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Bc-Open Sets in Topological Spaces

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ABSTRACT

In this paper, we introduce a new class of *b*-open sets called *Bc*-open, this class of sets lies strictly between the classes of *θ*-semi open and *b*-open sets. We also study its fundamental properties and compare it with some other types of sets and we investigate further topological properties of sets and we introduce and investigate new class of space named *Bc*-compact.

Keywords: Closed; *b*-Open; *Bc*-Open

1. Introduction

In 1937, regular open sets were introduced and used to define the semi-regularization space of a topological space. Throughout this paper, (X,τ) and (Y,σ) stand for topological spaces with no separation axioms assumed unless otherwise stated. For a subset *A* of *X*, the closure of A and the interior of A will be denoted by $Cl(A)$ and $Int(A)$, respectively. Stone [1] defined a subset *A* of a space *X* to be a regular open if $A = Int(Cl(A))$. Norman Levine [2] defined a subset *A* of a space X to be a semi-open if $ACl(Int(A))$, or equivalently, a set A of a space X will be termed semi-open if and only if there exists an open set U such that $U \subset A \subset Cl(U)$. Mashhour *et al.* [3] defined a subset *A* of a space *X* to be a preopen if $A \subset Int(Cl(A))$. Njastad [4] defined a subset *A* of a space *X* to be an α-open if $AInt(Cl(Int(A)))$. The complement of a semi-open (resp., regular open) set is said to be semi-closed [5] (resp., regular closed). The intersection of all semi-closed sets of *X* containing *A* is called the semi-closure [6] of *A*. The union of semi-open sets of *X* contained in *A* is called the semi-interior of *A*. Joseph and Kwack [7] introduced the concept of θ-semi open sets using semi-open sets to improve the notion of S-closed spaces. Also Joseph and Kwack [7] introduced that a subset A of a space X is called θ-semi-open if for each $x \in A$, there exists a semi-open set G such that $x \in G \subset Cl(G) \subset A$. It is well-known that, a space *X* is called $T1$ if to each pair of distinct points *x*, *y* of *X*, there exists a pair of open sets, one containing *x* but not *y* and the other containing *y* but not *x*, as well as is $T1$ if and only if for any point $x \in X$, the singleton set $\{x\}$ is closed. *A* space *X* is regular if for each $x \in X$ and each open set *G* containing *x*, there exists an open set *H* such that $x \subset H \subset Cl(H) \subset G.N.K$. Ahmed [8] defined a topological space (X,τ) to be s^{**}-normal if and only if for every semi-closed set *F* and every semi-open set *G* containing *F*, there exists an open set *H* such that $F \subset H \subset Cl(H) \subset G$. In 1968, Velicko [9], defined the concepts of δ-open and θ-open as, a subset A of a space X is called δ-open (resp., θ-open) if for each $x \in A$, there exists an open set G such that $x \in G \subset Int(Cl(G)) \subset A$ (resp., $x \in G \subset Cl(G) \subset A$). Di Maio and Noiri [10] introduced that a subset *A* of a space *X* is called semi-θ-open if for each $x \in A$, there exists a semi-open set *G* such that $x \in G \subset scl(G) \subset A$. The family of all open (resp., semi-open, α-open, preopen, *θ*-semi-open, semi-*θ*-open, *θ*-open, *δ*-open, regular open, semi-closed and regular closed) subsets of a topological space (X,τ) are denoted by τ (resp., $SO(X)$, $\alpha O(X)$, $PO(X)$, $\theta SO(X)$, $S\theta O(X)$, $\theta O(X)$, $\delta O(X)$, $RO(X)$, $SC(X)$ and $RC(X)$).

Definition 1.1. [11] A subset *A* of a space *X* is called *b*-open if $A \subseteq Int(Cl(A)) \cup Cl(Int(A))$. The family of all *b*-open subsets of a topological space (X,τ) is denoted by $BO(X,\tau)$ or (Briefly. $BO(X)$).

In 1999, J. Dontchev and T. Noiri [12] have shown the following lemma:

Lemma 1.2. For a subset A of a space (X,τ), the following conditions are equivalent:

1) $A \in RO(X)$.
2) $A \in \tau \cap SC(X)$.
3) $A \in \alpha O(X) \cap SC(X)$.
4) $A \in PO(X) \cap SC(X)$.

Theorem 1.3. [13] If X is s^{**}-normal, then $S\theta O(X) = \theta O(X) = \theta SO(X)$.

We recall that a topological space X is said to be extremally disconnected [14] if $Cl(G)$ is open for every open set G of X.

Definition 1.4. [15] A space X is called locally indiscrete if every open subset of X is closed.

Theorem 1.5. [13] A space X is extremally disconnected if and only if $\delta O(X)=\theta SO(X)$.

Theorem 1.6. [15] A space X is extremally disconnected if and only if $RO(X)=RC(X)$.

2. *Bc*-Open Sets

In this section, we introduce a new class of *b*-open sets called *Bc*-open sets in topological spaces.

