

Dirac-Born-Infeld-Einstein Theory with Weyl Invariance

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ABSTRACT

Weyl invariant gravity has been investigated as the fundamental theory of the vector inflation. Accordingly, we consider a Weyl invariant extension of Dirac-Born-Infeld type gravity. We find that an appropriate choice of the metric removes the scalar degree of freedom which is at the first sight required by the local scale invariance of the action, and then a vector field acquires mass. Then non-minimal couplings of the vector field and curvatures are induced. We find that the Dirac-Born-Infeld type gravity is a suitable theory to the vector inflation scenario.

Keywords: Modified Theories of Gravity; Weyl Invariance; Inflation

1. Introduction

The cosmological inflation is proposed as some resolutions for the important cosmological problems, e.g. the flatness, horizon and monopole problems. Most of successful models are based on classical scalar fields, although we have not observed such scalar bosons associated with the field.

The inflation can also be caused by other type of fields. The vector inflation has been proposed by Ford [1] and some authors [2-4]. It is emphasized that the massive vector field should non-minimally couple to gravity in such models [1-4].

The reason why the non-minimal coupling is important is as follows. Suppose the equation of motion for the vector field is given by

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} F^{\mu\nu}) - \left(m^2 - \frac{R}{6}\right) A^\nu = 0. \quad (1.1)$$

For the background field, we assume¹

$$ds^2 = -dt^2 + a^2(t) dx^2, \quad (1.2)$$

and $A_i (i=1,2,3)$ depends only on t and $A_0 \equiv 0$. Then we define $B_i \equiv A_i/a$, (1.1) becomes

$$\ddot{B}_i + 3\frac{\dot{a}}{a}\dot{B}_i + m^2 B_i = 0, \quad (1.3)$$

which is very similar to the equation for a homogeneous

scalar field in the Friedmann-Lemaître-Robertson-Walker universe. Moreover, the energy density is expressed as $\sim \dot{B}_i^2 + m^2 B_i^2$, which is also similar to the one for the scalar field. Thus, the slow evolution of the effective scalar field B_i can occur in the approximately isotropic inflating universe.

We have studied [5] Weyl invariant gravity [6-39] as a candidate for the theoretical model of the vector inflation. We found that the choice of the frame yields the mass of the Weyl gauge field, but the non-minimal coupling term is lost [5]. We come to the conclusion that we need further generalization of the gravitational theory.

In the different context, Deser and Gibbons considered Dirac-Born-Infeld (DBI)-Einstein theory [40] almost a decade ago, whose Lagrangian density is of the following type

$$\pm \sqrt{-\det(g_{\mu\nu} \pm \alpha R_{\mu\nu})}, \quad (1.4)$$

where $R_{\mu\nu}$ is the Ricci tensor and the α is a constant. Originally, electromagnetism of the DBI type has been considered as a candidate of the nonsingular theory of electric fields. Therefore the Dirac-Born-Infeld-Einstein theory as the highly-nonlinear theory is also expected as a theory of gravity suffered from no argument of singularity. The studies on the theory have been done by many authors [41-50]. Because of the nonlinearity in this theory, we expect the extension as the theory of gravity which realizes a successful vector inflation.

Consider the Weyl invariant D -dimensional extension of the Ricci curvature (see the next section) is

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¹Of course, only the vector field as the source cannot lead to the exactly isotropic expansion.

$$\tilde{R}_{\nu\sigma}[g, A] \equiv R_{\nu\sigma} + F_{\nu\sigma} - \left[(D-2)\nabla_\sigma A_\nu + g_{\nu\sigma}\nabla_\mu A^\mu \right] + (D-2)(A_\nu A_\sigma - A_\lambda A^\lambda g_{\nu\sigma}). \tag{1.5}$$

By simple replacement of the Ricci tensor by the Weyl invariant tensor in the action (1.4), the expansion

$$\sqrt{\det(1+A)} = 1 + \frac{1}{2}\text{tr}A + \frac{1}{8}((\text{tr}A)^2 - 2\text{tr}A^2) + \dots, \tag{1.6}$$

yields the terms $RA_\mu A^\mu$ and $R_{\mu\nu}A^\mu A^\nu$ and so on as well as R and $F_{\mu\nu}F^{\mu\nu}$. Other Weyl invariant terms are necessary, because the metric tensor must be combined with a scalar field which compensates the dimensionality. After the frame choice, the freedom of the scalar field is eaten by the vector field, then, the presence of the non-minimal terms mentioned above is still realized².

