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Energetic Nature of Rigidity

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ABSTRACT

The work is to present the energetic nature of the rigidity. It starts with the definition by introducing the notion of sensual magnitudes with the pyramidal structure of all surrounding magnitudes known by a human being. Next the selection of the subject is provided in view of a smooth categorization of magnitudes describing the reality. The adequate description of the considered mechanical phenomenon is presented by formulating general stiffness characteristics. There are several characteristics analyzed, both functional and parametric. An essential, quite a new one is the characteristic of stiffness energy measure which is the stiffness potential. The proper and gained stiffness potentials situated on stable and unstable potential fields have been analyzed. An example of using of this theory to practice is given. It has been referred to a cylindrical grinder case. The presented theory allowed describing the entire stiffness characteristics, including its initial very essential course which has been usually, though inequitably, extrapolated by a straight line segment coming out of zero point with zero coordinates.

Keywords: Rigidity; Coefficient of Stiffness; Stiffness Energy; Proper Stiffness Energy; Gained Stiffness Energy; Potential Field; Proper Stiffness Potential; Gained Stiffness Potential; Rigidity Work; Rigidity Force; Deflection

1. Introduction

The rigidity of solid or a material body system is the resistance to its deformation. In case of shape deformations the shape stiffness is considered; whereas in case of contact strains it should be referred to the contact stiffness.

Thus the stiffness is the magnitude which may be determined qualitatively by engaging senses (of touch and sight), forming in fact sensual, not a physical magnitude. Therefore the stiffness may be determined as small, big, very big, or immense. A determined measure may be ascribed to this sensual magnitude/stiffness. That measure is the coefficient of stiffness and it is a physical magnitude. It may be determined by measuring the component magnitudes and then calculating in accordance to a proper formula.

That physical magnitude, the coefficient of stiffness k, is defined as the first derivative (derivative of the first order) of the rigidity force S against the deflection f, then

$$k = \frac{\mathrm{d}S}{\mathrm{d}f} \tag{1}$$

This work determines generally the stiffness, by introducing the notion of the coefficient of stiffness; it does not divide it into springy, flexible, elastic, and plastic rigidity, or just flexibility, elasticity, plasticity. The considerations are provided with the load of system where no firm deformations occur.

Until now three categories of magnitudes have been separated, namely: sensual, physical, and material. There are also intellectual magnitudes, and the considered stiffness energy has such a character.

One should admit that right order/arrangement of all these magnitudes is of importance. It was considered also in the previous works [1-3]. The structure of these magnitudes: intellectual I, sensual S, physical P, and material M, is significant. It may be presented in the form of a pyramid (**Figure 1**), as this kind of lump presents terminological stratification in the best way. One could say this figure presents pyramidal, laminar structure of magnitudes of the external *human world*.

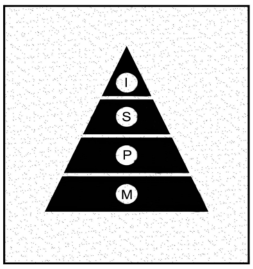

Figure 1. Pyramidal structure of magnitudes of the external human world [1-3].

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2. The Subject of Formulation

The selection of the subject is fully justified. First of all there is no correct terminological description of the phenomenon of shape deformation. To perform it in a right way, first a separation of all magnitudes is needed, *i.e.* intellectual, sensual, physical, and material, providing to: stiffness energy, rigidity, and stiffness measure of the described material system. One may refer now to the examples of material objects concerning their rigidity.

The stiffness (reversal to flexibility) is an essential feature of the system material elements, their connections, joints (kinematic pairs), as well as of each separate element. Dependent on the use of such a system (device), its stiffness is in different value ranges. The proper (initial) rigidity, with the force as a measure, is of importance for a link, node, and all material system.

Let us focus on the machine tools to evaluate their resistance to deformation of the component assemblies. This resistance, covering the rigidity, should be as high as possible. It is of special attention in the machine tools to be used for fine machining.

In [4] the rigidity of the working spaces of machining centres was assessed. The indexes determining the effect of the carrying machine tool system on the machining accuracy, related to the changing localization of cutting zone, were given. The evaluation results, as mentioned there, may be used to introduce changes in the design and technological process, to improve the rigidity of the machine tool in its working space.

The work [5] reveals that there are some methods to eliminate negative effect of the machine tool flexibility and particular its elements. They are: 1) selection of such a machining scheme which does not require a big rigidity of the technological system, 2) using some compensational systems which allow to decrease or compensate deformations of the machine tool elements, and 3) increasing the proper rigidity of the design components.

The contact rigidity in the grinding zone is a very important feature of grinders. The work [6] presents results of the studies of dependence of that feature on the quotient of velocities (ratio of the grinding wheel velocity to the workpiece velocity).

The study results presented in [7] show that the real thickness of the cut layer, while in-depth turning with the traverse feed, depends on the deformation of carrying assemblies of a machine tool. A mathematical model, to determine the cut layer regarding the flexibility of the machine tool, was derived.

The work [8] presents a theoretical model, describing the influence of a grinder rigidity on the cohesion of ground elements made of sintered Si_3N_4. The described verifying actions showed that there is a critical rigidity while grinding that material, distinctly dependent on the depth of grinding.

An extremely essential feature of a machine tool is its proper rigidity. It was shown in [9] that a high proper rigidity may be even replaced by a foundation. That idea was realized in relation to the machining centre to machine the workpieces of big overall dimensions (1600 × 1600 × 1200 mm). One should add that the machining on this centre is possible under one fixed position from five sides/directions at the same time.