Definition 2.1. A subset A of a space X is called *Bc*-open if for each $x\in A\in BO(X)$, there exists a closed set F such that $x\in F\subset A$. The family of all *Bc*-open subsets of a topological space (X,τ) is denoted by $BcO(X,\tau)$ or (Briefly. $BcO(X)$).

Proposition 2.2. A subset A of a space X is *Bc*-open if and only if A is *b*-open and it is a union of closed sets. That is $A=\cup F_{\alpha}$ where A is *b*-open set and F_{α} is closed sets for each α.

Proof. Obvious.

It is clear from the definition that every *Bc*-open subset of a space X is *b*-open, but the converse is not true in general as shown by the following example.

Example 2.3. Consider $X=\{a,b,c\}$ with the topology $\tau=\{\varphi,X,\{a\},\{b\},\{a,b\}\}$. Then the family of closed sets are: $\varphi,X,\{c\},\{a,c\},\{b,c\}$. We can find easily the following families:

$$BO(X)=\{\varphi,X,\{a\},\{b\},\{a,b\},\{a,c\},\{b,c\}\}$$

and

$$BcO(X)=\{\varphi,X,\{a,c\},\{b,c\}\}.$$

Then $\{a\}\in BO(X)$, but $\{a\}\notin BcO(X)$.

The next example notices that a *Bc*-open set need not be a closed set.

Example 2.4. Consider the space R with usual topology, if $A=(0,1]$ such that $(0,1]=\bigcup_{1}^{\infty}\left[\frac{1}{n},1\right]$, then A is *Bc*-open set, but it is not closed.

The following result shows that the arbitrary union of *Bc*-open sets in a topological space (X,τ) is *Bc*-open.

Proposition 2.5. Let $\{A\alpha:\alpha\in\Delta\}$ be a collection of *Bc*-open sets in a topological space X. Then $\cup\{A\alpha:\alpha\in\Delta\}$ is *Bc*-open.

Proof. Let $A\alpha$ be a *Bc*-open set for each α, then $A\alpha$ is b-open and hence $\cup\{A\alpha:\alpha\in\Delta\}$ is *b*-open. Let $x\in\cup\{A\alpha:\alpha\in\Delta\}$, there exist $\alpha\in\Delta$ such that $x\in A\alpha$. Since $A\alpha$ is *b*-open for each α, there exists a closed set F such that

$$x\in F\subset A\alpha\subset\cup\{A\alpha:\alpha\in\Delta\},$$

so $x\in F\subset\cup\{A\alpha:\alpha\in\Delta\}$, Therefore, $\cup\{A\alpha:\alpha\in\Delta\}$ is *Bc*-open set.

The following example shows that the intersection of two *Bc*-open sets need not be *Bc*-open set.

Example 2.6. Consider the space (X,τ) as in example 2.3, There $\{a,c\}\in BcO(X)$ and $\{b,c\}\in BcO(X)$, but $\{a,c\}\cap\{b,c\}=\{c\}\notin BcO(X)$.

From the above example we notice that the family of all *Bc*-open subsets of a space X is a supratopology and need not be a topology in general.

The following result shows that the family of all *Bc*-open sets will be a topology on X.

Proposition 2.7. If the family of all *b*-open sets of a space X is a topology on X, then the family of *Bc*-open is also a topology on X.

Proof. Clearly $\varphi,X\in BcO(X)$ and by Proposition 2.5 the union of any family of *Bc*-open sets is *Bc*-open. To complete the proof it is enough to show that the finite intersection of *Bc*-open sets is *Bc*-open set. Let A and B be two *Bc*-open sets then A and B are b-open sets. Since $BO(X)$ is a topology on X, so $A\cap B$ is *b*-open. Let $x\in A\cap B$, then $x\in A$ and $x\in B$, so there exists F and E such that $x\in F\subset A$ and $x\in E\subset B$ this implies that $x\in F\cap E\subset A\cap B$. Since any intersection of closed sets is closed, $F\cap E$ is closed set. Thus $A\cap B$ is *Bc*-open set. This completes the proof.

Proposition 2.8. The set A is *Bc*-open in the space (X,τ) if and only if for each $x\in A$, there exists a *Bc*-open set B such that $x\in B\subset A$.

Proof. Assume that A is *Bc*-open set in the (X,τ), then for each $x\in A$, put $A=B$ is *Bc*-open set containing x such that $x\in B\subset A$.

Conversely, suppose that for each $x\in A$, there exists a *Bc*-open set B such that $x\in B\subset A$, thus $A=\cup Bx$ where $Bx\in BcO(X)$ for each x, therefore A is *Bc*-open set.

In the following proposition, the family of *b*-open sets is identical to the family of *Bc*-open sets.

Proposition 2.9. If a space X is $T1$-space, then the families $BO(X)=BcO(X)$.

Proof. Let A be any subset of a space X and $A\in BO(X)$, if $A=\varphi$, then $A\in BcO(X)$. If $A\neq\varphi$, then for each $x\in A$. Since a space X is $T1$, then every singleton is closed set and hence $x\in\{x\}\subset A$. Therefore $A\in BcO(X)$. Hence $BO(X)\in BcO(X)$, but $BcO(X)\subset BO(X)$ generally, therefore $BO(X)=BcO(X)$.