In the next section, we review the Weyl invariant gravity with the vector field [11-14,16-21,25-37]. The expression (1.4) is generalized to the Weyl invariant one. The Lagrangian for a Weyl-invariant DBI gravity is proposed in Section 3. In Section 4, the necessity condition for the vector inflation is investigated. In Section 5, another possible inflationary scenario is provided. The last section is devoted to the summary and prospects.

2. Weyl's Gauge Gravity Theory

In this section, we review the Weyl's gauge transformation to construct the gauge invariant Lagrangian.

Consider the transformation of metric (in D dimensions)

$$g_{\mu\nu} \rightarrow g'_{\mu\nu} = e^{2\Lambda(x)} g_{\mu\nu}, \tag{2.1}$$

where $\Lambda(x)$ is an arbitrary function of the coordinates x^μ .

We can define the field with weight $d = -(D-2)/2$ which transforms as

$$\Phi \rightarrow \Phi' = e^{-(D-2)\Lambda(x)/2} \Phi. \tag{2.2}$$

In order to construct the locally invariant theory, we consider the covariant derivative of the scalar field

$$\tilde{\partial}_\mu \Phi \equiv \partial_\mu \Phi - \frac{D-2}{2} A_\mu \Phi, \tag{2.3}$$

where A_μ is a Weyl's gauge invariant vector field.

Under the Weyl gauge field transformation

$$A_\mu \rightarrow A'_\mu = A_\mu - \partial_\mu \Lambda(x), \tag{2.4}$$

we obtain the transformation of the covariant derivative of the scalar field as

$$\tilde{\partial}_\mu \Phi \rightarrow e^{-(D-2)\Lambda(x)/2} \tilde{\partial}_\mu \Phi. \tag{2.5}$$

The field strength of the vector field is given by

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{2.6}$$

²Note that $f(\tilde{R}[g, A])$ does not bring about substantial nonminimal couplings [5].

which is gauge invariant as

$$F_{\mu\nu} \rightarrow F'_{\mu\nu} = F_{\mu\nu}. \tag{2.7}$$

The modified Christoffel symbol is defined as

$$\tilde{\Gamma}^{\lambda}_{\mu\nu} \equiv \frac{1}{2} g^{\lambda\sigma} (\tilde{\partial}_\mu g_{\sigma\nu} + \tilde{\partial}_\nu g_{\sigma\mu} - \tilde{\partial}_\sigma g_{\mu\nu}), \tag{2.8}$$

where $\tilde{\partial}_\mu g_{\sigma\nu} \equiv \partial_\mu g_{\sigma\nu} + 2A_\mu g_{\sigma\nu}$. The modified curvature is given as follows:

$$\tilde{R}^{\mu}_{\nu\rho\sigma}[g, A] \equiv \partial_\rho \tilde{\Gamma}^{\mu}_{\nu\sigma} - \partial_\sigma \tilde{\Gamma}^{\mu}_{\nu\rho} + \tilde{\Gamma}^{\mu}_{\lambda\rho} \tilde{\Gamma}^{\lambda}_{\nu\sigma} - \tilde{\Gamma}^{\mu}_{\lambda\sigma} \tilde{\Gamma}^{\lambda}_{\nu\rho}. \tag{2.9}$$

The Ricci curvature in the Weyl invariant version is

$$\begin{aligned} \tilde{R}_{\nu\sigma}[g, A] &\equiv \tilde{R}^{\mu}_{\nu\mu\sigma}[g, A] \\ &= R_{\nu\sigma} + F_{\nu\sigma} + (\nabla_\sigma A_\nu - D\nabla_\sigma A_\nu - g_{\nu\sigma}\nabla_\mu A^\mu + \nabla_\sigma A_\nu) \\ &\quad + [A^\mu (g_{\nu\sigma} A_\mu - g_{\nu\mu} A_\sigma) - A_\nu (A_\sigma - DA_\sigma) \\ &\quad - A_\lambda A^\lambda (Dg_{\nu\sigma} - g_{\nu\sigma})] \\ &= R_{\nu\sigma} + F_{\nu\sigma} - [(D-2)\nabla_\sigma A_\nu + g_{\nu\sigma}\nabla_\mu A^\mu] \\ &\quad + (D-2)(A_\nu A_\sigma - A_\lambda A^\lambda g_{\nu\sigma}), \end{aligned} \tag{2.10}$$

where ∇ denotes the usual generally covariant derivative. Note that under the gauge transformation

$$\tilde{R}_{\nu\sigma}[g, A] \rightarrow \tilde{R}_{\nu\sigma}[g', A'] = \tilde{R}_{\nu\sigma}[g, A]. \tag{2.11}$$