Some more certificates of exemplary studies of rigidity of the component elements of the machine tools may be added to this analysis. The work [10] provides the study results of this feature in reference to the needle segments of the rolling guides of machine tools, the bedways made by 1NA-HYDREL Company. It was found that the rigidity of the bedways plays an important role in modeling and analysis of dynamics of the slide systems of machine tools, especially regarding the non-linearity.

The presented literature references show that the need to describe the mechanical phenomenon of deformation of the material systems is still of importance and valid. The necessity exists to describe this phenomenon more adequately and precisely. The resistance to deformations (rigidity) and the tendency to deformations (deformability) of these systems are their reverse features characterizing the considered phenomenon. The rigidity is, however, the feature which is more often introduced to evaluate the technological systems. It appears that also due to the exploitational reasons they should be more resistant to deformations. Thus the rigidity is to be the subject of considerations of this work.

Furthermore, at first the general indicative rigidity characteristics will be given as the basis to explain the title energetic nature of the rigidity. Its adequate presentation in the form of dependence of the rigidity force on the deflection results from the source differential description of the magnitude in the interstate spaces.

3. Adequate Description of the Considered Mechanical Phenomenon

At the source of cognitive way of all physical phenomena which occur with variable rate/intensity or velocity between the neighbouring energetic states there is a general differential equation, namely:

$$\mathrm{d}Z = \pm \frac{\partial Z}{\partial N} \mathrm{d}N \tag{2}$$

where: dZ—total differential of magnitudes forming dependent variables, dN—total differential of magnitudes forming independent variables, $\partial Z / \partial N$ —partial derivative of dependent magnitudes against independent ones. The symbols (±) are the algebraic operators with a determined function. The operator (+) fulfills a formal function, just confirming the physical significance of the described phenomenon, whereas the sign (–) provides a physical sense to a determined description. That general source differential equation has been presented in [11], referred to the real determined technological conditions with different initial conditions. These initial conditions determine a detail solution of this type of equation. They refer to the initial conditions of a phenomenon or process but not those connected with the beginning of measure of determined physical magnitudes. The measurement of these magnitudes is a necessary though not a sufficient condition. The sufficient condition is ascribed to their measurement at the very beginning of a phenomenon.

The record of the Equation (2), applied to the considered phenomenon, possesses the following configuration:

$$\mathrm{d}S = \pm \frac{\partial S}{\partial f}\mathrm{d}f \tag{3}$$

where the sign (+) concerns the progressively rising changes of the rigidity force, and (–) is considered in describing degressively rising changes of this magnitude. In the first case the rate of force changes rises with the increase of deflection, whereas the second one informs that the rate of changes will be decreasing respectively. One should add that the rate of changes of the considered magnitude is just the coefficient of stiffness defined by the formula (1).

4. General Rigidity Characteristics

The rigidity characteristics, being the dependence of the rigidity force on deflection (**Figure 2**), has a progressive exponential course in the direction of the system loading (**Figure 2(a)**); whereas the degressive course of this kind takes place in the direction of de-loading system (**Figure 2(b)**); as it results from the joining of these curves (**Figure 2(c)**) and they do not overlap forming a sort of the rigidity hysteresis loop.

These courses occur in a deformation-force space/zone (dotted area) limited by the deformation and force potential field. To say more strictly, it concerns the stable deformation potential field $(SPF)_f^0$ and unstable potential field of this type $(APF)_f^1$. They are the fields limiting the mentioned space in the direction of deflection.

In the force direction it is more complex in character and requires some extended explanations. Two energetic bands appear here (the spaces filled with horizontal lines). One of them concerns the proper rigidity energy (**Figure 2(a)**) and is limited by the bottom stable force potential field $(BSPF)_S^0$ and the upper stable force potential field $(TSPF)_S^0$, whereas the second one refers to the gained rigidity energy (**Figure 2(b)**) and is limited by the upper/top unstable force potential field $(TAPF)_S^1$ and the bottom stable force potential field $(BSPF)_S^0$.

The introduced notions of the rigidity energy are in full agreement with the definition of energy; the definition is not respected until now with the evidence being the actions of aliasing the energy with work. It is not necessary to develop many references in the field to support or confirm the existing notion gap. It is enough to mention some recent books covering this energetic quest [12-16].

The proper rigidity energy of the system is its readiness to perform the rigidity work whereas the gained rigidity energy of this system, in turn, is the ability to perform the rigidity work over it. At this stage, while defining both these energies, one may state that the proper energy is lesser than the gained energy. One should admit that they are intellectual magnitudes (see **Figure 1**), and not the physical ones. This is why these energies may be evaluated qualitatively only, indicating that: the first one (proper) is lesser than the second one (gained), or the second one being bigger than the first one.

A need for possessing measures of both these intellecttual magnitudes exists. These measures will be the physiccal magnitudes. The measure of the proper rigidity energy is the proper rigidity potential V_S^0, being the product of the initial proper rigidity force S_0 and the distance between the deformation potential fields, that is $(\Delta f)_{0-1}$. Thus

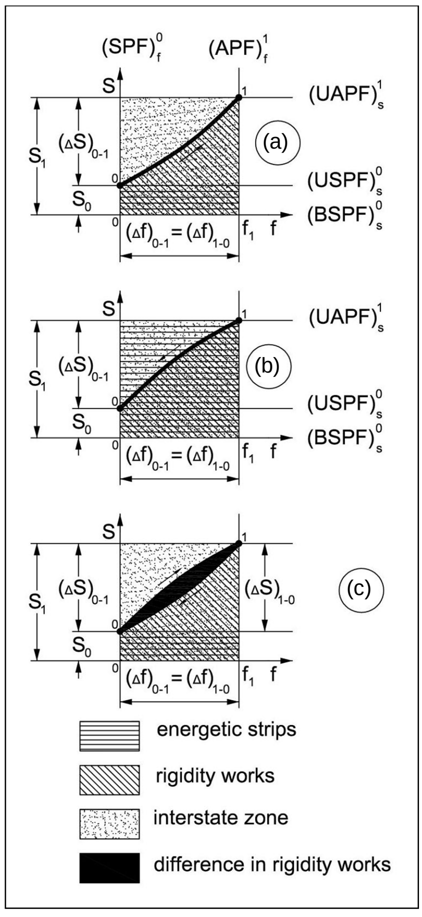

Figure 2. Indicative courses of the rigidity curves related to the directions of: loading (a), and de-loading (b) of the system; as well as to the both directions (c).