Proposition 2.10. Every θ-semi open set of a space X is *Bc*-open set.

Proof. Let A be a θ-semi open set in X, then for each $x\in A$, there exists a semi-open set G such that $x\in G\subset ClG\subset A$, so $\cup\{x\}\in\cup G\cup ClG\subset A$ for each

$x \in A$ implies that $A = \cup G$ which is semi-open set and $A = \cup ClG$ is a union of closed sets, by Proposition 2.2, *A* is *Bc*-open set.

The following example shows that the converse of the above Proposition may not be true in general.

Example 2.11. Since a space *X* with cofinite topology is *T*1, and then the family of *b*-open and *Bc*-open sets are identical. Hence any open set *G* is *Bc*-open but not θ-semi open.

The proof of the following corollaries is clear from their definitions.

Corollary 2.12. Every θ-open set is *Bc*-open.

Corollary 2.13. Every regular-closed is *Bc*-open set.

Proposition 2.14. If a topological space (X,τ) is locally indiscrete, then $SO(X) \subset BcO(X)$.

Proof. Let *A* be any subset of a space *X* and $A \in SO(X)$, if $A = \varphi$, then $A \in BcO(X)$. If $A \neq \varphi$, then $A \subset ClInt(A)$. Since *X* is locally indiscrete, then $Int(A)$ is closed and hence $Int(A) \subset A$, this implies that for each $x \in A$, $x \in xInt(A) \subset A$. Therefore, *A* is *Bc*-open set. Hence $SO(X) BcO(X)$.

Remark 2.15. Since every open set is semi-open, it follows that if a topological space (X,τ) is *T*1 or locally indiscrete, then $\tau \subset BcO(X)$.

Proposition 2.16. Let (X,τ) be a topological space, if *X* is regular, then $\tau \subset BcO(X)$.

Proof. Let *A* be any subset of a space *X*, and *A* is open, if $A = \varphi$, then $A \in BcO(X)$. If $A \neq \varphi$, since *X* is regular, so for each $x \in A \subset X$, there exists an open set *G* such that $x \in G \subset ClG \subset A$. Thus we have $x \in ClG \subset A$. Since $A \in \boldsymbol{\tau}$ and hence $A \in BO(X)$, therefore $\tau \in BcO(X)$.

Proposition 2.17. Let (X,τ) be an extremally disconnected space. If $A \in \delta O(X)$, then $A \in BcO(X)$.

Proof. Let $A \in \delta O(X)$. If $A = \varphi$, then $A \in BcO(X)$. If $A \neq \varphi$. Since a space *X* is extremally disconnected, then by Theorem 1.5, $\delta O(X) = \theta SO(X)$. Hence $A \in \theta SO(X)$. But $\theta SO(X) \subset BcO(X)$ in general. Therefore, $A \in BcO(X)$.

Corollary 2.18. Let (X,τ) be an extremally disconnected space. If $A \in RO(X)$, then $A \in BcO(X)$.

Proof. The proof is directly from Proposition 2.28 and the fact that $RO(X) \subset \delta O(X)$.

Proposition 2.19. Let (X,τ) be an s^{**}-normal space. If $A \in S\theta O(X)$, then $A \in BcO(X)$.

Proof. Let $A \in S\theta O(X)$. If $A = \varphi$, then $A \in BcO(X)$. If $A \neq \varphi$, since a space *X* is s^{**}-normal, then by Theorem 1.3, $S\theta O(X) = \theta SO(X)$. Hence $A \in \theta SO(X)$. But $\theta SO(X) \in BcO(X)$ in general. Therefore, $A \in BcO(X)$.

Proposition 2.20. For any subset *A* of a space (X,τ) and $BO(X) = SO(X)$. The following conditions are equivalent:

1) *A* is regular closed.
2) *A* is closed and *Bc*-open.
3) *A* is closed and *b*-open.
4) *A* is *α*-closed and *b*-open.
5) *A* is pre-closed and *b*-open.

Proof. Follows from Lemma 1.2.

Definition 2.21. A subset *B* of a space *X* is called *Bc*-closed if $X \setminus B$ is *Bc*-open. The family of all *Bc*-closed subsets of a topological space (X,τ) is denoted by $BcC(X,\tau)$ or (Briefly, $BcC(X)$).

Proposition 2.22. A subset *B* of a space *X* is *Bc*-closed if and only if *B* is a *b*-closed set and it is an intersection of open sets.

Proof. Clear.

Proposition 2.23. Let $\{B\alpha : \alpha \in \Delta\}$ be a collection of *Bc*-closed sets in a topological space *X*. Then $\cap\{B\alpha : \alpha \in \Delta\}$ is *Bc*-closed.

Proof. Follows from Proposition 2.5.

The union of two *Bc*-closed sets need not be *Bc*-closed as is shown by the following counterexample.

Example 2.24. In Example 2.3, the family of *Bc*-closed subset of *X* is: $BcC(X) = \{\varphi, X, \{a\}, \{b\}\}$. Here $\{a\} \in BcC(X)$ and $\{b\} \in BcC(X)$, but

$$\{a\} \cup \{b\} = \{a,b\} \notin BcC(X).$$

All of the following results are true by using complement.

