International Journal of Modern Nonlinear Theory and Application, 2012, 1, 118-124
http://dx.doi.org/10.4236/ijmnta.2012.14018 Published Online December 2012 (http://www.SciRP.org/journal/ijmnta)

Deriving the Exact Percentage of Dark Energy Using a Transfinite Version of Nottale's Scale Relativity

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Received September 19, 2012; revised November 10, 2012; accepted November 18, 2012

ABSTRACT

In this paper Nottale's acclaimed scale relativity theory is given a transfinite Occam's razor leading to exact predictions of the missing dark energy [1,2] of the cosmos. It is found that 95.4915% of the energy in the cosmos according to Einstein's prediction must be dark energy or not there at all. This percentage is in almost complete agreement with actual measurements.

Keywords: Dark Energy; Lorentz Factor; Scale Relativity; Cantor Set; Hausdorff Dimension; Hardy's Quantum Entanglement

1. Introduction

The mysterious major fundamental problem of cosmology and theoretical physics [1-20], *i.e.* the missing hypothetical dark energy is tackled and solved in the present paper.

Nottale's theory of scale relativity is a powerful Weyl-like general gauge theory with applications in high energy particle physics as well as cosmology [3,7,8]. In that respect it is quite similar to the mathematical and physical K and E-infinity theory [17-19]. The main difference comes only from the systematic use of logarithmic scaling in Nottale's scale relativity [3,7,8] where as E-infinity is exclusively based on a transfinite Weyl scaling [4,5]. In particular Nottale's theory gave up differentiability but not continuity [3,7,8]. By contrast E-infinity gave up both differentiability as well as continuity but preserved the cardinality of the continuum [4,5] using the geometry of random elementary Cantor sets [3-5,18]. Since the Hausdorff dimension of such random elementary Cantor sets is the golden mean and its powers, the scaling exponent of the theory are combinatorics of these infinitely many golden mean random Cantor sets [3-6,14-18]. In this regard we stress that it is generally wrong to think that discontinuity of space-time introduces something unphysical or a-physical to a theory because an empty set is physical and present in nature as it is present in the fundamental axioms of set theory upon which our entire mathematical methods are based.

In the present work we use Nottale's theory to give first an accurate approximate solution to the problem of the missing dark energy in the cosmos [1,2]. Subsequently we minimally deform Nottale's scale relativity [3,7,8] making it transfinitely almost exact. The so obtained results are in superb agreement with the cosmological measurements [1,2].

2. Scale Relativity—Preliminary Remarks

Scale relativity is a profound general theory which was developed toward the end of the eighties last century [3,7,8]. The theory builds heavily on the tradition of Einstein-Minkowski geometrization of physics and simultaneously makes extensive use of what at the time was the new science of nonlinear dynamics and the great pioneering spirit of deterministic chaos and fractal geometry. That way scale relativity combined the great ideas of Einstein with those of H. Weyl's original gauge theory [4], R. Feynman and Garnet Ord's proposal for a fractal space-time [3]. Scale relativity and fractal space-time sparked quite a revolution in the way we think about foundational problems and cutting edge research in theoretical physics and although it is not as visible as superstrings [9] or loop quantum mechanics [10], it is in no way less original or insightful. In fact, it may be more in a complimentary way as we will attempt to show in the present paper.

There are many parallels and equally differences between

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scale relativity and E-infinity Cantorian space-time theory [3-8]. Let us concentrate on the most important common aspect of the two theories. No doubt it is fractality and scaling. These in reverse order are the quintessence of the two theories from a conceptual view point. However when we look at the quantitative analytical treatment, then the two theories differ in slightly less than a minor way. This is because scale relativity employs logarithmic scaling more or less similar to the logarithmic scaling of the standard model of elementary particles and quantum field theory. One only needs to remember the logarithmic running of the coupling constant as a function of the energy scale used in the electroweak and strong interaction [9-13] to get the idea. Such logarithmic scaling is a powerful approximation to the unattainable non-perturbative exact solution but none the less, it is an approximation. By contrast E-infinity theory employs a golden mean based exact renormalization semi-group and exact golden mean scaling exponents [4-6,14]. None the less, without Weyl and Nottale's insight into the interrelation between gauge theory and fractals, we could not have developed E-infinity theory in its present form which depends crucially on the many excellent results of not only non-commutative geometry [17], superstrings [9], M-theory [15,16] and loop quantum gravity [10] but also in a fundamental way on Nottale's scale relativity and of course Ord's fractal space-time [3].

