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Dissipative Discrete System with Nearest-Neighbor Interaction for the Nonlinear Electrical Lattice

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ABSTRACT

A generalized dissipative discrete complex Ginzburg-Landau equation that governs the wave propagation in dissipative discrete nonlinear electrical transmission line with negative nonlinear resistance is derived. This equation presents arbitrarily nearest-neighbor nonlinearities. We analyze the properties of such model both in connection to their modulational stability, as well as in regard to the generation of intrinsic localized modes. We present a generalized discrete Lange-Newell criterion. Numerical simulations are performed and we show that discrete breathers are generated through modulational instability.

Keywords: Generalized Dissipative Discrete Complex Ginzburg-Landau Equation; Discrete Lange Newell-Criterion; Pulse Trains; Solitary Patterns

1. Introduction

During these last decades the behavior of nonlinear discrete systems has received considerable attention in many areas of physics. The nonlinear electrical transmission lines (NLTLs) are good examples of such systems. They are very convenient tools for studying quantitatively the fascinating properties of wave propagation in nonlinear dispersive media. Afshari and Hajimiri [1] have introduced and analyzed pulse narrowing and edge sharpening passive NLTL, using accumulation mode metal-oxide semiconductor varactors and the gradual scaling lines, showing simultaneous edge sharpening for both rising and falling edges in silicon. The experimental results show considerable improvement in the rise and fall times of the pulses. These lines can have applications in ultra-wideband systems, broadband signal generations, and high-speed serial communications [2-4]. The problem of a wide pulse degenerating into multiple pulses rather than a single pulse is solved by using a gradually scaled NLTL. The ability of solitons to propagate with small dispersion can be used as an effective means to transmit data, modulated as short pulses over long distances; one example of this is the ultra wideband impulse radio that has recently gained popularity [5]. More recently, the experimental, analytical and numerical study of a left-handed nonlinear electrical lattice have been performed by English *et al.* [6]. They found that the above system clearly supports backward wave propagation of plane waves, but also envelope solitons of the bright and dark type. From the viewpoint of NLTL experiments, pulse propagation [7] and envelope soliton formation [8] were recently studied (see also the review of Ref. [9]), while pertinent theoretical works, based on the use of a nonlinear Schrödinger (NLS) equation, allowed the description of bright [10] or dark [11] envelope solitons observed in the experiments. One of the best mechanism to generate solitonlike excitation is through the modulational instability (MI).

Thus far, discrete spatial solitons (nonlinear eigenstates) have been successfully demonstrated in NLTL [12-15]. Like every nonlinear system, a NLTL can exhibit an instability that leads to a self-induced modulation of input plane wave with the subsequence generation of localized pulses [16-19]. This phenomenon is known as a Benjamin-Feir modulational instability [20] and it is responsible of many physically interesting effects such as

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the formation of envelope solitons. In homogeneous nonlinear systems, MI may be considered as the leading mechanism for energy localization as well as the formation of traveling intrinsic localized modes [21,22]. The corresponding mathematical model is the NLS equation or the complex Ginzburg-Landau (CGL) equation with periodically varying dispersion and nonlinear coefficients. The CGL describes the long wavelength modulations (envelopes or amplitudes) of both travelling waves and homogeneous oscillations. This equation has a wide range of applications.

Dissipative phenomena in nonlinear media with complex parameters are attracting nowadays a great deal of attention. In the present work we shall address these problems with a twofold aim. From one side, we derive the discrete CGL (DCGL) equation with nearest-neighbor nonlinearities which governs the propagation of wave in the DNLTL. From the other side, we show that the derived equation can be used to explore interesting dynamical behaviors as generate nonlinear localized modes in the DNLTL. To this regard we investigate the MI as a mechanism of the generation of bright matter-waves in the DNLTL. Dissipation is one of the main forces acting against the formation of nonlinear coherent structures in extended systems. When dissipation is present in systems without additional gain mechanisms, typically all excitations decay into the regime of linear waves.