Proposition 2.25. If a space *X* is $T1$, then $BcC(X) = BC(X)$.

Proposition 2.26. For any subset *B* of a space *X*. If $B \in \theta SC(X)$, then $B \in BcC(X)$.

Corollary 2.27. Each θ-closed set is *Bc*-closed.

Corollary 2.28. Each regular open set is *Bc*-closed.

Proposition 2.29. If a topological space (X,τ) is locally indiscrete, then $SC(X) \subset BcC(X)$.

Proposition 2.30. Let (X,τ) be a topological space, if *X* is regular or locally indiscrete, then the family of closed sets is a subset of the family of *Bc*-closed sets.

Proposition 2.31. Let (X,τ) be any extremally disconnected space. If $B \in \delta C(X)$, then $B \in BcC(X)$.

Corollary 2.32. Let (X,τ) be an extremally disconnected space. If $B \in RC(X)$, then $B \in BcC(X)$.

Proposition 2.33. Let (X,τ) be a s^{**}-normal space. If $B \in S\theta C(X)$, then $B \in BcC(X)$.

Proposition 2.34. For any subset *B* of a space (X,τ) and $SC(X) = BC(X)$. The following conditions are equivalent:

1) *B* is regular open.
2) *B* is open and *Bc*-closed.
3) *B* is open and *b*-closed.
4) *B* is *α*-open and *b*-closed.
5) *B* is preopen and *b*-closed.

Diagram 1 shows the relations among $BcO(X)$, $BO(X)$, $RO(X)$, $RC(X)$, $\delta O(X)$, τ, $\alpha O(X)$, $\theta O(X)$ and $\theta SO(X)$.

$$
\begin{tikzcd}
RO(X) \arrow[d] & & RC(X) \arrow[d] \\
\delta O(X) \arrow[d] & \theta O(X) \arrow[l] \arrow[r] & \theta SO(X) \arrow[dd] \\
\tau \arrow[d] & & \\
\alpha O(X) \arrow[r] & BO(X) & BcO(X) \arrow[l]
\end{tikzcd}
$$

Diagram 1.

3. Some Properties of *Bc*-Open Sets

In this section, we define and study topological properties of *Bc*-neighborhood, *Bc*-interior, *Bc*-closure and *Bc*-derived of a set using the concept of *Bc*-open sets.

Definition 3.1. Let (X,τ) be a topological space and $x \in X$, then a subset *N* of *X* is said to be *Bc*-neighborhood of x, if there exists a Bc-open set *U* in *X* such that $x \in U \subset N$.

Proposition 3.2. In a topological space (X,τ), a subset *A* of *X* is *Bc*-open if and only if it is a *Bc*-neighbourhood of each of its points.

Proof. Let $A \subset X$ be a *Bc*-open set, since for every $x \in A, x \in A \subset A$ and *A* is *Bc*-open. This shows *A* is a *Bc*-neighborhood of each of its points.

Conversely, suppose that *A* is a *Bc*-neighborhood of each of its points. Then for each $x \in A$, there exists $Bx \in BcO(X)$ such that $Bx \subset A$. Then $A = \cup\{Bx : x \in A\}$. Since each *Bx* is *Bc*-open. It follows that *A* is *Bc*-open set.

Proposition 3.3. For any two subsets *A*, *B* of a topological space (X,τ) and $A \subset B$, if *A* is a *Bc*-neighborhood of a point $x \in X$, then *B* is also *Bc*-neighborhood of the same point x.

Proof. Let *A* be a *Bc*-neighborhood of $x \in X$, and $A \subset B$, then by Definition 2.1, there exists a *Bc*-open set *U* such that $x \in U \subset A \subset B$, this implies that *B* is also a *Bc*-neighborhood of *x*.

Remark 3.4. Every *Bc*-neighborhood of a point is *b*-neighborhood, it follows from every *Bc*-open set is *b*-open.

Definition 3.5. Let *A* be a subset of a topological space (X,τ), a point $x \in X$ is said to be *Bc*-interior point of A, if there exist a *Bc*-open set U such that $x \in U \subset A$. The set of all *Bc*-interior points of *A* is called *Bc*-interior of *A* and is denoted by $BcInt(A)$.

Some properties of the *Bc*-interior of a set are investigated in the following theorem.

Theorem 3.6. For subsets *A*, *B* of a space *X*, the following statements hold.

1) $BcInt(A)$ is the union of all *Bc*-open sets which are contained in *A*.

2) $BcInt(A)$ is Bc-open set in *X*.

3) A is Bc-open if and only if $A = BcInt(A)$.

4) $BcInt(BcInt(A)) = BcInt(A)$.

5) $BcInt(\phi) = \phi$ and $BcInt(X) = X$.

6) $BcInt(A) \subset A$.

7) If $A \subset B$, then $BcInt(A) \subset BcInt(B)$.

8) If $A \cap B = \phi$, then $BcInt(A) \subset BcInt(B)$.