$$V_S^0 = S_0 \cdot (\Delta f)_{0-1} = S_0 \cdot f_1 \quad (4)$$

That corresponds with the field of horizontally dashed area (see **Figure 2(a)**), *i.e.* the proper energetic band, or the band of the proper energy. One should add that the magnitude S_0 is at the same time the intensity of stable deformation potential field $(SPF)_f^0$. Thus the potential is the product of the intensity of the mentioned potential field and the distance between the deformation potential field, with the distance being the measure of deformation resistance of the deformation-force space. (Here, the analogy to the Ohm's law may be noticed.)

And now, getting to the gained rigidity energy: its measure is the gained rigidity potential V_S^1, being the product of the gained rigidity force S_1 and the distance between the deformation potential fields, that is $(\Delta f)_{1-0}$. Therefore

$$V_S^1 = S_1 \cdot (\Delta f)_{1-0} = S_1 \cdot f_1 \quad (5)$$

It may be noticed, the band of the gained energy is considerably bigger than the band of the proper energy and it covers the entire deformation-force zone. The notion of the rigidity tension U_S is introduced now, being the potential difference of V_S^1 and V_S^0. Thus, after taking into account the formulae (4) and (5), one obtains

$$U_S = V_S^1 - V_S^0 = (S_1 - S_0) f_1 = \Delta S \cdot f_1 \quad (6)$$

It results evidently from these considerations that the energy measure (the more to say the energy, being the intellectual magnitude) is not the same as the work, although both these magnitudes have the same measure unit. The contents of these both magnitudes are generous and much differentiated.

The oblique dashed fields (see **Figure 2**) present the areas referred to the rigidity works. In the direction of the system loading (**Figure 2(a)**) this kind of work is bigger than the rigidity measure of the proper energy. However, in the direction of de-loading of the system (**Figure 2(b)**) there is a reverse situation, because the gained energy measure is greater than the work. The difference of the rigidity works (**Figure 2(c)**), corresponding with the blackened area, denotes the rigidity hysteresis, as resulting mainly from the fact that in the direction of the system loading a determined external stimulus acts, whereas in the reverse direction, the direction of return of the system into the stable state, this stimulus (due to sufficiently big the gained energy) does not work. In other words, at first the system collected/cumulated the energy, and then it was giving it out.

5. Creation of the Rigidity Characteristics

At first the rigidity characteristics will be described, corresponding with the direction of the system loading. That is an exponential, progressively rising curve which illustrates the dependence of the rigidity force S on the deflection f (**Figure 3**).

This curve is the envelope of right-angled triangles of which the deformation leg is constant and equals to the so called deformation constant Θ, with the force leg varying respectively. The initial point of the curve is on the intersection of the stable potential fields of: the stable deformation potential field $(SPF)_6^0$ and the upper/top stable force potential field $(TSPF)_S^6$. The final point of this curve is situated on the intersection of the unstable potential fields of: the first unstable deformation potential field $(APF)_f^1$ and the first upper/top unstable force potential field $(TAPF)_S^1$. The mentioned potential fields are the limits of the deformation-force space of which the dimension in the deformation direction is $(\Delta f)_{0-1}$, and in the force direction is $(\Delta S)_{0-1}$.

The force leg of this moving/shifting right-angled triangle is, as it was mentioned, respectively variable that results from the changing position of tangent to the curve of rigidity force. For ΔS the length of this leg equals $\Delta S + (\Delta S)_{0-1}$.

Now one may come up to the integration of Equation (3), denoting limits of the integrals from the total differentials. It means that

$$\int_{S_0+\Delta S}^{S_0+2\Delta S+(\Delta S)_{0-1}} \mathrm{d}S = \frac{\partial S}{\partial f} \int_f^{f+\Theta} \mathrm{d}f \quad (7)$$

and

$$\Delta S + (\Delta S)_{0-1} = \frac{\mathrm{d}S}{\mathrm{d}f}\Theta \quad (8)$$

or

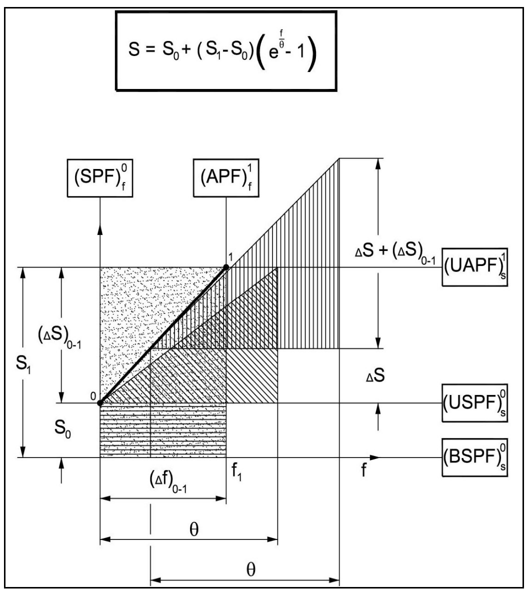

Figure 3. Illustration of scheme of creation of the rigidity characteristics in the direction of the system loading.

$$\frac{\mathrm{d}S}{\Delta S+(\Delta S)_{0-1}}=\frac{1}{\Theta}\mathrm{d}f \tag{9}$$

One may notice the partial derivative was substituted by the quotient of the total differentials. One could do that as the total differentials have been clearly determined by introducing the limits of their integrals. It is worthy admitting that the sign (+), the positive algebraic operator, has been regarded because the changes in the rigidity force are rising progressively.