3. Weyl Invariant Lagrangian

Although we can use the Weyl invariant Ricci tensor $\tilde{R}_{\mu\nu}$ in the DBI gravity, we should note that the metric tensor in the action is not Weyl invariant (which is shown in (2.1)). Thus, we use a combination $\Phi^{4/(D-2)} g_{\mu\nu}$ instead of the metric tensor. The scalar Φ compensates the dimension of the metric. Now the use of $\tilde{R}_{\mu\nu}$ and $\Phi^{4/(D-2)} g_{\mu\nu}$ in the DBI type action leads to the theory of gravity, a vector field, and unexpectedly, a scalar field.

The introduction of the compensating scalar field tells us the action is far from general one. The monomial of the type of the kinetic term, in other words, two coordinate derivatives of the scalar field can be considered, while the curvature includes also two derivatives with no contraction. The possible monomials are

$$\Phi^{-2} \tilde{\partial}_\mu \Phi \tilde{\partial}_\nu \Phi \text{ and } \tilde{\nabla}_\mu (\Phi^{-1} \tilde{\partial}_\nu \Phi). \tag{3.1}$$

Another notice is in order. The decomposition of a rank two tensor shows that there are three irreducible ones; an anti-symmetric tensor, a traceless symmetric tensor and a trace part.

Now, we must introduce the following independently Weyl invariant tensors into the determinant in the DBI theory:

$$\begin{aligned} &\Phi^{4/(D-2)} g_{\nu\sigma}, \tilde{R}_{\nu\sigma}^S [g, A], \tilde{R}[g, A] g_{\nu\sigma}, F_{\nu\sigma}, \\ &\Phi^{-2} \tilde{\partial}_\nu \Phi \tilde{\partial}_\sigma \Phi, \Phi^{-2} g^{\lambda\mu} \tilde{\partial}_\lambda \Phi \tilde{\partial}_\mu \Phi g_{\nu\sigma}, \\ &\tilde{\nabla}_\sigma (\Phi^{-1} \tilde{\partial}_\nu \Phi) + \tilde{\nabla}_\nu (\Phi^{-1} \tilde{\partial}_\sigma \Phi), \tilde{\nabla}^\mu (\Phi^{-1} \tilde{\partial}_\mu \Phi) g_{\nu\sigma}, \end{aligned} \tag{3.2}$$

where

$$\begin{aligned} &\tilde{R}_{\nu\sigma}^S [g, A] \\ &= R_{\nu\sigma} - \left[\frac{D-2}{2} (\nabla_\sigma A_\nu + \nabla_\nu A_\sigma) + g_{\nu\sigma} \nabla_\mu A^\mu \right] \\ &+ (D-2) (A_\nu A_\sigma - A_\lambda A^\lambda g_{\nu\sigma}), \end{aligned} \tag{3.3}$$

and

$$\begin{aligned} &\tilde{R}[g, A] \equiv g^{\nu\sigma} \tilde{R}_{\nu\sigma}^S [g, A] \\ &= R - 2((D-1)\nabla_\mu A^\mu - (D-1)(D-2)A_\mu A^\mu). \end{aligned} \tag{3.4}$$

We choose those as symmetric tensors are not traceless³.

Our model of Weyl invariant DBI gravity is described by the Lagrangian density

$$L = -\sqrt{-\det M_{\mu\nu}} + (1-\lambda) \sqrt{-\det (\Phi^{4/(D-2)} g_{\mu\nu})}, \tag{3.5}$$