9) $BcInt(A) \cup BcInt(B) \subset BcInt(A \cup B)$.

10) $BcInt(A \cap B) \subset BcInt(A) \cap BcInt(B)$.

Proof. 7) Let $x \in X$ and $x \in BcInt(A)$, then by Definition 3.5, there exists a *Bc*-open set U such that $x \in U \subset A \subset B$ implies that $x \in U \subset B$. Thus $x \in BcInt(B)$.

The other parts of the theorem can be proved easily.

Proposition 3.7. For a subset *A* of a topological space (X,τ), then $BcInt(A) \subset bInt(A)$.

Proof. This follows immediately since all Bc-open set is *b*-open.

Definition 3.8. Let *A* be a subset of a space *X*. A point $x \in X$ is said to be *Bc*-limit point of *A* if for each *Bc*-open set U containing $x, U \cap (A \setminus \{x\}) \neq \varphi$. The set of all *Bc*-limit points of *A* is called a *Bc*-derived set of *A* and is denoted by $BcD(A)$.

Proposition 3.9. Let *A* be a subset of *X*, if for each closed set *F* of *X* containing *x* such that $F \cap (A \setminus \{x\}) \neq \varphi$, then a point $x \in X$ is *Bc*-limit point of *A*.

Proof. Let *U* be any *Bc*-open set containing *x*, then for each $x \in U \in BO(X)$, there exists a closed set F such that $x \in F \subset U$. By hypothesis, we have $F \cap (A \setminus \{x\}) \neq \varphi$. Hence $U \cap (A \setminus \{x\}) \neq \varphi$. Therefore, a point $x \in X$ is *Bc*-limit point of *A*.

Some properties of *Bc*-derived set are stated in the following theorem.

Theorem 3.10. Let *A* and *B* be subsets of a space *X*. Then we have the following properties:

1) $BcD(\varphi) = \varphi$.

2) If $x \in BcD(A)$, then $x \in BcD(A \setminus \{x\})$.

3) If $A \subset B$, then $BcD(A) \subset BcD(B)$.

4) $BcD(A) \cup BcD(B) \subset BcD(A \cup B)$.

5) $BcD(A \cap B) \subset BcD(A) \cap BcD(B)$.

6) $BcD(BcD(A)) \setminus ABcD(A)$.

7) $BcD(A \cup BcD(A)) \subset A \cup BcD(A)$.

Proof. We only prove 6), 7), and the other parts can be proved obviously.

6) If $x \in BcD(BcD(A)) \setminus A$ and U is a *Bc*-open set containing *x*, then $U \cap (BcD(A) \setminus \{x\}) \neq \varphi$. Let $y \in U \cap (BcD(A) \setminus \{x\})$. Then, since $y \in BcD(A)$ and $y \in U, U \cap (A \setminus \{y\}) \neq \varphi$. Let $z \in U \cap (A \setminus \{y\})$. Then, $z \neq x$ for $z \in A$ and $x \notin A$. Hence, $U \cap (A \setminus \{x\}) \neq \varphi$. Therefore, $x \in BcD(A)$.

7) Let $x \in BcD(A \cup BcD(A))$. If $x \in A$, the result is obvious. So, let $x \in BcD(A \cup BcD(A)) \setminus A$, then, for *Bc*-

open set U containing $x, U \cap (A \cup BcD(A)) \setminus \{x\} \neq \varphi$. Thus, $U \cap (A \setminus \{x\}) \neq \varphi$ or $U \cap (BcD(A) \setminus \{x\}) \neq \varphi$. Now, it follows similarly from 1) that $U \cap (A \setminus \{x\}) \neq \varphi$. Hence, $x \in BcD(A)$. Therefore, in any case, $(BcD(A) \cup BcD(A)) \subset A \cup BcD(A)$.

Corollary 3.11. For a subset A of a space X, then $bD(A) \subset BcD(A)$.

Proof. It is sufficient to recall that every *Bc*-open set is *b*-open.

Definition 3.12. For any subset A in the space X, the *Bc*-closure of A, denoted by $BcCl(A)$, is defined by the intersection of all *Bc*-closed sets containing A.

Proposition 3.13. A subset A of a topological space X is *Bc*-closed if and only if it contains the set of its *Bc*-limit points.

Proof. Assume that A is *Bc*-closed and if possible that x is a *Bc*-limit point of A which belongs to $X \setminus A$, then $X \setminus A$ is *Bc*-open set containing the *Bc*-limit point of A, therefore $A \cap X \setminus A \neq \varphi$, which is a contradiction.

Conversely, assume that A contains the set of its *Bc*-limit points. For each $x \in X \setminus A$, there exists a *Bc*-open set U containing x such that $A \cap U = \varphi$, that is $x \in U \subset X \setminus A$ by Proposition 2.8, $X \setminus A$ is *Bc*-open set and hence A is *Bc*-closed set.

Proposition 3.14. Let A be a subset of a space X, then $BcCl(A) = ABcD(A)$.

Proof. Since $BcD(A) \subset BcCl(A)$ and $A \subset BcCl(A)$, then $A \bigcup BcD(A) \subset BcCl(A)$.

