

# Interconverting Models of Gray Matrix and Geographic Coordinates Based on Space Analytic Geometry\*

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## ABSTRACT

With the progress and development of science, the synchronous satellite as the high-tech product is taken something more and more seriously by all countries. Converting gray value matrix into geographic coordinates becomes more and more important. This paper establishes the models for interconverting row and column of gray value matrix which detected by the infrared detectors on synchronous satellites and geographic coordinates based on the related theorems and properties of space analytic geometry. We draw satellite cloud image with the data of gray matrix in MATLAB, convert coastline's latitude and longitude coordinates into row and column coordinates of a gray matrix according via these models when the sub-satellite point's row and column coordinates and geographic coordinates are given, then add coast-lines to the original satellite cloud image with the transformation data. We find that coast-lines completely match with the bump characteristics of satellite cloud image, and that verify the correctness of the models.

**Keywords:** Space Analytic Geometry; Satellite Cloud Image; Geographic Coordinates; Gray Value Matrix; Coastline

## 1. Introduction

Satellite cloud image plays an important role in the scientific research of mastering atmospheric circulation, medium-and long-term weather forecast and disastrous weather and so on. It is drawn with grayscale data which converted from the temperature data over the Earth detected by infrared detector on geosynchronous satellite. It captures the distribution of clouds in the atmosphere, to find the weather system and verify the correctness of surface weather maps that have been drawn. Sub-satellite point represented by geographic latitude and longitude is the point at which a line between the satellite and the centre of the Earth intersects the Earth's surface. Infrared detector scans sampling in accordance with stepping angle (north-south direction) and row scanning angle. The satellite probe data is a gray value matrix, each element of the matrix corresponds to a sampling point of the Earth or extraterrestrial. Established the models that interconvert non-negative elements of grayscale matrix and geographic coordinates, taking up north down south left west right east as the rule, is not only a way to estimate the distance between each probe point, but also a method to the add coast-lines to a satellite cloud image accurately. In meteorological research and service, it is often posi-

tioning and reading the value of geographic coordinates in satellite cloud image, the low resolution and small image make positioning accuracy in a low level. Therefore it produces positioning error, which has an influence on forecast accuracy and research accuracy. Calculating the conversion formulas have attracted much attention, in particular among people interested in the remote sensing technology and meteorology.

Zhang Qingshan [1] uses linear regression to estimate satellite spin axis vector in the nephogram imaging process, calculating the exact position and attitude of the satellite on the basis of geographic coordinates, transforming the nephogram coordinates to geographic coordinates. Wang Guanghui & Chen Biao [2] use the method of rotating spherical coordinates system to calculate longitude and latitude of the 2048th sampling point in scanline scanning by High Resolution Infrared Radiation Sounder. Zhang Jinjin & Liang Hongyou [3] design algorithms whose objective is to converse plane coordinates to latitude and longitude coordinates use of IDL. Dong Minglun & Zhou Jiong [4] educe the relationship between Cartesian conversions and Geographic conversions according to visual imaging principle of synchronous meteorological satellite. George P. Gerdan & Rodney E. Deakin [5] review and analyze six published techniques of Cartesian-to-geographic conversion, which are regarded as representative of the numerous published methods, found that *Lin & Wang's* method is appreciably

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The surface of the ellipsoid may be parameterized in several ways. One choice which singles out the “z”-axis is:

$$\begin{cases} x = r \cos \varphi \sin(\lambda_0 - \lambda) \\ y = r \cos \varphi \cos(\lambda_0 - \lambda) \\ z = r \sin \varphi \end{cases} \quad (2)$$

From right-angled triangle EHS we get:

$$\tan(k\alpha) = \frac{x}{c - y}$$

From right-angled triangle LHS we get:

$$\tan(l\alpha) = \frac{z}{c - y}$$

Build equations:

$$\begin{cases} x = (c - y) \tan(k\alpha) \\ z = (c - y) \tan(l\alpha) \\ y = a \sqrt{1 - \left(\frac{x^2}{a^2} + \frac{z^2}{b^2}\right)} \end{cases} \quad (3)$$

From Equations (2) and (3) we work out:

$$\begin{cases} k = \frac{1}{\alpha} \arctan\left(\frac{r \cos \varphi \sin(\lambda_0 - \lambda)}{c - r \cos \varphi \cos(\lambda_0 - \lambda)}\right) \\ l = \frac{1}{\alpha} \arctan\left(\frac{r \sin \varphi}{c - r \cos \varphi \cos(\lambda_0 - \lambda)}\right) \end{cases} \quad (4)$$

where  $r = \frac{1}{\left(\frac{\cos \varphi}{a}\right)^2 + \left(\frac{\sin \varphi}{b}\right)^2}$ .