with

$$\begin{aligned} M_{\mu\nu} &\equiv \Phi^{4/(D-2)} g_{\mu\nu} - \alpha_1 \tilde{R}_{\mu\nu}^S [g, A] - \alpha_2 \tilde{R}[g, A] g_{\mu\nu} \\ &+ \beta F_{\mu\nu} + \gamma_1 \Phi^{-2} \tilde{\partial}_\mu \Phi \tilde{\partial}_\nu \Phi + \gamma_2 \Phi^{-2} g^{\lambda\sigma} \tilde{\partial}_\lambda \Phi \tilde{\partial}_\sigma \Phi g_{\mu\nu} \\ &- \gamma_3 \left[\tilde{\nabla}_\mu (\Phi^{-1} \tilde{\partial}_\nu \Phi) + \tilde{\nabla}_\nu (\Phi^{-1} \tilde{\partial}_\mu \Phi) \right] \\ &- \gamma_4 g^{\lambda\sigma} \tilde{\nabla}_\lambda (\Phi^{-1} \tilde{\partial}_\sigma \Phi) g_{\mu\nu}, \end{aligned} \tag{3.6}$$

where $\alpha_1, \alpha_2, \beta, \gamma_1, \gamma_2, \gamma_3, \gamma_4$ and λ are dimensionless constants⁴.

Furthermore the Lagrangian density can be expressed by the new metric conformally related to the original one and new variables. Here we choose

$$\hat{g}_{\mu\nu} \equiv f^{-2} \Phi^{4/(D-2)} g_{\mu\nu}, \tag{3.7}$$

and

$$\hat{A}_\mu \equiv A_\mu - \frac{2}{D-2} \partial_\mu \ln \Phi. \tag{3.8}$$

Note that a mass scale f was introduced here. By using the new metric and vector field, we rewrite the each term in the determinant of the Lagrangian as

³Judging from the number of fields and derivatives, the term $\Phi^{-4/(D-2)} g^{\mu\nu} F_{\nu i} F_{\sigma\mu}$ is allowed in the same order. But this term is different from others in the point that it includes two kinds of fields except for the metric. Therefore we discarded this marginally possible term here.

⁴If we demand that the terms with lowest derivatives in the expansion (1.6) look like the Lagrangian of scalar-tensor theory, we must choose as $\alpha_1 + 4\alpha_2 > 0$ and $\gamma_1 + 4\gamma_2 + 4\gamma_3 + 8\gamma_4 > 0$, for $D = 4$.

$$\begin{aligned} &\Phi^{4/(D-2)} g_{\mu\nu} = f^2 \hat{g}_{\mu\nu}, \tilde{R}_{\nu\sigma}^S [g, A] = \tilde{R}_{\nu\sigma}^S [\hat{g}, \hat{A}], \\ &\tilde{R}[g, A] g_{\nu\sigma} = \tilde{R}[\hat{g}, \hat{A}] \hat{g}_{\nu\sigma}, F_{\nu\sigma} = \hat{F}_{\nu\sigma}, \\ &\Phi^{-2} \tilde{\partial}_\nu \Phi \tilde{\partial}_\sigma \Phi = \left(\frac{D-2}{2} \right)^2 \hat{A}_\nu \hat{A}_\sigma, \\ &\Phi^{-2} g^{\lambda\mu} \tilde{\partial}_\lambda \Phi \tilde{\partial}_\mu \Phi g_{\nu\sigma} = \left(\frac{D-2}{2} \right)^2 \hat{g}^{\lambda\mu} \hat{A}_\lambda \hat{A}_\mu \hat{g}_{\nu\sigma}, \\ &\tilde{\nabla}_\sigma (\Phi^{-1} \tilde{\partial}_\nu \Phi) + \tilde{\nabla}_\nu (\Phi^{-1} \tilde{\partial}_\sigma \Phi) \\ &= (D-2) \left[-\frac{1}{2} (\hat{\nabla}_\nu \hat{A}_\sigma + \hat{\nabla}_\sigma \hat{A}_\nu) + 2\hat{A}_\nu \hat{A}_\sigma - \hat{g}^{\lambda\mu} \hat{A}_\lambda \hat{A}_\mu \hat{g}_{\nu\sigma} \right], \\ &g^{\lambda\mu} \tilde{\nabla}_\lambda (\Phi^{-1} \tilde{\partial}_\mu \Phi) g_{\nu\sigma} \\ &= (D-2) \left[-\hat{\nabla}^\mu \hat{A}_\mu - (D-2) \hat{g}^{\lambda\mu} \hat{A}_\lambda \hat{A}_\mu \right] \hat{g}_{\nu\sigma}. \end{aligned} \tag{3.9}$$