On the other hand. To show that $BcCl(A) \subset A \bigcup BcD(A)$, since $BcCl(A)$ is the smallest *Bc*-closed set containing A, so it is enough to prove that $A \cup BcD(A)$ is *Bc*-closed. Let $x \notin A \cup BcD(A)$. This implies that $x \notin A$ and $x \notin BcD(A)$. Since $x \notin BcD(A)$, there exists a *Bc*-open set Gx of x which contains no point of A other than x but $x \notin A$. So Gx contains no point of A, which implies $Gx \subset X \setminus A$. Again, Gx is a *Bc*-open set of each of its points. But as Gx does not contain any point of A, nopoint of Gx can be a *Bc*-limit point of A. Therefore, nopoint of Gx can belong to $BcD(A)$. This implies that $Gx \subset X \setminus BcD(A)$. Hence, it follows that

$$x \in Gx \subset X \setminus A \cap X \setminus BcD(A) \subset X \setminus (A \cup BcD(A)).$$

Therefore, $A \cup BcD(A)$ is *Bc*-closed. Hence $BcCl(A) \subset A \cup BcD(A)$. Thus

$$BcCl(A) = A \cup BcD(A).$$

Corollary 3.15. Let A be a set in a space X. A point $x \in$ is in the *Bc*-closure of A if and only if $A \bigcap U \neq \varphi$ for every *Bc*-open set U containing x.

Proof. Let $x \notin BcCl(A)$. Then $x \notin \cap F$, where F is *Bc*-closed with $A \subset F$. So $x \in X \setminus \cap F$ and $X \setminus \cap F$ is a *Bc*-open set containing x and hence

$$(X \setminus \cap F) \cap A \subset (X \setminus \cap F) \cap (\cap F) = \varphi.$$

Conversely, suppose that there exists a *Bc*-open set containing x with $A \cap U = \varphi$. Then $A \subset X \setminus U$ and $X \setminus U$ is a *Bc*-closed. Hence $x \notin BcCl(A)$.

Proposition 3.16. Let A be any subset of a space X. If $A \cap F \neq \varphi$ for every closed set F of X containing x, then the point x is in the *Bc*-closure of A.

Proof. Suppose that U be any *Bc*-open set containing x, then by Definition 2.1, there exists a closed set F such that $x \in F \subset U$. So by hypothesis $A \cap F \neq \varphi$ implies $A \cap U \neq \varphi$ for every *Bc*-open set U containing x. Therefore $x \in BcCl(A)$.

Here we introduce some properties of *Bc*-closure of the sets.

Theorem 3.17. For subsets A, B of a space X, the following statements are true.

1) The *Bc*-closure of A is the intersection of all *Bc*-closed sets containing A.

2) $A \subset BcCl(A)$.

3) $BcCl(A)$ is Bc -closed set in X

4) A is *Bc*-closed set if and only if $A = BcCl(A)$.

5) $BcCl(BcCl(A)) = BcCl(A)$.

6) $BcCl(\phi) = \phi$ and $BcCl(X) = X$.

7) If $A \subset B$, then $BcCl(A) \subset BcCl(B)$.

8) If $BcCl(A) \cap BcCl(B) = \phi$, then $AB = \phi$.

9) $BcCl(A) \cup BcCl(B) \subset BcCl(A \cup B)$.

10) $BcCl(A \cap B) \subset BcCl(A) \cap BcCl(B)$.

Proof. Obvious.

Proposition 3.18. For any subset A of a topological space X. The following statements are true.

1) $X \setminus BcCl(A) = BcInt(X \setminus A)$.

2) $X \setminus BcInt(A) = BcCl(X \setminus A)$.

3) $BcCl(A) = X \setminus BcInt(X \setminus A)$.

4) $BcInt(A) = X \setminus BcCl(X \setminus A)$.

Proof. We only prove 1), the other parts can be proved similarly. For any point $x \in X$, $x \in X \setminus BcCl(A)$ implies that $x \notin BcCl(A)$, then for each $G \in BcO(X)$ containing $x, A \cap G = \varphi$, then $x \in G \subset X \setminus A$. Thus $x \in BcInt(X \setminus A)$.

Conversely, by reverse the above steps, we can prove this part.

Remark 3.19. If A is a subset of a topological space X. Then

$$\begin{aligned} n\theta sInt(A) &\subset BcInt(A) \subset bInt(A) \subset A \subset bCl(A) \\ &\subset BcCl(A) \subset \theta BCl(A). \end{aligned}$$

Proof. Obvious.

4. *Bc*-Compactness

In this section, we introduce and investigate new class of space named *Bc*-compact.

Definition 4.1. A filter base $\mathfrak{F}$ in a topological space

(X,τ) *Bc*-converges to a point $x \in X$ if for every *Bc*-open set *V* containing *x*, there exists an $F \in \mathfrak{F}$ such that $F \subset V$.

Definition 4.2. A filter base $\mathfrak{F}$ in a topological space (X,τ) *Bc*-accumulates to a point $x \in X$ if $F \cap V \neq \varphi$, for every $\mathfrak{F}$-open set *V* containing (X,τ) and every $F \in \mathfrak{F}$.