A second important aspect of scale relativity is that it gives the Planck length the same status which only the velocity of light enjoys in Einstein's theory of relativity. Such a proposal seems at first sight to be controversial because unlike the speed of light, the Planck length cannot be measured experimentally in any realistic set up [7, 12,13]. Never the less, it seems to us that the theory of varying speeds of light [12,13] which was clearly influenced by Nottale's scale relativity and gives a convincing mathematical argument for Planck energy invariance while preserving the Lorentzian symmetry group invariance although velocity and energy could be made arbitrarily much larger than the light velocity and the Planck energy without violating both [12,13].

In the present work we will adopt scale relativity theory in substantially its original form to determine the dark energy content of the universe [1,2] which boils down to revising Einstein's energy mass relation in order to extend its applicability to the realm of quantum gravity [7, 8,10]. Subsequently we will show how Nottale's scale relativity could gradually be made transfinitely exact and obtain the same result obtained using E-infinity theory and the fractal 11 dimensional M-theory [15,16]. We start in the next section with a few explicit examples to illustrate the main ideas of scale relativity in the light of the competing theories such as E-infinity theory and Heterotic superstrings [9].

3. Calculus in Non-Commutative Geometry, Scale Relativity and Cantorian Fractal Space-Time

3.1. Background

To be able to use our calculus as developed by Newton and Leibniz, smoothness and of course continuity are absolutely indispensible. However fractals are not smooth and Cantor sets are totally disjoint. In such a case and where continuity is assumed as in Nottale's scale relativity and partially in Ord's anti-Bernoulli stochastic fractal space-time, one could resort to the non-standard analysis developed by Robinson as done initially by Nottale [8] or use a form of quantum calculus as done by Ord [3,7,8,18]. These methods break down completely in the case of non-continuity and Cantor sets. From a classical pure quantum view point, only a generalization of the Heisenberg truly non-continuous view point rather than Schrödinger's pseudo continuous theory is to resort to non-classical measure theoretical methods, K-theory and categories [17]. In non-commutative geometry A. Connes replaced differentiation of real or complex variables by a Poisson bracket of the form [17]

$$Df = [F, f] = Ff - fF \tag{1}$$

while the opposite or reciprocal operation, *i.e*. integration is replaced by the Dixmier trace [17]

$$DixT = Tr_w(T). \tag{2}$$

Here we follow the standard notation used also in [17].

3.2. Scaling in E-Infinity Theory

In E-infinity Cantorian fractal space-time on the other hand we discovered quite early that the H. Weyl failed original gauge theory is valid in space with no scale at all such as the infinite dimensional but hierarchical Cantor set modelling E-infinity space-time [4,5]. Consequently scaling down is analogous to differentiation while scaling up is analogous to the opposite operation namely integration. It is as simple a duality as that between adding and subtracting or multiplying and dividing. Maybe a simple example makes the idea clearer. Let us take the low energy electromagnetic coupling constant

$$\bar{\alpha}_0 = 137 + k_0 = 137 + \phi^5 (1 - \phi^5) = 137.082039325,$$

where ϕ^5 is Hardy's generic probability of quantum entanglement and $\phi = (\sqrt{5} - 1)/2$. Now we scale $\bar{\alpha}_0$ up twice using $(1/\phi)^2 = 2 + \phi$. This is the opposite number so to speak of "integrating" $\bar{\alpha}_0$ twice. That way one finds [4]

$$\bar{\alpha}_0\left(\frac{1}{\phi}\right)^2 = 358.8854382. \quad (3)$$

We note that pure gravity $G_p^{(d)} = \frac{d(d-3)}{2}$ for $d = 8$ dimensions and the Riemann tensor $R^{(n)} = \frac{n^2(n^2-1)}{12}$ for $n = 4$ are equal [4]

$$R^{(4)} = G_p^{(8)} = 20. \quad (4)$$

The strong interaction is given by the compactified Lie symmetry group $SL(2,7)$ [4,5]

$$\begin{aligned} |S(2,7)|_c &= |SL(2,7)| + 16k \\ &= 336 + 2.88543824 \cong 339 \end{aligned} \quad (5)$$