In this paper, we take sub-satellite point as the initial point to compute the row and column coordinates of the four cardinal directions: north, east, south and west. Therefore, we divide the ellipse into four quadrants taking sub-satellite point as origin, and discuss in four cases to converse  $(k, l)$  to  $(i, j)$ . In the first quadrant, the formula can be:

$$\begin{cases} i = I + k \\ j = J - l \end{cases} \quad (5)$$

when  $i > I, j < J$ .

Put (4) into (5), that is:

$$\begin{cases} i = I + \frac{1}{\alpha} \arctan\left(\frac{a \cos \varphi \sin(\lambda_0 - \lambda)}{c - a \cos \varphi \cos(\lambda_0 - \lambda)}\right) \\ j = J - \frac{1}{\alpha} \arctan\left(\frac{a \sin \varphi}{c - a \cos \varphi \cos(\lambda_0 - \lambda)}\right) \end{cases} \quad (6)$$

when  $i > I, j < J$ .

Similarly, we can get the other three quadrants' formulas:

The second quadrant:

$$\begin{cases} i = I - \frac{1}{\alpha} \arctan\left(\frac{a \cos \varphi \sin(\lambda_0 - \lambda)}{c - a \cos \varphi \cos(\lambda_0 - \lambda)}\right) \\ j = J - \frac{1}{\alpha} \arctan\left(\frac{a \sin \varphi}{c - a \cos \varphi \cos(\lambda_0 - \lambda)}\right) \end{cases} \quad (7)$$

when  $i < I, j < J$ .

The third quadrant:

$$\begin{cases} i = I - \frac{1}{\alpha} \arctan\left(\frac{a \cos \varphi \sin(\lambda_0 - \lambda)}{c - a \cos \varphi \cos(\lambda_0 - \lambda)}\right) \\ j = J + \frac{1}{\alpha} \arctan\left(\frac{a \sin \varphi}{c - a \cos \varphi \cos(\lambda_0 - \lambda)}\right) \end{cases} \quad (8)$$

when  $i < I, j > J$ .

The fourth quadrant:

$$\begin{cases} i = I + \frac{1}{\alpha} \arctan\left(\frac{a \cos \varphi \sin(\lambda_0 - \lambda)}{c - a \cos \varphi \cos(\lambda_0 - \lambda)}\right) \\ j = J + \frac{1}{\alpha} \arctan\left(\frac{a \sin \varphi}{c - a \cos \varphi \cos(\lambda_0 - \lambda)}\right) \end{cases} \quad (9)$$

when  $i > I, j > J$ .

The four Formulae (6)-(9) above are the models that converse longitude and latitude coordinates into row and column coordinates.

Conversion from geographic coordinates to row and column coordinates is a simple operation. Inverse computations, on the other hand, are a little complex, now we want to use  $(i, j)$  to deduct  $(\varphi, \lambda)$ . Consider the case of the second quadrant for example: put the first two equations of (3) into the third equation of (3), we have:

$$(c - y)^2 \left[ \frac{\tan^2(k\alpha)}{a^2} + \frac{\tan^2(l\alpha)}{b^2} \right] + \frac{y^2}{a^2} = 1 \quad (10)$$

Let  $\frac{\tan^2(k\alpha)}{a^2} + \frac{\tan^2(l\alpha)}{b^2} = d$ , we get

$(c - y)^2 d + \frac{y^2}{a^2} = 1$ , the solutions to this quadratic equation can be found directly from the quadratic formula.

Solve for  $y$ :  $y = \frac{a^2 cd \pm a \sqrt{a^2 d - c^2 d + 1}}{a^2 d + 1}$ . As  $y > 0$ , so

we take  $y = \frac{a^2 cd + a \sqrt{a^2 d - c^2 d + 1}}{a^2 d + 1}$ .