The work is organized as follows. In Section 2, the analytical model based on the DNLTL is presented and we derived the DCGL equation with nearest-neighbor nonlinearities. Then, we present a qualitative analysis concerning MI and we propose the generalized Lange-Newell criterion. In Section 3, since the discrete breathers solutions with small amplitudes are very close to plane waves, we focus on the generation of nonlinear excitations induced by MI. Finally, conclusions are drawn in Section 4.

2. The DCGL Equation with Nearest-Neighbor Nonlinearities in the Nltl

Many schematic electrical lattices have already been consider in the litterature. Recently, a one dimensional bi-inductance lattices which act as band-pass filters has been considered [23]. Authors of [12,13] consider an original capacitor which has the purpose to block dc current from flowing through the resistor and inductor to the ground in the case where the driver contained a dc voltage offset. This description is only correct in the linear and weakly nonlinear regime, but does not hold in the fully nonlinear regime. The main point (captured both by the experimental and numerical traces) is that the blocking capacitor does not alter the linear and weakly nonlinear properties of the lattice, but that it certainly does affect the strongly nonlinear regime in the dynamics. But, the attenuation of waves is due to dissipative effects of the medium in which they travel. We are interested in the study of the propagation of nonlinear localized modes in a nonlinear transmission line by doping the line with negative nonlinear resistance. Analytical results and numerical simulation have shown that the attenuated wave recovers its amplitude on a short distance of the doped domain. So, the original purpose of this negative nonlinear resistance is the particular functional form of the capacitor which should amplify the waves after attenuation. It has been shown that the wave conserves its pulse form when crossing the amplification domain [24].

So, here we consider a nonlinear network of N cells as illustrated in **Figure 1**. Each cell contains a linear inductor L_s in the series and shunt branch and a linear inductor L_p in parallel with a nonlinear capacitor C_p in the shunt branch. This capacitor is the well-known bias-dependent responsible for nonlinearity of the system. Its capacitance is assumed to be expanded as a power series of the local signal voltage V_n, which appears across the nonlinear capacitor of the nth cell

$$C_p\left(V_n+V_b\right)=C_{0s}\frac{1}{2}\left(1-2\alpha V_n+3\beta V_n^2\right), \qquad (1)$$

where, C_{0p} is a constant corresponding to the capacitance of the nonlinear diode at the dc bias-voltage V_b. The nonlinear parameters α and β are assumed to be positive constants. In Equation (1), we keep nonlinear coefficient up to the second order for the following reasons. First, the polynomial approximation of the C-V curve and corresponding fit are justified if the voltage amplitude is small enough. Second, in this voltage range, to reduce the equation of motion to an ordinary differential equation, it is sufficient to take into account these two terms, only, to balance the first-order dispersion term. Having in mind that the compactification of solitary wave results from the nonlinear dispersion of the system, we have to choose the dispersion element properly in order to assure

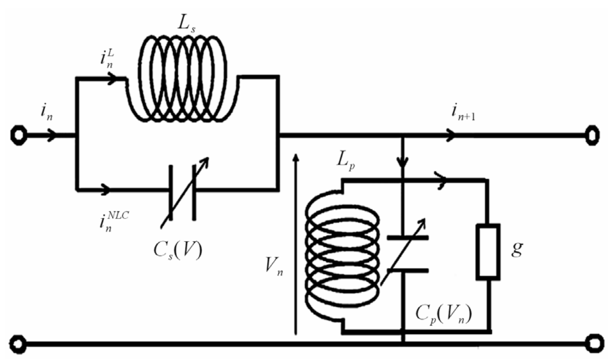


Figure 1. Schematic representation of the NLTL.