We now can write $M_{\mu\nu}$ as

$$\begin{aligned} M_{\mu\nu} &= f^2 g_{\mu\nu} - \alpha_1 R_{\mu\nu} - \alpha_2 R g_{\mu\nu} + \beta F_{\mu\nu} \\ &+ \gamma'_1 A_\mu A_\nu + \gamma'_2 g^{\rho\sigma} A_\rho A_\sigma g_{\mu\nu} \\ &+ \gamma'_3 (\nabla_\mu A_\nu + \nabla_\nu A_\mu) + \gamma'_4 \nabla^\rho A_\rho g_{\mu\nu}, \end{aligned} \tag{3.10}$$

where the ‘‘hat’’s are dropped and dimensionless constants are

$$\begin{aligned} \gamma'_1 &= -(D-2)\alpha_1 + \left(\frac{D-2}{2} \right)^2 \gamma_1 - 2(D-2)\gamma_3, \\ \gamma'_2 &= (D-2)\alpha_1 + (D-1)(D-2)\alpha_2 \\ &+ \left(\frac{D-2}{2} \right)^2 \gamma_2 + (D-2)\gamma_3 + (D-2)^2 \gamma_4, \\ \gamma'_3 &= \frac{D-2}{2} (\alpha_1 + \gamma_3), \\ \gamma'_4 &= \alpha_1 + 2(D-2)\alpha_2 + (D-2)\gamma_4. \end{aligned}$$

We can rewrite the Lagrangian as

$$\begin{aligned} L &= -\sqrt{-\det M_{\mu\nu}} + (1-\lambda) f^D \sqrt{-g} \\ &= -\sqrt{-g} \sqrt{\det M_\nu^\mu} + (1-\lambda) f^D \sqrt{-g}. \end{aligned} \tag{3.11}$$

This is the candidate Lagrangian for the vector inflation.

4. Cosmology of Weyl’s Gauge Gravity

In this section, we apply our Weyl invariant DBI theory of gravity to cosmology in four dimensions ($D = 4$).

We take the metric for the homogeneous flat universe as

$$ds^2 = -dt^2 + a_1^2(t) dx^2 + a_2^2(t) dy^2 + a_3^2(t) dz^2 \tag{4.1}$$

and, moreover, we assume the approximate isotropy $a_1 \approx a_2 \approx a_3 = a(t)$.

We consider that only $A_1(t)$ is homogeneously evolving, and $A_2 = A_3 = A_0 = 0$.

By these ansätze, we look for the condition that the vector field behaves much like a scalar field at classical homogeneous level. Substituting the ansätze, we find

$$M_0^0 = f^2 - 3\alpha_1 \frac{\ddot{a}}{a} - 6\alpha_2 \left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 \right] + \gamma_2' \frac{1}{a^2} A_1^2, \quad (4.2)$$

$$M_1^0 = -\beta \dot{A}_1 - \gamma_3' \left(\dot{A}_1 - 2 \frac{\dot{a}}{a} A_1 \right), \quad (4.3)$$

$$M_0^1 = -\beta \frac{\dot{A}_1}{a^2} + \gamma_3' \frac{1}{a^2} \left(\dot{A}_1 - 2 \frac{\dot{a}}{a} A_1 \right), \quad (4.4)$$

$$M_1^1 = f^2 - \alpha_1 \left[\frac{\ddot{a}}{a} + 2 \left(\frac{\dot{a}}{a} \right)^2 \right] - 6\alpha_2 \left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 \right] + (\gamma_1' + \gamma_2') \frac{1}{a^2} A_1^2, \quad (4.5)$$

$$M_2^2 = M_3^3 = f^2 - \alpha_1 \left[\frac{\ddot{a}}{a} + 2 \left(\frac{\dot{a}}{a} \right)^2 \right] - 6\alpha_2 \left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a} \right)^2 \right] + \gamma_2' \frac{1}{a^2} A_1^2. \quad (4.6)$$

After some calculations, we can subtract the part of the Lagrangian which includes bilinear and higher-order of the vector field A_1 . We find that if the parameters are chosen as

$$\beta^2 = \frac{1}{2} (5\alpha_1 \gamma_1' + 12\alpha_2 \gamma_1' + 12\alpha_1 \gamma_2' + 48\alpha_2 \gamma_2'), \quad (4.7)$$

and

$$(\gamma_3')^2 = -\frac{1}{2} \alpha_1 \gamma_1', \quad (4.8)$$

the vector-field part becomes

$$a^3 \left[\frac{1}{2} (\beta^2 - \gamma_3'^2) \dot{B}_1^2 - \frac{f^2}{2} (\gamma_1' + 4\gamma_1') B_1^2 - \frac{1}{8} (-\gamma_1'^2 + 4\gamma_1' 4\gamma_2' + 8\gamma_2'^2) B_1^4 + \dots \right], \quad (4.9)$$

where $B_1 = A_1/a$.

