, the low confidence ones should always happen, so we endue the sub-measurement matrix with different power value $1/\mathrm{pw}_k, k = 1, 2, ..., u$, a high power is set to the matrix with a high confidence and vice versa, which can improve performance the reconstruction of image.

For the convenience of denotation, $ASS\left\langle i, \boldsymbol{C} \right\rangle$ denotes that let the *i*-th row of $\boldsymbol{A}$ be the value of $\boldsymbol{C}$, so the measurement matrix's fusion formula can be represented as:

$$\begin{aligned}\boldsymbol{A} = \sum_{i=1}^{m} \boldsymbol{ASS} \Big\langle i, \{ & [\boldsymbol{A}_1(j_1(i),:) + \boldsymbol{A}_1(k_1(i),:)]/\mathrm{pw}_1 |_{j_1(i) \neq k_1(i)}, \\ & [\boldsymbol{A}_2(j_2(i),:) + \boldsymbol{A}_2(k_2(i),:)]/\mathrm{pw}_2 |_{j_2(i) \neq k_2(i)}, \ldots, \\ & [\boldsymbol{A}_u(j_u(i),:) + \boldsymbol{A}_u(k_u(i),:)]/\mathrm{pw}_u |_{j_u(i) \neq k_u(i)} \} \Big\rangle \end{aligned} \tag{8}$$

Where, for the different i, at least the value of $j_p(i)$ or $k_p(i)$, $p = 1, 2, ..., u$ should be changed.

By the equation (8), $\boldsymbol{y} = \boldsymbol{Ax}$ and $\boldsymbol{y}_i = \boldsymbol{A}_i \boldsymbol{x}_i$, we can obtain the measurements' vector fusion formula as follow:

$$\boldsymbol{y}(i) = \sum_{v=1}^{u} \frac{1}{\mathrm{pw}_v} [\boldsymbol{y}_v(j_v(i)) + \boldsymbol{y}_v(k_v(i))] \tag{9}$$

Where, $i = 1, 2, ..., m$; the relation of $j_v(i)$ and $k_v(i)$ is same as equal(8).

4. Experimental Results

The proposed distributed CS based-on block measurements fusion(BMF-DCS) sampling and reconstruction algorithms were implemented using *Matlab* software with version 7.8.0 in PC Computer. For making a nice comparison, we refer to the resulting implementations as the Block CS for image based-on sub-images joint (SIJ-BCS) . In the numerical experiments, we choose three images *Dock* , *Mountain* and *Cameraman* for the experimental object, those test image is of 256×256 pixels and its measurement ratio is 0.6. To evaluate directional transforms for CS reconstruction, we deploy the DCT matrix is as sparse matrix within both the SIJ-BCS framework and proposed BMF-DCS in this paper, and the orthogonal matching pursuit reconstructing algorithm is applied to solve the optimization problem.

Fig.6-Fig.8 illustrates the several example visual results. The numerical experiments demonstrate that both the SIJ-BCS and BMF-BCS can reconstruct the original image with similar good visual qualities for *Dock, Mountain and Cameraman*. However, We note that the reconstruction images from BMF-BCS are smoother than that from SIJ-BCS. That is ,the reconstructed images by the SIJ-BCS method have many noises in the edges and blurred picture , but the reconstructed images using the BMF-BCS method is of smooth in the edges.

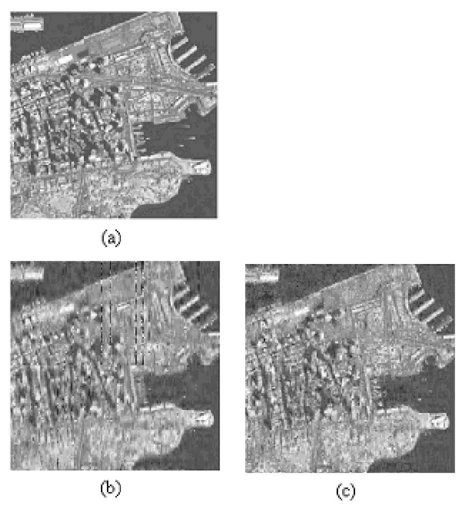


Figure 6. Reconstruction of Dock image. (a) Original image; (b) SIJ-BCS reconstruction; (c) BMF-DCS reconstruction.