that the resulting network will satisfy this requirement. It has been pointed out by Comte and Marquié [25] that the introduction of the nonlinear resistor in the series branch of the nonlinear transmission line modelling the propagation of fluxons in reaction-diffusion chain can create a nonlinear dispersion and then the compactification of kink solitons. Here, the introduction of the voltage dependence in the capacitor in the series branch can make circuits that perform a variety of tasks and probably the compactification of envelope solitary waves. Thus, the capacitance-voltage relationship in the series branch is Taylor expanded to second order and reads:

$$C_s(V) = C_{0s}\frac{1}{2}\left(1 - 2\eta V + 3\lambda V^2\right), \qquad (2)$$

where V is the voltage across the nonlinear capacitor with the zero-voltage value C_{0s}. So, the operating point of this capacitor corresponds to the zero-voltage value. In order to take in to account the dissipation of the network, the conductance g is connected in parallel with C_p and L_p, respectively. The conductance g accounts for the dissipation of the inductor L_p in addition to the loss of the nonlinear capacitor C. The corresponding conductance g is given by [24]

$$g(V_n) = \alpha_1' - \beta_1 V_n. \qquad (3)$$

The linear dispersion relation of the line is a typical band pass filter:

$$\omega^2 = \frac{\omega_0^2 + 4u_0^2\sin^2(k/2)}{1 + 4C_{0r}\sin^2(k/2)} \qquad (4)$$

where, $C_{0r} = C_{0s}/C_{0p}$, $\omega_0^2 = 1/L_pC_{0p}$ and $u_0^2 = 1/L_sC_{0p}$, are the dimensionless capacitance and characteristic frequencies of the system. The corresponding linear spectrum has a gap $f_0 = \omega_0/2\pi$ and it is limited by the cut-off frequency $f_{\max} = \omega_{\max}/2\pi$, with $\omega_{\max} = \left(\omega_0^2 + 4u_0^2\right)/\left(1 + 4C_{0r}\right)$ due to lattice effects. The linear dispersion curve of the network is plotted in **Figure 2** as a function of the wave vector k (rad/cell). From Equation (4), one can derive the following group velocity:

$$V_g(k) = \frac{d\omega}{dk} = \frac{\left(u_0^2 - C_{0r}\omega_0^2\right)\sin(k)}{\left(1 + 4C_{0r}u_0^2\sin^2(k/2)\right)^2\omega} \qquad (5)$$

This group velocity is represented in **Figure 3**. We restrict our study to slow temporal variations in the envelope. As we shall see, it provides a deep and useful insight into the full dissipative dynamics of the nonlinear electrical line and leads to pattern formation.

Applying Kirchhoff's laws to this system leads to the following set of differential equations governing wave propagation in the network

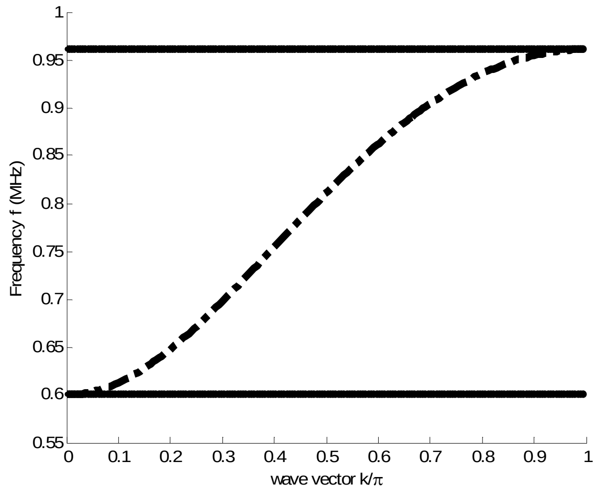


Figure 2. Linear dispersion curve of the lattice: frequency $f = \omega/2\pi$ (MHz) as a function of wave vector k (rad/cell). The characteristic frequencies of the network and reduced capacitance are $\omega_0 = 3.77 \times 10^6$ rad/s, $u_0 = 2.58 \times 10^6$ rad/s and $C_{0r} = 0.03$, respectively.