Furthermore, by integrating both sides of the Equation (9), one obtains the result

$$\ln\left[\Delta S+(\Delta S)_{0-1}\right]=\frac{1}{\Theta}f+C^{*} \tag{10}$$

that is

$$\Delta S+(\Delta S)_{0-1}=e^{\frac{f}{\Theta}+C^{*}}=e^{C^{*}}\cdot e^{\frac{f}{\Theta}}=Ce^{\frac{f}{\Theta}} \tag{11}$$

After regarding that for $f=0$ the magnitude $\Delta S=0$, one obtains

$$C=(\Delta S)_{0-1} \tag{12}$$

and after substituting (12) to (11)

$$\Delta S=(\Delta S)_{0-1}\left(e^{\frac{f}{\Theta}}-1\right) \tag{13}$$

that is (see **Figure 3**)

$$S=S_0+\Delta S=S_0+(\Delta S)_{0-1}\left(e^{\frac{f}{\Theta}}-1\right) \tag{14}$$

or

$$S=S_0+(S_1-S_0)\left(e^{\frac{f}{\Theta}}-1\right) \tag{15}$$

One may determine now the second coordinate of the point 1, that is the deflection corresponding with the end of the phenomenon of the system deformation. That result is obtained by introducing the force $S=S_1$ and deflection $f=f_1$ to the Equation (15). Therefore

$$f_1=\Theta\ln 2 \tag{16}$$

The characteristics of the rigidity system, corresponding with the direction of its de-loading (**Figure 4**), is degressively rising in the nature. This curve is the envelope of right-angled triangles of which the deformation leg is constant and equals to the so called the deformation constant Θ (introduced above), and the force leg varies respectively.

The deformation constant Θ is on the nominal potential field (NPF), being the asymptote to which the second, apparent part of the curve (dashed line) is tending. Along this field the mentioned triangle with its leg is moving.

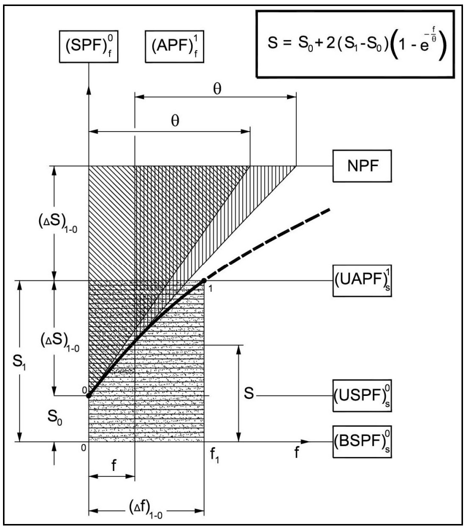

Figure 4. Illustration of scheme of creation of the rigidity characteristics description in the direction of the system de-loading.

Now one may start integrating the Equation (3), marking the limits of integrals from the total differentials and regarding with this the negative sign of the algebraic operator. Therefore

$$\int_{S}^{S_0+2(\Delta S)_{1-0}}\mathrm{d}S=-\frac{\partial S}{\partial f}\int_{f}^{f+\Theta}\mathrm{d}f \tag{16}$$

and further

$$S_0+2(\Delta S)_{1-0}-S=-\frac{\mathrm{d}S}{\mathrm{d}f}(f+\Theta-f) \tag{17}$$

which is

$$S_0+2(\Delta S)_{1-0}-S=-\frac{\mathrm{d}S}{\mathrm{d}f}\Theta \tag{18}$$

or

$$\frac{\mathrm{d}S}{S_0+2(\Delta S)_{1-0}-S}=-\frac{1}{\Theta}f \tag{19}$$

By integrating both sides of the Equation (19) one obtains the following solution

$$\ln\left[S_0+2(\Delta S)_{1-0}-S\right]=-\frac{1}{\Theta}f+C^{*} \tag{20}$$

that is

$$S_0+2(\Delta S)_{1-0}-S=e^{C^{*}-\frac{f}{\Theta}}=e^{C^{*}}\cdot e^{-\frac{f}{\Theta}}=Ce^{-\frac{f}{\Theta}} \tag{21}$$

After taking into account the initial conditions, meaning that for $f=0$, $S=S_0$, one obtains

$$C=2(\Delta S)_{1-0} \tag{22}$$

and after substituting (22) to (21)

$$S = S_0 + 2(\Delta S)_{1-0}\left(1-e^{-\frac{f}{\Theta}}\right) \tag{23}$$

Regarding further that $(\Delta S)_{1-0} = S_1 - S_0$, the Equation (23) may be recorded as follows

$$S = S_0 + 2(S_1 - S_0)\left(1-e^{-\frac{f}{\Theta}}\right) \tag{24}$$

Thus two basic functional characteristics have been introduced, corresponding with the directions of: the loading (15), and de-loading (24) of the system. Now further characteristics, being the formulae on the coefficient of stiffness k and the rigidity work/labour L_S, may be determined.