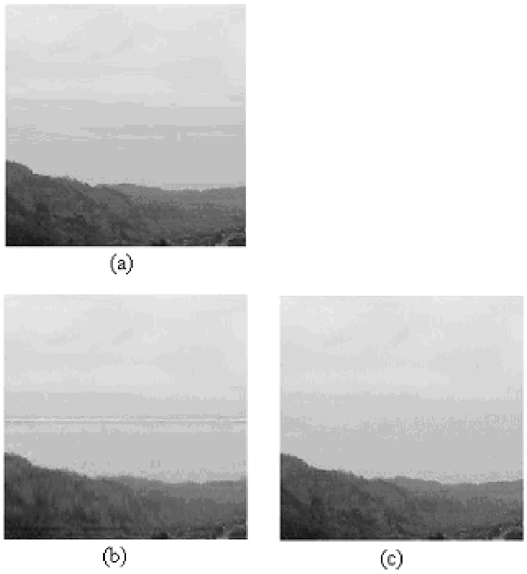


Figure 7: Reconstruction of *Mountain* image. (a) Original image; (b) SIJ-BCS reconstruction; (c) BMF-DCS reconstruction.

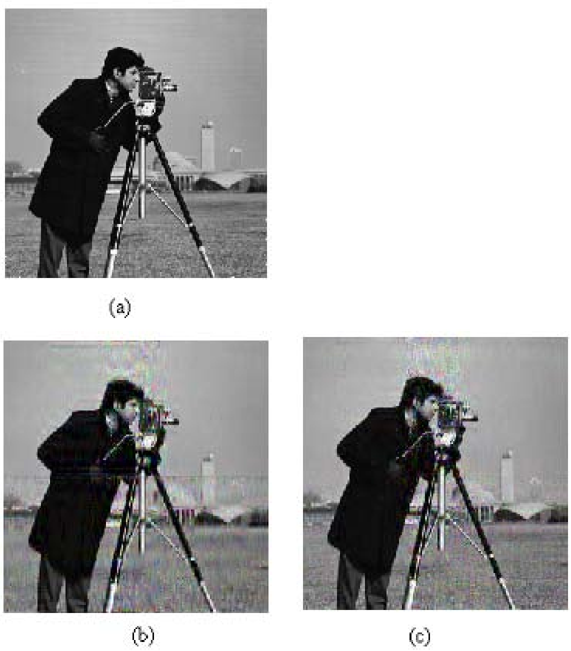


Figure 8: Reconstruction of *Cameraman* image. (a) Original image; (b) SIJ-BCS reconstruction; (c) BMF-DCS reconstruction.

To evaluate the performance of reconstruction by qualitatively, We employ the processing time of reconstruction image in the decoder (TOD) as the calculate cost, and use the number of data that stored in the decoder (NDD) to be represent the storage quantity, and the power signal-to-noise ratio(PSNR) is used for evaluation of the construction quality. Here, the PSNR is given by:

$$\mathrm{PSNR} = 10 \times \log\{M \times N \times f_{\max}^2 / \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\boldsymbol{g}(x,y) - \boldsymbol{f}(x,y)]^2\} \quad (10)$$

Table 1 tabulates the TOD, PSNR and NDD results of both our algorithms (MBF-BCS) and SIJ-BCS reconstruction algorithms on three 256×256 pixels natural images *Dock*, *Mountain* and *Cameraman*.

From table 1, we can see that, for the complex image Dock and Cameraman with symmetrical sparsity, these two methods perform similarly, the BMF-BCS reconstruction PSNR is higher than the SIJ-BCS only by 1dB. For the simple image Mountain with asymmetric sparsity, the PSNR of BMF-BCS yields about 4.5 dB improvements.

Additionally, The SIJ-BCS method's processing time of the decoder reaches more than 100 times the BMF-BCS methods, this result indicates that the BMF-BCS largely reduces the calculate cost of the decoder. In some sense, we can use some cheap equipments to achieve the work that achieved by the dear equipment before. The last column of Table 1, value of NDD shows that near half amount of processing data by the BMF-BCS method is that by the SIJ-BCS method in the decoder, it imply our method can also reduce the cost of the equipment .