and we could infer that scaling up "ergo pseudo integrate" of $\bar{\alpha}_0$ results in obtaining all the 339 gluons as well as gravity. In fact we could be more accurate and write that [4,5]

$$\begin{aligned} |S(2,7)|_c &= |SL(2,7)| + |SU(2)| - (k_0 + k^2) \\ &= 336 + 3 - \frac{1}{2(4+\phi^3)} \end{aligned} \quad (6a)$$

where

$$k_0 = \phi^5(1-\phi^5) = 0.082039325 \quad (6b)$$

and

$$k = \phi^3(1-\phi^3) = 2\phi^5 = 0.1803398 \quad (6c)$$

are Hardy type transfinite quantum entanglement corrections [6,14].

In addition we have [4]

$$|SL(2, n=7)| = n(n^2-1) = (7)(48) = 336 \quad (7)$$

for the quarks-like state of the strong force [4,5] and finally [4]

$$|SU(n=2)| = n^2 - 1 = 3 \quad (8)$$

for the electroweak force.

To sum up the insight of this section we say that while in scale relativity calculus is replaced by non-standard analysis and logarithmic scaling, in our approach we need only the golden mean scaling operation down scaling replaces differentiation and up scaling replacing integration. Before the end of this part however we give a very important and instructive down scaling which starts from the number of the first level of massless particles like quantum states in a transfinite Heterotic string theory. We can determine the Ambjorn-Loll [20] extremely important spectral dimension of quantum gravity D_s = 4.019999.

We know that the classical value of N_o in Heterotic strings is found from (504)(16) = 8064. This is actually the multiplication of the holographic boundary $|SL(2,7)| = 336$ with the instanton number n = 24. However, in the exact transfinite theory we have 336 → 336 + 16k and 24 → 26 + k. Therefore the exact is [4,5]

$$N_0 = (26+k)(336+k) = 8872.135962. \quad (9)$$

This is nothing but up scaling of the modular space $M(80)$ with dim $M(80)$ = 80 using $\frac{1}{\phi^5} = 11 + \phi^5$ of a fractal 11 dimensional M-theory space [4,16,17]

$$N_0 = (10)(80)(11+\phi^5) = 8872.135962. \quad (10)$$

Differentiating, *i.e.* scaling using ϕ sixteen times, one finds [4,20]

$$D_S = (N_0)\phi^{16} = 4.01999 \quad (11)$$

exactly as the value found by Ambjorn and Loll using an efficient computer [20]. It is really curious to see that digital computers are far more accurate than calculus when it comes to high energy physics. However, golden mean computers are even far more accurate and efficient than digital computers [4]. Next we look at a simple example of how Nottale's theory deals with scaling and non-standard analysis.

3.3. Some Examples from Scale Relativity Calculus

Let us start with the fundamental optimized coupling constant of scale relativity, namely unification coupling [3,7,8]

$$\bar{\alpha}_g = 4\pi^2 = 39.4784176. \quad (12)$$

To bring this value in line with numerical experiments as well as E-Infinity's exact prediction of the non-super symmetric grand unification of all fundamental forces except for gravity, we must realize the following and change things accordingly:

1) The factor 4 stands in reality for the four topological dimensions of space-time. It must therefore be changed to the fractal-Hausdorff dimension of the core of space-time, *i.e.* to that of a Hilbert 4D cube 4 + ϕ^3 = 4.2360679.

2) Second π^2 must be changed to one of its transfinite opposite numbers. In this case π^2 = 9.869604401 must be changed: $\pi^2 \to 10$

From the above we find

$$\begin{aligned} &\bar{\alpha}_g(\text{transfinitely exact}) \\ &= (4+\phi^3)(10) = 42 + 2k = 42.36067 \end{aligned} \quad (13)$$

where $\phi = \frac{\sqrt{5}-1}{2}$, $k = \phi^3\left(1-\phi^3\right) = 2\phi^5 = 0.1803398$.