6. Further Characteristics of the Rigidity System

The consecutive functional characteristics are formed by the dependences of the coefficient of rigidity k on the deflection f, being the first derivatives (derivatives of the first order) of the rigidity force S against the latter parameter that has been reflected by the definition formula (1) given above. Therefore that kind of characteristics, referred to the direction of loading, takes the form

$$k = \frac{S_1 - S_0}{\Theta} e^{\frac{f}{\Theta}} = k_0 e^{\frac{f}{\Theta}} \tag{25}$$

and in reference to the direction of de-loading of the system

$$k = \frac{2(S_1 - S_0)}{\Theta} e^{-\frac{f}{\Theta}} = 2k_0 e^{-\frac{f}{\Theta}} = k_1 e^{-\frac{f}{\Theta}} \tag{26}$$

The illustration of both of these dependences (**Figure 5**) indicates that for a determined deflection f^* that coefficient takes uniform, the same value k^*.

That deflection may be determined by comparing the formulae (25) and (26), respectively, that is

$$k_0 e^{\frac{f^*}{\Theta}} = k_1 e^{-\frac{f^*}{\Theta}} \tag{27}$$

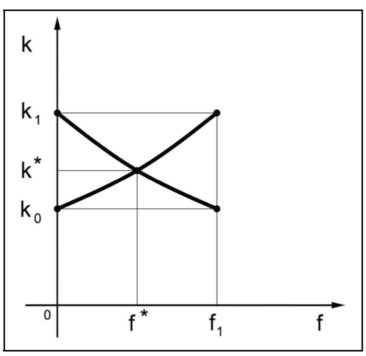

Figure 5. Courses of dependences of the rigidity coefficients k_0, k_1 on the deflection f.

The solution of that equation with one unknown f^* takes the following form

$$f^* = \frac{1}{2}\Theta \ln \frac{k_1}{k_0} \tag{28}$$

By substituting to the equation, for instance (25), one obtains

$$\begin{aligned} k^* &= k_0 e^{\frac{\Theta \ln \frac{k_1}{k_0}}{2\Theta}} = k_0 e^{\frac{1}{2}\ln\frac{k_1}{k_0}} = k_0 e^{\ln\left(\frac{k_1}{k_0}\right)^{\frac{1}{2}}} = k_0 \left(\frac{k_1}{k_0}\right)^{\frac{1}{2}} \\ &= k_0 \sqrt{\frac{k_1}{k_0}} = \sqrt{\frac{(k_0)^2 k_1}{k_0}} = \sqrt{k_0 k_1} \end{aligned} \tag{29}$$

Thus the coefficient of rigidity k^* is the square root of the product of coefficients k_0 and k_1, that is the geometrical mean of these coefficients. Furthermore, by regarding $k_1 = 2k_0$, one obtains

$$k^* = \sqrt{2(k_0)^2} = \sqrt{2}k_0 \tag{30}$$

Now let us consider the following functional characteristics, being the formulae on the rigidity work

$$L_S = \int_0^{f_1} S(f)\,\mathrm{d}f \tag{31}$$

Therefore the rigidity work in the direction of the system loading is expressed by the formula

$$\begin{aligned} L_S &= \int_0^{f_1} \left[S_0 + (S_1 - S_0)\left(e^{\frac{f}{\Theta}} - 1\right)\right] \mathrm{d}f \\ &= f_1(2S_0 - S_1) + \Theta(S_1 - S_0)e^{\frac{f_1}{\Theta}} \end{aligned} \tag{32}$$

and in the direction of the system de-loading

$$\begin{aligned} L_S &= \int_0^{f_1} \left[S_0 + 2(S_1 - S_0)\left(1 - e^{-\frac{f}{\Theta}}\right)\right] \mathrm{d}f \\ &= f_1(2S_1 - S_0) + 2\Theta(S_1 - S_0)e^{-\frac{f_1}{\Theta}} \end{aligned} \tag{33}$$

7. Example of Using the Presented Theory

The presented theory has been used to the adequate description of the rigidity characteristics of the tailstock (loose headstock) and the fixed headstock of the workpiece with the assemblies being the design links of the system of thread grinder of type MM-582. In **Table 1**, there are the measurement results of the rigidity force S and the deflections of these links of the grinder, *i.e.* y_w, z_w, y_k, z_k; they have been excerpted from the work [17]. The measurements of deflections, related to the particular rigidity forces, have been performed in the planes perpendicular to the workpiece axes, in the directions: vertical z, and horizontal y; thus the above markings the

Table 1. Results of measurements of rigidities and deflections of the loose headstock and fixed headstock of thread grinder of type MM-582 [17].

R	•	×	o	+
	y_w	z_w	y_k	z_k
N	10^{-6} m			
20	0.8	0.5	1.0	0.8
30	1.5	1.0	2.0	1.2
40	2.0	1.2	2.5	1.6
50	2.4	1.6	3.2	2.0
60	2.8	1.8	4.0	2.5
80	3.8	2.5	5.4	3.0
100	4.5	3.0	6.2	3.8

deflections of: y_w—the deflection of the fixed headstock in horizontal direction, z_w—the deflection of the fixed headstock in vertical direction, y_k—the deflection of the loose headstock in horizontal direction, z_k—the deflection of the loose headstock in vertical direction. One should add that all these results are to determine the characteristics of links of the system in the direction of its loading.

The graphical illustration of the set of experimental points (**Figure 6**) has been substituted by the straight lines coming out of one, zero point of the coordination system: the rigidity force—deflection. One may notice here very simplified data handling of measurements of all these magnitudes. Position of the lines did not result from a statistical handling of the material, but was determined approximately. That way (approximate) was treated also the approximation and the extrapolation of the phenomenon of deformation of both these links of the grinder.

It appears the set of experimental points indicates distinctly that the courses of deflections are exponential in their character and rising degressively. In the coordination system: deflection—rigidity force, the system which should be considered, the courses of the rigidity force will be exponential and rising progressively.