Build equations:

$$\begin{cases} x = (c - y) \tan(k\alpha) \\ z = (c - y) \tan(l\alpha) \\ \varphi = \arctan \frac{z}{\sqrt{x^2 + y^2}} \\ \lambda = \lambda_0 + \arctan \frac{x}{y} \end{cases} \quad (11)$$

Let  $\tan(k\alpha) = m, \tan(l\alpha) = n$ , from (10) we obtain  $y = \frac{a^2cd + a\sqrt{a^2d - dc^2 + 1}}{a^2d + 1}$ , put it into (11), we get (12)

where

$$m = \tan((I - i)\alpha), \quad n = \tan((J - j)\alpha) \quad d = \frac{m^2}{a^2} + \frac{n^2}{b^2}.$$

The other three equations can be obtained by the same method. We do not list all the cases in this paper.

### 3. A Numerical Example

In this section, we present our experiment results. It may be helpful to illustrate these models in a constructed example simple enough that an analytical approach is possible. The results reported here are based on  $2288 \times 2288$  gray value matrix detected by a satellite detection which is located at  $0^\circ\text{N}$  latitude and  $86.5^\circ\text{E}$  longitude, that is  $Q(\lambda_0, 0) = Q(86.5, 0)$ . Given the distance between satellite and the center of Earth is 42,164,000 m, that is  $c = 42164000$  m. The detection scans by the stepping angle of 140 micro-arcs, that is  $\alpha = 0.00014^\circ$ . Given  $a = 6378136.5$  m,  $b = 6356751.8$  m.

We select arbitrarily 40 sampling points on the column 1145 to test the models. Use (7) to convert the sampling points' row and column coordinates into longitude and latitude coordinates, then we want to use the results to calculate the row and column coordinates via (6)-(9), results as follow:

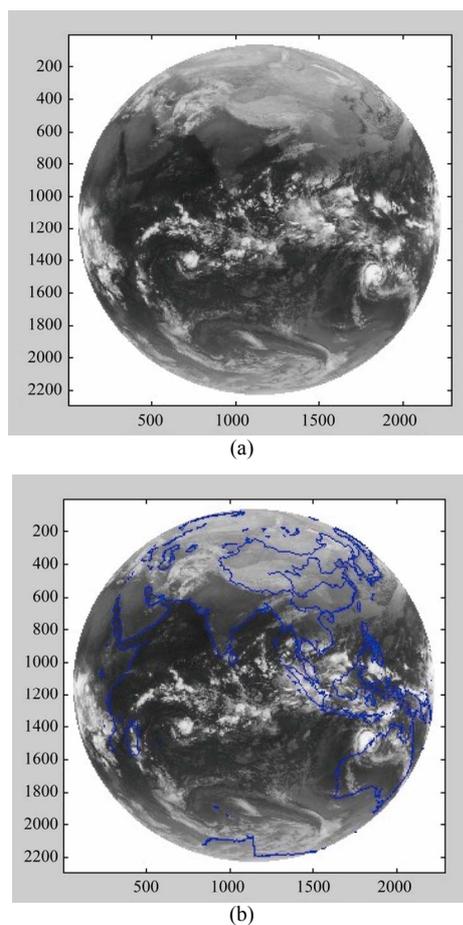
From **Table 1** it can be seen that after converting the row and column coordinates into geographical coordinates, then converting back, there exists error of 1 - 3 pixels between the row coordinates before and after conversion in nine of the sampling points. The rest coordinates are wholly consistent. The error of 1 - 3 pixels is

rounding error which falls within the acceptable range. And results proved that the models are reasonable.

In reference [4], the calculated results exists 0.3 pixel error between the rows and 4.6 pixel between the columns, the models of this paper based on previous studies have higher accuracy.

Draw satellite cloud image with the data of gray value matrix in MATLAB [6], as shown in **Figure 2(a)**, add coast-lines into satellite cloud image, shown in **Figure 2(b)**.

Convert the data of coastline's longitude and latitude



**Figure 2. (a) Satellite cloud image; (b) Satellite cloud image with coast-lines.**

$$\begin{cases} \varphi = \arctan \frac{n \left[ c - \frac{a^2cd + a\sqrt{a^2d - dc^2 + 1}}{a^2d + 1} \right]}{\sqrt{m^2 \left( \frac{c - a\sqrt{a^2d - dc^2 + 1}}{a^2d + 1} \right)^2 - \left( \frac{a^2cd + a\sqrt{a^2d - dc^2 + 1}}{a^2d + 1} \right)^2}} \\ \lambda = \lambda_0 + \arctan \frac{m \left( c - a\sqrt{a^2d - dc^2 + 1} \right)}{a^2cd + \sqrt{a^2d - dc^2 + 1}} \end{cases} \quad (12)$$