There is a very simple and elementary way to show that this is the exact value as well as how to obtain the super symmetric $\bar{\alpha}_{gs}$ which we know to be $\bar{\alpha}_{gs} = 26 + k$. The value of inverse electromagnetic coupling at low energy $\bar{\alpha}_0 = 137 + k_0 = 137.0820393$ should be divided equally among the number of fundamental equations at a certain energy scale. When gravity is out and the electrical force and the magnetic force are counted as one force, then we have only 3 fundamental forces with a fractal weight due to the fractality of space-time equal to $3 + \phi^3 = 3.23606799$. The common coupling or unification grand unification coupling is thus

$$\bar{\alpha} = \frac{\bar{\alpha}_0}{3+\phi^3} = \frac{137}{3+\phi^3} = 42 + 2k \tag{14}$$

exactly as anticipated. Now if we admit gravity and count electrical force and magnetic force as two forces, then the number is 5 and the fractal weight is $5 + \phi^3$. Consequently the total unification inverse coupling of all fundamental forces becomes

$$\bar{\alpha} = \frac{\bar{\alpha}_0}{5+\phi^3} = 26 + k = 26.18033989. \tag{15}$$

Finally in the case of only 4 fundamental forces the unification coupling is given by

$$\bar{\alpha} = \frac{\bar{\alpha}_0}{4+\phi^3} = 32 + 2k = 32.18033989. \tag{16}$$

This $\bar{\alpha} = 32.18033989.$ is what we include approximately in our renormalization equation of unification using the logarithmic scaling as in Nottale's theory of scale relativity. We see this clearly from [18]

$$\bar{\alpha}_u = \bar{\alpha}_3 + \bar{\alpha}_4 + \delta \ln\left(\frac{M_u}{M_x}\right) \tag{17}$$

where $\bar{\alpha}_3 = 9$, $\bar{\alpha}_4 = \bar{\alpha}_{QG} = 1$, $M_u = 10^{16}$ GeV, $M_x = 91$ GeV, $\delta = 1/n$, *i.e.* $\delta = 1$ for non-super symmetric interaction, $\delta = 1/2$ for super symmetric interaction [18] and

$$\begin{aligned}\bar{\alpha}_0 &= \left(\bar{\alpha}_1\right)\left(\frac{1}{\phi}\right) + \left(\bar{\alpha}_2 = \frac{\bar{\alpha}_1}{2}\right) + \bar{\alpha}_3 + \bar{\alpha}_4 \\ &= 137 + k_0 = 137.082039325\end{aligned} \tag{18}$$

for $\bar{\alpha}_1 = 60$ and $\bar{\alpha}_2 = 30$. Now inserting in the logarithmic term one finds [18]

$$\ln\left(\frac{10^{16}\text{ GeV}}{91\text{ GeV}}\right) = 32.33050198 \approx \frac{\bar{\alpha}_0}{4+\phi^3} \tag{19}$$

exactly as anticipated. Here 10^{16} GeV is the mass of the GUT monopole and 91 GeV is the mass of the electroweak unification. In fact Nottale's scale relativity has generalized this logarithmic scaling and used Levy-Gillmann operators skillfully to achieve his result which although not exact, paved the way for our work and for the exceptionally beautiful work of Magueijo and Smolin [12,13] on varying speed of light theory (VSL). In the next section we will show how E-infinity as well as scale relativity can resolve the mysterious dark energy problem [1,2].

4. Resolution of the Missing Hypothetical Dark Energy Using Scale Relativity and E-Infinity

Scale relativity puts the running value of $\bar{\alpha}_0$ at 10^{16} GeV of scale relativity [3,7,8] for $\bar{\alpha}_{GUT} = 105$. Clearly at $\bar{\alpha}_{GUT}$ we have everything except gravity. Scaling 105 logarithmically and squaring it gives us now a measure for the error in Einstein's special relativity energy mass resolution when applied at ultra high energy and distances. That way we find the scaling exponent needed for $E = m_0 c^2$, namely

$$\begin{aligned}\lambda &= \frac{1}{\left(\ln \bar{\alpha}_{GUT}\right)^2} = \frac{1}{\left(4.65396036\right)} \\ &= \frac{1}{21.65934694} = 0.04616944.\end{aligned} \tag{20}$$

Einstein's energy-mass equation now reads as follows:

$$E = \gamma m_0 c^2 \tag{21}$$

where

$$\gamma = \frac{1}{21.65934694}.$$

The corresponding dark energy is therefore

$$E_{(\text{dark})} \cong \left(1 - \frac{1}{21.65934694}\right)(100) = 95.383\%. \tag{22}$$