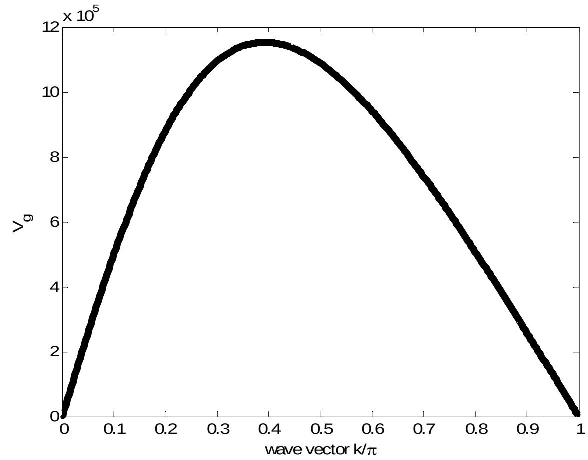


Figure 3. Group velocity.

$$\begin{aligned}&\frac{d^2V_n}{dt^2} + u_0^2\left(2V_n - V_{n-1} - V_{n+1}\right) + \omega_0^2 V_n \\ &+\alpha_1\frac{dV_n}{dt} - \beta_1\frac{dV_n^2}{dt} - \alpha\frac{d^2V_n^2}{dt^2} + \beta\frac{d^2V_n^3}{dt^2} \\ &= C_{0r}\frac{d^2}{dt^2}\Big\{\left(V_{n-1} - 2V_n + V_{n+1}\right) - \eta\Big[\left(V_{n-1} - V_n\right)^2 \\ &\quad -\left(V_n - V_{n+1}\right)^2\Big] + \lambda\Big[\left(V_{n-1} - V_n\right)^3 - \left(V_n - V_{n+1}\right)^3\Big]\Big\}\end{aligned} \qquad (6)$$

with, $\alpha_1 = \alpha_1'/C_{0p}$ and $\beta_1 = \beta_1'/C_{0p}$. For this purpose, restricting moreover our study to weak amplitude and slow temporal variations of the wave envelope, we look for a solution of Equation (6) in the form

$$V_n = \varepsilon\psi_n(T)e^{-i\omega t} + \varepsilon\psi_n^*(T)e^{i\omega t}, \qquad (7)$$

where ε is small parameter ($\varepsilon \ll 1$) and $T=\varepsilon^2 t$, ψ_n is unknown complex envelope function, ψ_n^* stands for complex conjugate and ω denoting frequency. Inserting this relation in Equation (6), we collect solutions of order $\left[\varepsilon,\ e^{-i\omega t}\right]$ which give a relation between the wave function at different site of the lattice. Thereafter, one can write the relation at order $\left[\varepsilon^2,\ e^{-0i\omega t}\right]$, using the dispersion relation [Equation (4)] and equations resulting from the above different order, one obtains the following equation:

$$\begin{aligned}&i\varphi_{n\tau}+\left(P_r+iP_i\right)\left(\phi_{n-1}-2\phi_n+\phi_{n+1}\right)-i\gamma_r\phi_n-\gamma_i\phi_n\\&-\left(Q_r+iQ_i\right)\left|\phi_n\right|^2\phi_n=D\Big[\left(\left|\phi_{n-1}\right|^2\varphi_{n-1}+\left|\phi_{n+1}\right|^2\phi_{n+1}\right)\\&+2\left(\varphi_{n-1}+\varphi_{n+1}\right)\left|\varphi_n\right|^2-2\left(\left|\phi_{n-1}\right|^2+\left|\phi_{n+1}\right|^2\right)\phi_n\Big]\end{aligned} \tag{8}$$

where the complex coefficients of Equation (8) are given by

$$P_r=1$$

$$P_i=\frac{2\alpha_1\omega\left(u_0^2-C_{0r}\omega^2\right)}{\left(\omega_0^2-\omega^2+2u_0^2-2C_{0r}\omega^2\right)\left(2u_0^2-C_{0r}\left(\omega^2+\omega_0^2\right)\right)}$$