In the next part of the work the latter system was adopted as in this system the rigidity coefficients may be determined, being the derivative of the rigidity force against the deflection, and defined analytically by the formula (1). The first system (see **Figure 6**) allows to determine the reversal matter to the rigidity force k, which is the coefficient of flexibility c, namely

$$c = \frac{1}{k} = \frac{\mathrm{d}f}{\mathrm{d}S} \tag{34}$$

Taking advantage of the presented theory, first of all the values of coordinates y_w^*, z_w^*, y_k^*, z_k^*, should be determined. These coordinates determine the position of straight line, tangent to this searched, adequate course of the rigidity force. These coordinates result from the formula (16), then

$$y_w^* = \frac{y_w}{\ln 2} \tag{35}$$

$$z_w^* = \frac{z_w}{\ln 2} \tag{36}$$

$$y_k^* = \frac{y_k}{\ln 2} \tag{37}$$

$$z_k^* = \frac{z_k}{\ln 2} \tag{38}$$

The results of calculations of these coordinates (**Table 2**) indicate that they are bigger as to their values than the experimental coordinates. The values of coordinates y_w^*, z_w^*, y_k^*, z_k^*, should be now handled statistically, by approximating them in accordance to the following dependences:

$$S = S_0 + k_0 y_w^* \tag{39}$$

$$S = S_0 + k_0 z_w^* \tag{40}$$

$$S = S_0 + k_0 y_k^* \tag{41}$$

$$S = S_0 + k_0 z_k^* \tag{42}$$

For generally recorded that kind of the linear dependence

$$y = a + bx \tag{43}$$

the coefficients a and b are determined in accordance to the following formulae:

$$b = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} (x_i)^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \tag{44}$$

$$a = \frac{1}{n}\left(\sum_{i=1}^{n} y_i - b\sum_{i=1}^{n} x_i\right) \tag{45}$$

which are the result of using the rule of the least sum square of deviations of the experimental values from

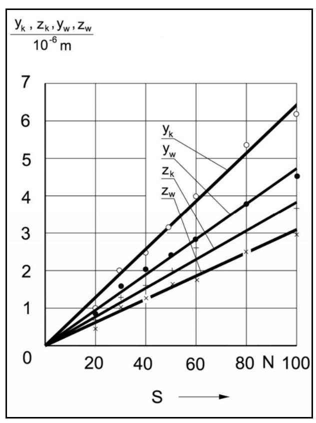

Figure 6. Graphical illustration of the set of experimental points [17].

Table 2. Results of measurements and calculations of magnitudes characterizing the design links of the investigated grinder.

R	•		×		o		+	
	y_w	y_w^*	z_w	z_w^*	y_k	y_k^*	z_k	z_k^*
N	10^{-6} m							
20	0.8	1.2	0.5	0.7	1.0	1.4	0.8	1.2
30	1.5	2.2	1.0	1.4	2.0	2.9	1.2	1.7
40	2.0	2.9	1.2	1.7	2.5	3.6	1.6	2.3
50	2.4	3.5	1.6	2.3	3.2	4.6	2.0	2.9
60	2.8	4.0	1.8	2.6	4.0	5.8	2.5	3.6
80	3.8	5.5	2.5	3.6	5.4	7.8	3.0	4.3
100	4.5	6.5	3.0	4.3	6.2	8.9	3.8	5.5

theoretical ones, resulting from the position of the linear regression function. That rule has been described in detail in references, such as [18-21].

Comparison of the magnitudes needed for calculation of the coefficients of linear regression, the initial rigidity force S_0 and the initial coefficient of stiffness k_0 in the formulae (39), (40), (41), and (42), has been presented in **Table 3**.

For example, for the rigidity force S and deflection y_w^*, that is the tangent shifted on the direction of tangent to the searched curve, tangent in the initial point, with the coefficients S_0 and k_0 described by the formulae

$$k_0 = \frac{n\sum_{i=1}^{n}\left(y_w^*\right)_i S_i - \sum_{i=1}^{n}\left(y_w^*\right)_i \sum_{i=1}^{n} S_i}{n\sum_{i=1}^{n}\left(y_w^*\right)_i^2 - \left[\sum_{i=1}^{n}\left(y_w^*\right)_i\right]^2} \tag{46}$$

$$S_0 = \frac{1}{n}\left[\sum_{i=1}^{n} S_i - k_0 \sum_{i=1}^{n}\left(y_w^*\right)_i\right] \tag{47}$$

Therefore

$$k_0 = \frac{7\times1711 - 25.8\times380}{7\times115.5-\left(25.8\right)^2} = 15.2 \quad \text{N}\cdot\mu\text{m}^{-1} \tag{48}$$

$$S_0 = \frac{1}{7}\left(380 - 15.2\times25.8\right) = -1.8 \quad \text{N} \tag{49}$$

Thus

$$S = -1.8 + 15.2 y_w^* \tag{50}$$

One may notice, the initial rigidity force S_0 has a negative value. The regression line (**Figure 7**) crosses the axis of abscissae, that is the axis of deflections y_w, y_w^*, in a determined point.

The coordinate of this point y_w^0 may be calculated by substituting the rigidity force $S=0$ to (50). Therefore

$$\left(y_w^*\right)_{S=0} = y_w^0 = \frac{1.8}{15.2} = 0.12 \quad \mu\text{m} \tag{51}$$

Table 3. Comparison of magnitudes needed to calculate the values of the coefficients of linear regression.