Before giving an exact interpretation for this approximate result let us first revise the numerics. The value which should have been used for $\bar{\alpha}_{GUT}$ is (10) $\left(D_F^{11}\right)$ which means [4,5]

$$\begin{aligned}\bar{\alpha}_{GUT} &= (10)\left(\frac{1}{\phi^5}\right) = (10)(11.09016995) \\ &= 110.9016995.\end{aligned} \tag{23}$$

Logarithmic scaling and squaring then leads to

$$\begin{aligned}\frac{1}{\lambda} &= \left(\ln 110.9016995\right)^2 = \left(4.70864419\right)^2 \\ &= 22.17133038 \approx 22.\end{aligned} \tag{24}$$

The result is almost the exact one, namely (22 + k) where k = 0.18033989 as we can show using exact methods

In other words $\lambda \approx \frac{1}{22}$ is the reciprocal value of the non-visible "dark" dimension of our Bosonic section of the transfinite version of Heterotic string theory. That means for "dark" dimensions we have [9]

$$\begin{aligned} D_{(\text{dark})} &= \text{The total number of the dimensions} \\ &\quad -\text{space-time dimensions} \qquad (25) \\ &= (26+k)-(4) = 22+k = 22.18033939. \end{aligned}$$

E-infinity scaling reaches the exact result without logarithmic scaling. Let us first recall that the entire Heterotic superstrings dimensional hierarchy is readily found for $\bar{\alpha}_0$ for a Cooper pair as follows, starting from [4, 5,16-18]

$$\left(\frac{\bar{\alpha}_0}{2}\right)(\phi)^n = (68.54101966)(\phi)^n \qquad (26)$$

and setting $n = 1, 2, 3, \cdots$ one finds [4]

$$\begin{aligned} &42+2k = 42.36067977 \\ &26+k = 26.18033939 \\ &16+k = 16.18033939 \\ &10 \\ &6+k = 6.18033939 \\ &4-k = 3.819660122. \end{aligned}$$

Setting $X \pm k \approx X$ one finds the classical Heterotic string dimensional hierarchy 26, 16, 10, 6 and 4. This was a down scaling of $\frac{\bar{\alpha}_0}{2}$. Now the up scaling leads to the following $\left(\frac{\bar{\alpha}_0}{2}\right)\left(\frac{1}{\phi}\right)^n$. For $n = 1$ one finds [4]

$$\left(\frac{\bar{\alpha}_0}{2}\right)\left(\frac{1}{\phi}\right) = \left(11+\phi^5\right)(10) = 110.9016995 = \bar{\alpha}_{GUT}. \quad (27)$$

Dividing through all the five interactions using the $D_T = 5$ one finds [4]

$$\frac{\bar{\alpha}_{GUT}}{5} = 22+k = 22.18033989. \qquad (28)$$

This is of course the exact result and shows the high quality of accuracy in the Nottale method. Should we have used the fractal weight $5+\phi^3$ rather than 5 we would have found [4,5]

$$\frac{\bar{\alpha}_{GUT}}{5+\phi^3} = 21+k = 21.18033989. \qquad (29)$$

In the first case we look at an Einstein 4 dimensional space-time with 22 + k "dark" dimensions while in the second case we have a 5 dimensional Klein-Kaluza space-time with only 21 + k "dark" dimensions. Based on this analysis our tangible space is exactly four dimensional topologically and $4+\phi^3 = 4.23606799$ Hausdorffly. However it is the larger $11+\phi^5$ core of our space which encapsules the $4+\phi^3$ smaller core which decides on Hardy's quantum entanglement [6,14] being exactly $\frac{1}{11+\phi^5} = \phi^5$ and also decides on the reduction factor or the scaling exponent $\lambda = 2\phi^5 = \frac{1}{22+k}$ of Einstein's equation $E = m_0 c^2$. The scaled new quantum relativity or effective quantum gravity equation

$$E = \left(\frac{1}{2}\right)\left(\frac{1}{11+\phi^5}\right)\left(m_0 c^2\right) = \frac{m_0 c^2}{22+k} \qquad (30)$$

predicts that we have a missing dark energy of exactly $E_{(\text{dark})}$ = 95.49150281%, almost the same as in the approximate scale relativity analysis following Nottale's theory. This reduction could be interpreted in a variety of intuitive ways which will be discussed in the conclusion of the paper.