As for image block CS method based on sub images joint, it ignores the correlations among the sub-images, whose reconstructing original images in manner of united them .Therefore, the block CS method can only reconstruct the sparser images with high precision and the less sparser sub-images with low precisions. But the new method proposed in this paper can reconstruct the whole image with high precision because of the fault-tolerant data fusion used in the decoder to mine the correlation of the sub-images.

5. Conclusion

In this paper, we propose a novel distributed CS method based-on block measurements fusion in which we use the block measurement matrix and its vectors fusion to synthesis the whole measurements, then to reconstruct the original images, that need no joint sub-image. The several numerical experimental results show our algorithm can reconstruct the original image with a high precision, and largely reduce the storage and calculation cost for image reconstruction in the decoder. Compared with

Table 1. Comparisons of reconstruction TOD, PSNR and NDD measured by different methods.

		TOD/s	PSNR/dB	NDD
Dock	SIJ-BCS	4.336	17.25	65536
	BMF-BCS	0.031	18.34	39321
Mountain	SIJ-BCS	4.255	32.26	65536
	BMF-BCS	0.026	36.72	39321
Camera-man	SIJ-BCS	4.628	21.57	65536
	BMF-BCS	0.028	22.35	39321

existing schemes, the proposed new one contains a much lower data storage and a much lower calculation cost as well as high quality of image reconstruction. We think it is of practical potentials in the network communication as well as realtime object recognition domain.

REFERENCES

[1] Varma,K.Bell.A.JPEG2000-choices and tradeoffs for encoders[J].Signal Processing Magazine, IEEE Nov.2004 Volume：21,Issue:6:70-75.

[2] DONOHO David L. Compressed Sensing[J]．IEEE Trans Inform Theory,2006,52:1289-1306．

[3] CANDÉS E, ROMBERG J, TAO T. Robust Uncertainty Principles: Exact Signal Reconstruction from Highly Incomplete Frequency Information[J]. IEEE Transactions on Information Theory,2006, 52(2): 489-509.

[4] BARANIUK R, CEVHER V, DUARTE M, et al. Model-based Compressive Sensing[J].IEEE Trans. Inform. ThEory,2010, 56(4):1982-2001.

[5] Baron, D., Wakin, M.B., Duarte, M.F. Sarvotham,S., Baraniuk, R.G. Distributed compressed sensing.(2005) Available at http://www.dsp.rice.edu/cs.

[6] M. Duarte, M. Davenport, D. Takhar, J. Laska, T. Sun, K. Kelly, and R. Baraniuk, "Single-pixel imaging via compressive sampling," IEEE Signal Processing Magazine, vol. 2, no. 25, pp. 83–91, 2008.

[7] Sungkwang Mun, James E. Fowler. Block Compressed Sensing of images using directional transforms. Proceedings of the International Conference on Image Processing, 2009, 3021-3024.

[8] Baraniuk R. Compressive sensing[J]. IEEE Signal Processing Magazine, 2007, 24(4): 118-121

[9] BOURGAIN J, DILWORTH S, FORD K., et al. Explicit Constructions of Rip Matrices and Related Problems[J]. Duke Math. J,2011,159:145-185.

[10] M. Davenport ,M. Wakin. Analysis of orthogonal matching pursuit using the restricted isometry property. IEEE Trans Inform Theory, 2010,56(9):4395-4401.

[11] M. Davenport ,M. Wakin. Analysis of orthogonal matching pursuit using the restricted isometry property. IEEE Trans Inform Theory, 2010,56(9):4395-4401.

[12] T. Blumensath , M. Davies. Iterative hard thresholding for compressive sensing.Appl Comput Harmon Anal,2009, 27(3):265-274.

[13] T. Blumensath and M. Davies. Gradient pursuits. IEEE Trans. Signal Processing, 2008,56(6):2370-2382.

[14] E. Candes, B. Recht. Exact matrix completion via convex optimization. Found Comput Math, 2009,9(6):717-772.

[15] Chartrand R．Exact reconstruction of sparse signals via non-convex minimization[J].IEEE Signal Processing Letters,2007,14(10),707- 710．

[16] Gan L.Block compressed sensing of natural images[C]//Proc Int Conf on Digital Signal Processing, Cardiff, UK, 2007．

[17] Yaakov Tsaig, David L.Donoho. Extensions of Compressed Sensing. Signal Processing, 86(2006) 549-571.