Proposition 4.3. Let $\mathfrak{F}$ be a filter base in a topological space (X,τ). If $\mathfrak{F}$ *Bc*-converges to a point $x \in X$, then $\mathfrak{F}$ rc-converges to a point *x*.

Proof. Suppose that $\mathfrak{F}$ *Bc*-converges to a point $x \in X$. Let *V* be any regular closed set containing *x*, then $V \in BcO(X)$. Since $\mathfrak{F}$ *Bc*-converges to a point $x \in X$, there exists an $F \in \mathfrak{F}$ such that $F \subset V$. This shows that $\mathfrak{F}$ *rc*-converges to a point *x*.

In general the converse of the above proposition is not necessarily true, as the following example shows.

Example 4.4. Consider the space $(R,\cup)$. Let $\mathfrak{F} = \{R,[0-\varepsilon,0+\varepsilon]:\varepsilon > 0 \in R\}$. Then $\mathfrak{F}$ *rc*-converges to 0, but $\mathfrak{F}$ does not *Bc*-converges to 0, because the set $(0-\varepsilon,0+\varepsilon)$ is *Bc*-open containing 0, there exist no $F \in \mathfrak{F}$ such that $F \subset (0-\varepsilon,0+\varepsilon)$.

Corollary 4.5. Let $\mathfrak{F}$ be a filter base in a topological space (X,τ). If $\mathfrak{F}$ *Bc*-accumulates to a point $x \in X$, then $\mathfrak{F}$ *rc*-accumulates to a point *x*.

Proof. Similar to Proposition 4.3.

Proposition 4.6. Let $\mathfrak{F}$ be a filter base in a topological space (X,τ) and *E* is any closed set containing x. If there exists an $F \in \mathfrak{F}$ such that $F \subset E$, then $\mathfrak{F}$ *Bc*-converges to a point $x \in X$.

Proof. Let V be any *Bc*-open set containing x, then for each $x \in V$, there exists a closed set *E* such that $x \in E \subset V$. By hypothesis, there exists an $F \in \mathfrak{F}$ such that $F \subset E \subset V$ which implies that $F \subset V$. Hence $\mathfrak{F}$ *Bc*-converges to a point $x \in X$.

Proposition 4.7. Let $\mathfrak{F}$ be a filter base in a topological space (X,τ) and *E* is any closed set containing x, such that $F \cap E \neq \varphi$ for each $F \in \mathfrak{F}$, then $\mathfrak{F}$ is *Bc*-accumulation to a point $x \in X$.

Proof. The proof is similar to Proposition 4.6.

Definition 4.8. We say that a topological space (X,τ) is *Bc*-compact if for every *Bc*-open cover $\{V\alpha:\alpha \in \Delta\}$ of *X*, there exists a finite subset $\Delta 0$ of Δ such that $X = \cup\{V\alpha:\alpha \in \Delta 0\}$.

Theorem 4.9. If every closed cover of a space *X* has a finite subcover, then *X* is *Bc*-compact.

Proof. Let $\{V\alpha:\alpha \in \Delta\}$ be any *Bc*-open cover of *X*, and $x \in X$, then for each $x \in V\alpha(x), \alpha \in \Delta$, there exists a closed set $F\alpha(x)$ such that $x \in F\alpha(x) \subset V\alpha(x)$. So the family $\{F\alpha(x): x \in X\}$ is a cover of *X* by closed set, then by hypothesis, this family has a finite subcover such that

$$X = \{F\alpha(xi): i = 1,2,\cdots,n\} \subset \{V\alpha(xi): i = 1,2,\cdots,n\}.$$

Therefore, $X = \{V\alpha(xi): i = 1,2,\cdots,n\}$. Hence *X* is *Bc*-compact.

Proposition 4.10. If a topological space (X,τ) is *b*-compact, then it is *Bc*-compact.

Proof. Let $\{V\alpha:\alpha \in \Delta\}$ be any *Bc*-open cover of *X*. Then $\{V\alpha:\alpha \in \Delta\}$ is *b*-open cover of *X*. Since *X* is *b*-compact, there exists a finite subset $\Delta 0$ of Δ such that $X = \cup\{V\alpha:\alpha \in \Delta 0\}$. Hence *X* is *Bc*-compact.

Proposition 4.11. Every *Bc*-compact $T1$-space is *b*-compact.

Proof. Suppose that *X* is T_1 and *Bc*-compact space. Let $\{V\alpha:\alpha \in \Delta\}$ be any *b*-open cover of *X*. Then for every $x \in X$, there exists $\alpha(x) \in \Delta$ such that $x \in V\alpha(x)$. Since *X* is T_1, by Since *X* is *Bc*-compact, so there exists a finite subset $\Delta 0$ of Δ in *X* such that $X = \cup\{V\alpha:\alpha \in \Delta 0\}$. Hence *X* is *b*-compact.

The next corollary is an immediate consequence of Proposition 4.10 and 4.11.

Corollary 4.12. Let *X* be a $T1$-space. Then *X* is *Bc*-compact if and only if *X* is *b*-compact.