It is instructive for a deep understanding of the present work to ponder the implication of a comparison between Nottale's theory of scale relativity and El Naschie's E-infinity theory which is summarized in **Table 1**. In **Table 2**, we give another instructive comparison between working in the bulk and working with the holographic boundary to derive the scaling which elevates Einstein's special relativity equation to an effective quantum gravity equation.

5. Discussion

Following the picture adopted by Heterotic string theory compactified on a Calabi-Yau manifold, every point in our spacetime is joined to a Calabi-Yau 6 dimensional real manifold containing internal symmetry and compactified dimensions [19]. On this account we would have all in all (4)(6) = 24 dimensions and adding the string

Table 1. Comparison in calculating the Lorentz factor using scale relativity and E-infinity.

	Scale Relativity (Nottale)	E-infinity (El Naschie)
Grand unification coupling	$\left(\ln(5\bar{\alpha}_0)\right)^2 = 42.6411$ where $\bar{\alpha}_0 = 137.08203932$	$(5\bar{\alpha}_0)\frac{\phi}{10} = 42.3606$ $= (4+\phi^3)(10)$ where $\bar{\alpha}_0 = 137.08203932$
$\gamma \equiv$ Lorentz factor	$(\ln \bar{\alpha}_{GUT})^2$ $= (\ln(111))^2$ $= 22.1796$	$(\bar{\alpha}_0)\left(\frac{1}{\phi}\right)/10$ $= 22+k$ $= 26+k-4$ $= 22.18033$

Table 2. Comparison in predicting the Lorentz factor for dark energy step by step using the bulk and using the holographic boundary [4,5].

Bulk $\equiv \lvert E_8 E_8 \rvert = 496$	Holographic boundary $\equiv SL(2,C)$
$\frac{1}{\sqrt{\lvert E_8 E_8 \rvert - SU(3)SU(2)U(1)}}$	$\frac{R^{(4)} - D^4}{SL(2,7) + R(4) - D^4}$
$\frac{1}{\sqrt{496-12}}$	$\frac{20-4}{336+(20-4)}$
$\frac{1}{\sqrt{484}}$	$\frac{16}{336+16}$
$\frac{1}{\sqrt{(22)^2}}$	$\frac{1}{352/16}$
$\frac{1}{22}$	$\frac{1}{22}$

world sheet to it arrives at the 24 + 2 = 26 Bosonic dimension. These dimensions move in the opposite direction of another 16 Fermionic dimensions from which one finds 26 – 16 = 10 super symmetric dimensions. However in our transfinite version of Heterotic strings we do not need the 2 dimensional world sheet to arrive at 26. This is because the Hausdorff dimension of our core space is not 4 but $4+\phi^3 = 4.23606799$ and the 6 dimensions of the Calabi-Yau manifold [19] are not 6 but 6 + k = 6.18033898. Consequently the total dimension is given by

$$D_s(\text{Heterotic}) = (4+\phi^3)(6+k) = 26+k = 26.18033989. \tag{31}$$

Now Einstein's energy-mass equation was based on a mere 4 dimensional flat non-fractal, non-fuzzy Euclidean manifold. Subtracting these 4 dimensions from D_S = 26 + k we are left with 26 + k − 4 = 22 + k hidden dimensions.

This is a wonderfully simple and intuitive picture and is numerically identical with our analysis which was based on superficially completely different theories such as Nottale's scale relativity [3,7,8] or E-infinity theory [3-6]. It is now clear that $E = m_0 c^2$ must be scaled using

$$\lambda = \gamma = \frac{1}{22+k} = 0.0450849718 \tag{32}$$

which fully agrees with the measurement of WMAP and supernova analysis by predicting that exactly 95.4915028% of the energy of the cosmos must be dark energy [1,2].