**Table 1. Results of converting coordinates.**

Row	Column	Longitude	Longitude	Row after converting	Column after converting
100	1145	86.5	65.82926	101	1145
230	1145	86.5	49.9728	233	1145
342	1145	86.5	41.1886	345	1145
455	1145	86.5	33.91336	457	1145
566	1145	86.5	27.61518	567	1145
599	1145	86.5	25.85278	600	1145
650	1145	86.5	23.2053	651	1145
675	1145	86.5	21.93694	676	1145
697	1145	86.5	20.83494	697	1145
700	1145	86.5	20.68563	700	1145
713	1145	86.5	20.04117	713	1145
742	1145	86.5	18.61763	742	1145
750	1145	86.5	18.22815	750	1145
761	1145	86.5	17.69476	761	1145
790	1145	86.5	16.29969	790	1145
813	1145	86.5	15.20387	813	1145
829	1145	86.5	14.44661	829	1145
844	1145	86.5	13.74017	844	1145
856	1145	86.5	13.17732	856	1145
895	1145	86.5	11.36085	895	1145
903	1145	86.5	10.99046	903	1145
922	1145	86.5	10.11349	922	1145
930	1145	86.5	9.74532	930	1145
950	1145	86.5	8.827475	950	1145
976	1145	86.5	7.639261	976	1145
986	1145	86.5	7.183597	986	1145
998	1145	86.5	6.637687	998	1145
1000	1145	86.5	6.54679	1000	1145
1002	1145	86.5	6.455919	1002	1145
1011	1145	86.5	6.047289	1011	1145
1050	1145	86.5	4.281411	1050	1145
1111	1145	86.5	1.530465	1111	1145
1120	1145	86.5	1.125251	1120	1145
1124	1145	86.5	0.945185	1124	1145
1130	1145	86.5	0.67511	1130	1145
1135	1145	86.5	0.450065	1135	1145
1140	1145	86.5	0.22503	1140	1145
1145	1145	86.5	0	1145	1145
1150	1145	86.5	-0.22503	1149	1145
1151	1145	86.5	-0.40506	1152	1145

into row and column coordinates, then add coast-lines into satellite cloud image. We can discover that coast-lines exactly match with the bump characteristics of satellite cloud image. It illustrates the models are reasonable.

#### 4. Conclusions

This paper has presented the methods with analytical solution of interconverting row and column of gray value matrix and geographical coordinates based on the related theorems and properties of space analytic geometry, finally gives acceptable results of a numerical example to verify the correctness of the models.

This work can also be easily applied in satellite image registration and so on. Our future work involves using these models in cloud motion models developed at satellite cloud image and analyzing its feasibility.

#### REFERENCES

- [1] Q. S. Zhang, "Method of Static Meteorological Satellite Cloud Image Landmark Navigation," China Academic Journal Electronic Publishing House, Beijing, 1984, pp. 14-20.
- [2] G. H. Wang, B. Chen and W. P. He, "Research of Longitude and Latitude of AVHRR 2048 Based on Polar Coordinate Transformation," *Journal of Qingdao University*, Vol. 16, No. 3, 2003, pp. 45-48.
- [3] J. J. Zhang, H. Y. Liang, X. F. Chen, X. F. Gu, T. Yu and H. B. Ge, "Transformation from Image Panel Coordinate to Latitude and Longitude Coordinate," *Geospatial Information*, Vol. 8, No. 1, 2010, pp. 63-66.
- [4] M. L. Dong and Q. Zhou, "Transforming Relationship between Cartesian Coordinates and Geographical Coordinates and Error Analysis," *Marine Forecasts*, Vol. 28, No. 2, 2011, pp. 48-54.
- [5] G. P. Gerdan and R. E. Deakin, "Transforming Cartesian Coordinates  $X, Y, Z$  to Geographical Coordinates  $\varphi, \lambda, h$ ," *The Australian Surveyor*, Vol. 44, No. 1, 1999, pp. 55-63.
- [6] D. Teng, P. Wang, Y.-G. Che and D. Wang, "Realization Method of Electronic Chart Based on Coaseline with MATLAB7.0," *Laser Journal*, Vol. 31, No. 5, 2010, pp. 10-11.

[1] Q. S. Zhang, "Method of Static Meteorological Satellite