(a)

i	R	•			×		
		X_1	X_2	X_3	X_4	X_5	X_6
1	20	1.2	1.4	24	0.7	0.5	14
2	30	2.2	4.8	66	1.4	2.0	42
3	40	2.9	8.4	116	1.7	2.9	68
4	50	3.5	12.3	175	2.3	5.3	115
5	60	4.0	16	240	2.6	6.8	156
6	80	5.5	30.3	440	3.6	13.0	288
7	100	6.5	42.3	650	4.3	18.5	430
Σ	380	25.8	115.5	1711	16.6	49	1113

(b)

i	R	o			+		
		X_1	X_2	X_3	X_4	X_5	X_6
1	20	1.4	2.0	28	1.2	1.4	24
2	30	2.9	8.4	87	1.7	2.9	51
3	40	3.6	13.0	144	2.3	5.3	92
4	50	4.6	21.2	230	2.9	8.4	145
5	60	5.8	33.6	348	3.6	13.0	216
6	80	7.8	60.8	624	4.3	18.5	344
7	100	8.9	79.2	890	5.5	30.3	550
Σ	380	35	218.2	2351	21.5	79.8	1422

Remark: $x_1 = \left(y_w^*\right)_i$, $x_2 = \left(y_w^*\right)_i^2$, $x_3 = \left(y_w^*\right)_i \cdot R_i$, $x_4 = \left(z_w^*\right)_i$, $x_5 = \left(z_w^*\right)_i^2$, $x_6 = \left(z_w^*\right)_i \cdot R_i$.

The following conclusion arises that the considered node of the grinder, which is the fixed headstock, has a determined clearance in horizontal direction. Thus for the described curve to possess the physical significance, not having negative values, it should be presented in a reverse coordinates system, that is the deflection—rigidity force, and then elaborate/handle the experimental results in accordance with other theory, adopted to the description of the degressive exponential curve. The essence of this theory has been presented above in this work.

The authors have focused the attention on the adequate description of the dependence $S=\varphi\left(f\right)$ for the sets of experimental points of which the statistical handling reveals the positive initial rigidity force, reflecting the initial tension of the design link. This is why the course of further actions is neglected and it is connected with handling the results revealing lack of a physical meaning/sense $\left(S_0 \langle 0\right)$ in the system the rigidity force—deflecttion. That mentioned phenomenon is inordinate anyway and at least this is why its description has none a practical justification.

Coming to the description of the effects of loading of the link (the fixed headstock) in the vertical direction (**Figure 8**), one may calculate the coefficients S_0 and k_0

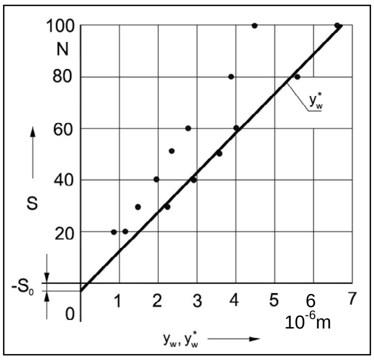

Figure 7. Dependence of the rigidity force *S* on the deflections y_w, y_w^*.

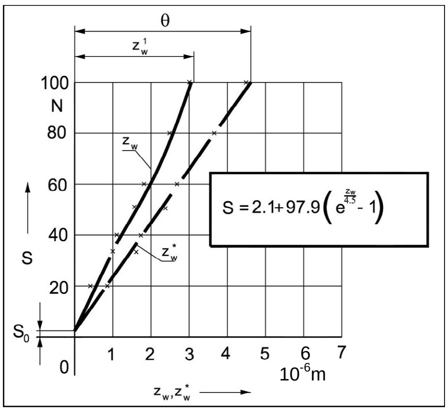

Figure 8. Dependence of the rigidity force *S* on the deflections z_w and z_w^*.

occurring in the formula (40).

These coefficients are $S_0 = 2.1$ N, and $k_0 = 22$ N·μm^{-1}, respectively. Therefore

$$S = 2.1 + 22 z_w^* \quad (52)$$

The deformation constant Θ corresponds with the coordinate z_w^* for $S_1 = 100$ N, so

$$\Theta = \left(z_w^*\right)_{S=100} = \frac{100 - 2.1}{22} = 4.5 \ \ \mu\text{m} \quad (53)$$

Now substituting the values of parameters S_0, S_1, and Θ to the formula (15), one obtains the following configuration of the dependence $S = \phi\left(z_w\right)$

$$S = 2.1 + 97.9\left(e^{\frac{z_w}{4.5}} - 1\right) \quad (54)$$

where, as may be noticed, the symbol z_w, relating to the deflection of spindle in the vertical direction has been introduced in the place of general denotation of the deflection *f*.

The coordinate z_w^1 (see **Figure 8**) may be calculated from the formula (16), substituting with this symbol the deflection f_1. Therefore

$$z_w^1 = 4.5 \times \ln 2 = 45 \times 0.693 = 3.1 \ \ \mu\text{m} \quad (55)$$

The measure of the proper rigidity energy, that is the rigidity potential V_S^0, is calculated according to the formula (4), with z_w^1 substituted instead of f_1. Therefore

$$V_S^0 = 2.1 \times \left(3.1 \times 10^{-6}\right) = 6.5 \times 10^{-6} \ \text{N} \cdot \text{m} = 6.5 \times 10^{-6} \ \text{J} \quad (56)$$

The measure of the gained rigidity energy, that is the gained rigidity potential V_S^1, is described by the formula (5). Thus, regarding that $f_1 = z_w^1$, one obtains

$$\begin{aligned} V_S^1 &= 100 \times \left(3.1 \times 10^{-6}\right) = 310 \times 10^{-6} \ \text{N} \cdot \text{m} \\ &= 310 \times 10^{-6} \ \text{J} \end{aligned} \quad (57)$$