To gain a deeper insight into the roots of scale relativity we should apply the original energy mass relation of scale relativity directly to the problem of dark energy. Even a fleeting glance at these equations reveal that they are in almost one to one correspondence with Einstein's equation and are also the inspiration to Magueijo-Smolin's beautiful energy-mass Planck length invariant equation. Following Nottale's notation we have [7,8]

$$E = (m_0 c^2)(\gamma_N) = (m_0 c^2)\left(\frac{1}{2}\right)\left(\ln\frac{\lambda_0}{\lambda}\right) \Big/ \sqrt{1-\left(\ln\frac{\lambda_0}{\lambda}\right)^2 \Big/ \left(\ln\frac{\lambda_0}{\Lambda}\right)^2}. \tag{33}$$

Here Λ is the Plank length $(\approx 10^{-33}\ \text{cm})$. From Sigalotti's analysis of the classical relativistic transition we know that when we set $(m_0 c^2) = 1$, then one finds that

$$E = \left(\frac{1}{2}\right)(\phi^6) \Big/ \sqrt{1-(\phi^{12}/\phi^{11})}\,(m_0 c^2) = \left(\frac{1}{2}\right)\frac{\phi^6}{\sqrt{1-\phi}}(m_0 c^2) = \frac{\phi^5}{2}. \tag{34}$$

Exactly as in the previous analysis which means a reduction of 95.49150281% in energy which matches almost exactly the missing dark energy measurements [1, 2].

There is even an outrageously simple way of arriving at $\lambda \cong \frac{1}{22}$ from semi classical considerations as suggested by El Naschie. The argument goes as follows. Special relativity is a one degree of freedom theory where the photon is the only elementary particle involved. The standard model however has 12 elementary photon-like particles. Thus we have here a factor of 12 − 1= 11 involved. Inserting in Newton's kinetic energy we find our previous result

$$E = \left(\frac{1}{2}\right)\left(\frac{1}{11}\right) m_0 c^2 = \frac{1}{22} m_0 c^2 = \lambda m_0 c^2. \tag{35}$$

6. Conclusions

Scale relativity gives yet another very constructive mental picture to understand what the missing dark energy means apart of answering the quantum question quantitatively with remarkable accuracy. Scale relativity is completely embedded in the scale invariance of fractal geometry [3-8]. We do not need to go from general relativity via quantum mechanics to arrive at quantum gravity. We could do the same by starting with special relativity however after freeing it from traditional prejudice and putting it in the right space-time setting, namely fractal geometry. The Lorentz factor does the rest and one finds that Einstein's celebrated equation maintains its form and the change is a mere down scaling by a minimal Lorentz factor equal to half of the value of Hardy's quantum entanglement [6,14]. In turn this causes a reduction of almost 95.5% of the classically predicted energy. This is what we call missing dark energy. It is the energy which would have been there if the space-time fabric were smooth, continuous and without holes. However actual space-time at quantum scales and surprisingly again at intergalactic scale displays a wild Cantorian fractal geometry and topology. It is a T-duality which we saw in the unification program at the Plank length, yet this time the surprising quantum effect of entanglement is showing its power at the Hubble length scale. The main equation obtained in the present work which is

$$E_{QR} = \lambda E_S = \lambda m_0 c^2 \quad (36)$$

is gauge invariant in the widest sense possible, meaning that it is almost invariant to the use of any mathematically and physically reasonably meaningful theory. It is a very robust result not affected by minor details of theoretical modeling. Thus we may show here in the conclusion what on reflection should have been presented in the introduction at the very beginning:

Special relativity implies three strange effects [11]:

1) length contraction;
2) time delineation;
3) mass increase.

All these classically feeble effects become noticeable only as the speed approaches the speed of light c [11]. We handle this semi-classically, *i.e.* using common sense by introducing a boost $(1+\beta)$ and anti boost $(1-\beta)$. Thus we have 1) $X \to (1-\beta)X$, where X is space co-ordinate, 2) $t \to (1+\beta)t$, where t is ordinary time and 3) $m_0 \to (1+\beta)m_0$, where m_0 is non-relativistic mass [11]. Inserting in Newton's kinetic energy $E = \left(\frac{1}{2}\right)\left(m_0 v^2\right)$ we find

$$E_{QR} = \left(\frac{1}{2}\right)(1+\beta)\left(\frac{1-\beta}{1+\beta}\right)^2 \left(m_0 c^2\right). \quad (37)$$

Setting $\beta = \frac{2}{\sqrt{5}+1}$ we find our previous result $E \cong \frac{1}{22} m_0 c^2$. Thus in one stroke we reconciled and fused together classical mechanics with relativity and quantum mechanics via the non-classical geometry of fractals [3-8]. This is magically beautiful.

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