A simple case is realized when $\alpha_2 = \gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0$, or these parameter take small values in comparison with α_1 . Then the parameter is α_1 only. Equations (4.7) and (4.8) tell us $\gamma_1' = -\gamma_2' = -2\alpha_1$, $\gamma_3' = \alpha_1$ and $\beta^2 = 7\alpha_1^2$. In this case, this is so simple that the effective mass for B_1 may be large. The tuning is possible; say, the choice of γ_4 does not affects (4.8) and makes the change in the effective mass.

An elaborate tuning may give the potential which induces the chaotic inflation [51]. In the next section, however, we show another simple inflation scenario.

5. A Simple Cosmological Scenario

The chaotic inflation in the model can occur by tuning of the parameters. We should remember that the model involves the higher-derivative gravity. Therefore another kind of inflation is worth to be considered.

First let us suppose the flat space. Then the potential, or the energy density for the constant B_1 , can be easily written down as

$$V = \sqrt{(f^2 + \gamma_2' B_1^2)^3 (f^2 + (\gamma_1' + \gamma_2') B_1^2)}. \quad (5.1)$$

Although other choices are possible, we consider here a simple choice as $\gamma_1' = 0$ and $\gamma_2' < 0$ ⁵. In this case, unfortunately, the previous conditions (4.7, 4.8) cannot be satisfied simultaneously, because $\alpha_1 + 4\alpha_2 > 0$ for the positive coefficient of the Einstein-Hilbert term in the action. Then the potential is

$$V = (f^2 - |\gamma_2'| B_1^2)^2. \quad (5.2)$$

This is the simplest potential. In the true vacuum, the vector field “condensates” and a “natural” choice $\lambda = 1$ leads to vanishing cosmological constants⁶!

This simplest version also has an inflationary phase. That is, for $B_1 = 0$, the scale factor behaves as $a(t) \approx e^{Ht}$ where $H^2 = f^2 / (3(\alpha_1 + 4\alpha_2))$.

Unfortunately, this phase is stabilized by the non-minimal coupling between curvatures and the vector field, because the effective potential in this phase becomes

$$V = \sqrt{|\gamma_2'|^2 B_1^4 (|\gamma_2'|^2 B_1^4 + 4(\gamma_3')^2 H^2 B_1^2)}. \quad (5.3)$$

The exit of the de Sitter phase is problematic, like the other higher-derivative models. Though the additional matter fields may play important roles, we will perform further study on them elsewhere.

6. Summary and Outlook

The Weyl invariant DBI gravity is a candidate for a model which causes an inflationary universe. If the vector inflation can explain the possible anisotropy in the early universe, we may seriously investigate the Weyl invariant DBI gravity.

Here we examined slow development of the massive vector field. The inflation along with a fast evolution is shown to be possible in the DBI inflation, where the scalar degrees of freedom which originate from string (field) theory or D brane theory [52,53]. The similar scenario is feasible in our model, though the higher-derivatives make the detailed analysis difficult. Anyway, numerical calcu-

⁵Note that γ_2' can be tuned by take an appropriate value for γ_4 .

⁶The parameters γ' s can be taken to be sufficiently small so that no “antigravity” emerges.

lations and large simulations will be needed to understand the minute meaning of the Weyl invariant DBI gravity, because the local inhomogeneity in the spatial directions as well as the strength of vector fields is important for thorough understanding in the early cosmology.

Finally, we think that some marginally related subjects are in order. The higher-dimensional cosmology in the Weyl invariant DBI gravity is worth studying because of its rich content. Incidentally, DBI gravity in three dimensions is eagerly studied [54-57], which is related to New Massive Gravity [58,59]. We think that the Weyl invariant extension of the lower-dimensional theory is also of much mathematical interest.

7. Note Added

After completing this manuscript, we become aware of the paper “Higgs mechanism for New Massive gravity and Weyl invariant extensions of higher derivative theories” by Dengiz and Tekin [60]. They investigated a Weyl-invariant DBI gravity in three dimensions.

We also become aware of two recent papers about the cosmology of Weyl invariant theory [61,62].

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