The coefficient of stiffness *k* is described by the formula (25). Therefore

$$k = 22 e^{\frac{z_w}{4.5}} \ \ \text{N} \cdot \mu\text{m}^{-1} \quad (58)$$

It appears that after substituting $f_1 = \Theta \ln 2$ to it, that is $z_w = z_w^1 = 4.5 \times 0.693 = 3.1$ the following relation occurs

$$k_1 = 22 \times 2 = 44 \ \ \text{N} \cdot \mu\text{m}^{-1} \quad (59)$$

then

$$k_1 = 2 k_0 \quad (60)$$

One may also calculate the rigidity work, in accordance to the formula (32), then

$$\begin{aligned} L_S &= 3.1 \times \left(2 \times 2.1 - 100\right) + 4.5 \times \left(100 - 2.1\right) \cdot e^{\frac{3.1}{4.5}} \\ &= 580 \ \text{N} \cdot \mu\text{m} = 580 \cdot 10^{-6} \ \text{N} \cdot \mu\text{m} = 580 \times 10^{-6} \ \text{J} \end{aligned} \quad (61)$$

By handling the results of measurements (**Figure 9**) of the rigidity forces *S* and deflections of the loose headstock in the horizontal direction, that is y_k, one obtains $S_0 = 2.3$ N, $k_0 = 10.4$ N·μm^{-1}, then

$$S = 2.3 + 10.4 y_k^* \quad (62)$$

The deformation constant $\Theta = 9.4$ μm. Therefore

$$S = 2.3 + 97.7\left(e^{\frac{y_k}{3.4}} - 1\right) \quad (63)$$

The values of further parameters of the characteristics of this design node are equal: $y_k^1 = 6.5$ μm, $V_S^0 = 15 \cdot 10^{-6}$ J, $V_S^1 = 650 \cdot 10^{-6}$ J coefficients of stiffness *k* are described by the formula

$$k = 10.4 e^{\frac{y_k}{9.4}} \ \ \text{N} \cdot \mu\text{m}^{-1} \quad (64)$$

and the rigidity work equals $L_S = 1207.5 \times 10^{-6}$ J.

The results of measurement of the rigidity force *S* and the deflection z_k, as well as the results of calculation of the magnitude z_k^* (**Figure 10**), handled in accordance to the above presented scheme, have led to the following

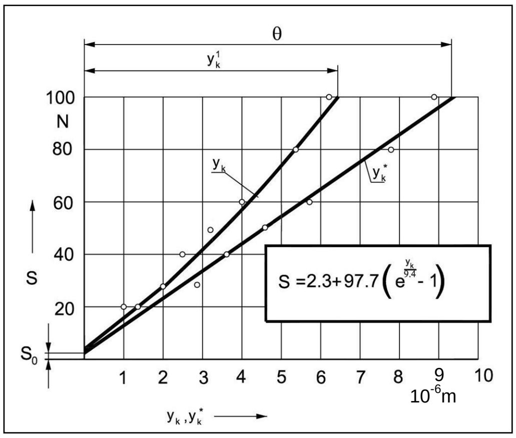

Figure 9. Dependence of the rigidity force *S* on the deflections y_k and y_k^*.

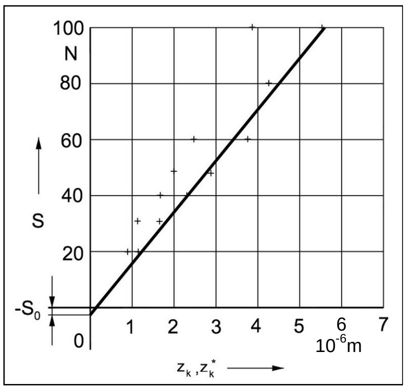

Figure 10. Dependence of the rigidity force *S* on the deflections z_k and z_k^*.

results: $S_0 = -2.5$ N, $k_0 = 18.5$ N·μm^{-1}. The regression line for the set of points of coordinates z_k^*, S_0 takes the following form:

$$S = -2.5 + 18.5 z_k^* \quad (65)$$

Here also, as it appears, the initial rigidity force has a negative value, that means the occurrence of a clearance in this design link, that is in the loose headstock. Further handling of the results, similar like before (see **Figure 7**), has none a practical justification. That statistic handling of the experimental material/matter made it possible to reveal the clearance occurring in this subjective link. Any further actions should be directed rather to stiffen this link.

8. Summary

In the frames of the summary it is worthy indicating that simplification of a description of the real systems leads to the hiding of some valuable information.

The rigidity problems, referred to the essential nodes of the grinder, have been analysed thoroughly. That approach made it possible to disclose the adequate rigidity characteristics of nodes both on the side of fixed headstock as well as of the loose headstock. It was found these characteristics possess the configuration of exponential curves. The phenomenon of deformation of these nodes has been considered in detail; that made it possible to obtain some valuable utilitarian information. In the light of the cognitive approach it appears any aspiration to linearise the real non-linear characteristics is not justified. To say more, the tendency to substitute them by a straight line, coming through zero point, is incorrect.

Moreover, in this work it was stated that the rigidity has the energetic nature. An adequate notion of the rigidity energy has been introduced, by determining its measures for the initial and final energetic states of the studied design nodes. They are the notions of the proper rigidity potential and the gained potential of this kind. It has been clarified the essential and fundamental difference between the energy and work, here in reference to the rigidity energy and work. *De facto* it has a broader meaning and refers to each kind of energy. The identifying the energy with work is erroneous and should not have place on the ground of science. Also the energy measures, as stated, also cannot be identified with the work.

The method of the rigidity investigation presented in this work has values both cognitive and utilitarian. It may be used to study the effect of the rigidity on the course and results of machining, as well as the exploitational properties of tools cutting determined materials on determined machine tools. This method may be used also for the assessment of technical condition of the machine tools, concerning their rigidity. Also the research on the recipient's request and/or the reliability compliance tests could be related to the rigidity aspect.

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