Proposition 4.13. Let a topological space (X,τ) be locally indiscrete. If *X* is *Bc*-compact then *X* is s-compact.

Proof. Follows from Proposition 2.14.

Proposition 4.14. If a topological space (X,τ) is *Bc*-compact, then it is *rc*-compact.

Proof. Let $\{V\alpha:\alpha \in \Delta\}$ be any regular closed cover of *X*. Then $\{V\alpha:\alpha \in \Delta\}$ is a *Bc*-open cover of *X*. Since *X* is *Bc*-compact, there exists a finite subset $\Delta 0$ of Δ such that $X = \cup\{V\alpha:\alpha \in \Delta 0\}$. Hence *X* is *rc*-compact.

Proposition 4.15. Let a topological space (X,τ) be regular. If *X* is *Bc*-compact, then it is compact.

Proof. Let $\{V\alpha:\alpha \in \Delta\}$ be any open cover of *X*. By Proposition 2.16, $\{V\alpha:\alpha \in \Delta\}$ forms a *Bc*-open cover of *X*. Since *X* is *Bc*-compact, there exists a finite subset $\Delta 0$ of Δ such that $X = \cup\{V\alpha:\alpha \in \Delta 0\}$. Hence *X* is compact.

Proposition 4.16. Let *X* be an almost regular space. If *X* is *Bc*-compact, then it is nearly compact.

Proof. Let $\{V\alpha:\alpha \in \Delta\}$ be any regular open cover of *X*. Since *X* is almost regular space, then, for each $x \in X$ and regular open $V\alpha(x)$, there exists an open set *Gx* such that $x \in Gx \subset Cl(Gx) \subset V\alpha(x)$. But $Cl(Gx)$ is regulaclosed for each $x \in X$, this implies that the family $\{Cl(Gx): x \in X\}$ is *Bc*-open cover of *X*, since *X* is *Bc*-compact, then there exists a subfamily $\{Cl(G(xi)): i = 1,2,\cdots,n\}$ such that $X = \bigcup_{i=1}^{n} Cl(G(xi)) \subset \bigcup_{i=1}^{n} V\alpha(xi)$. Thus *X* is nearly compact.

REFERENCES

[1] M. H. Stone, “Applications of the Theory of Boolean Rings

to Topology," *Transactions of the American Mathematical Society*, Vol. 41, No. 3, 1937, pp. 375-481. doi:10.1090/S0002-9947-1937-1501905-7

[2] N. Levine, "Semi-Open Sets and Semi-Continuity in Topological Spaces," *American Mathematical Monthly*, Vol. 70, No. 1, 1963, pp. 36-41. doi:10.2307/2312781

[3] A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deeb, "On Precontinuous and Week Precontinuous Mappings," *Proceedings of Mathematical and Physical Society of Egypt*, Vol. 53, 1982, pp. 47-53.

[4] O. Njastad, "On Some Classes of Nearly Open Sets," *Pacific Journal of Mathematics*, Vol. 15, No. 3, 1965, pp. 961-970. doi:10.2140/pjm.1965.15.961

[5] S. N. El-Deeb, I. A. Hasanein, A. S. Mashhour and T. Noiri, "On p-Regular Spaces," *Bulletin Mathématique de la Société des Sciences Mathématiques de Roumanie*, Vol. 27, No. 4, 1983, pp. 311-315.

[6] S. G. Crossley and S. K. Hildebrand, "Semi-Closure," *Texas Journal of Science*, Vol. 22, No. 2-3, 1971, pp. 99-112.

[7] J. E. Joseph and M. H. Kwack, "On S-Closed Spaces," *Bulletin of the American Mathematical Society*, Vol. 80, No. 2, 1980, pp. 341-348.

[8] N. K. Ahmed, "On Some Types of Separation Axioms," M.Bc. Thesis, Salahaddin University, Arbil, 1990.

[9] N. V. Velicko, "H-Closed Topological Spaces," *American Mathematical Society*, Vol. 78, No. 2, 1968, pp. 103-118.

[10] G. Di Maio and T. Noiri, "On s-Closed Spaces," *Indian Journal of Pure and Applied Mathematics*, Vol. 18, No. 3, 1987, pp. 226-233.

[11] D. Andrijevic, "On b-Open Sets," *Matematički Vesnik*, Vol. 48, No. 3, 1996, pp. 59-64.

[12] J. Dontchev and T. Noiri, "Contra-Semicontinuous Functions," *Mathematica Pannonica*, Vol. 10, No. 2, 1999, pp. 159-168.

[13] R. H. Yunis, "Properties of θ-Semi-Open Sets," *Zanco Journal of Pure and Applied Sciences*, Vol. 19, No. 1, 2007, pp. 116-122.

[14] M. H. Stone, "Algebraic Characterizations of Special Boolean Rings," *Fundamenta Mathematicae*, Vol. 29, No. 1, 1937, pp. 223-302.

[15] J. Dontchev, "Survey on Preopen Sets," *The Proceedings of the Yatsushiro Topological Conference*, 22-23 August 1998, pp. 1-18.