$$Q_i=\frac{3\lambda\alpha\beta_1\omega C_{0r}\left(u_0^2-C_{0r}\omega^2\right)^2}{8C_{0r}\eta^2\left(2u_0^2-C_{0r}\left(\omega^2+\omega_0^2\right)\right)\left(\omega_0^2-\omega^2\right)}$$

$$\begin{aligned}Q_r=&\frac{2}{\left(2u_0^2-C_{0r}\left(\omega^2+\omega_0^2\right)\right)}\\&\left[9\beta\omega^2+\frac{3\lambda\omega^2C_{0r}\left(\omega_0^2-\omega^2+2u_0^2-2C_{0r}\omega^2\right)\left(\omega_0^2-\omega^2\right)}{\left(u_0^2-C_{0r}\omega^2\right)^2}\right.\\&\left.+\frac{\left(4\alpha^2\omega^2-\beta_1^2\right)\left(u_0^2-C_{0r}\omega^2\right)^2}{16C_{0r}^2\eta^2\omega^2\left(\omega_0^2-\omega^2\right)^2}\right]\end{aligned}$$

$$\gamma_r=-2P_i;\ \gamma_i=2P_r;\ D=\frac{6\lambda C_{0r}\omega^2}{2u_0^2-C_{0r}\left(\omega^2-\omega_0^2\right)}$$

Equation (8) is the DCGL equation with nearest-neighbor nonlinearities. Note that the DCGL equation has been phenomenologically proposed to describe frustrated states in a linear array of vortices [26,27]. Also, it reproduces reasonably well characteristics of the turbulent regime below the percolation threshold. Percolation has been found to be a useful concept for the description of turbulence, and the results suggest that non adiabatic effects, such as discrete nature of the system, play a role in the system. From a physical point of view, it is of interest to study the effects of including nearest-neighbor nonlinearities terms than cubic in the equation on discrete solitons. These terms appear in different physical contexts such as Bose gases with hard core interactions in the Tonks-Girardeau regime [28] and low dimensional Bose-Einstein condensates in which quintic nonlinearities in the NLS equation are used to model three-body interactions [29]. A self-focusing cubic-quintic NLS equation is also used in nonlinear optics as a model for photonic crystals [30].

In particular when the nearest-neighbor parameter D = 0, Equation (8) becomes the well-known DCGL equation [31], and for $P_i = Q_i = \gamma_r = D = 0$, Equation (8) is reduced to the usual (nonintegrable) discrete nonlinear Schrödinger equation [32,33].

Modulation instability is a generic nonlinear phenomenon governing nonlinear wave propagation in dispersive media. It refers to a weak space-time dependence (modulation) of the wave amplitude, due to intrinsic medium nonlinearity, however weak. Under the effect of external perturbations (e.g., noise), the wave amplitude (the envelope) may potentially grow, eventually leading to energy localization via the formation of localized structures (envelope solitons) [34].

To analyze MI, which is responsible for energy localization, we seek a solution of Equation (8) in the form of plane wave disturbed as follow

$$\phi_n=\left[\phi_0+B_n(\tau)\right]\exp\left[i\left(kn-\omega t\right)\right] \tag{9}$$

where ϕ_0 is the initial complex constant amplitude, k and ω are, respectively, the wave number and the angular frequency of the carrier wave. The quantity $B_n(\tau)$ is the perturbation assumed to be small in comparison with the amplitude of the carrier wave. It would be important to ask what happens to the plane waves when the amplitude increases sufficiently so that the nonlinearity occurs. In the linear approximation an equation for $B_n(\tau)$ yields the dispersion relation for the evolution of small perturbations,

$$\left[\Omega-\lambda_r-i\lambda_i\right]^2=\chi_r+i\chi_i, \tag{10}$$

where λ_r, λ_i, χ_r, and χ_i are defined in the Appendix. The frequency Ω can be written as

$$\Omega=\left[\lambda_r\pm\sqrt{\frac{\left(\chi_r+|\chi|\right)}{2}}\right]+i\left[\lambda_i\pm\sqrt{\frac{\left(-\chi_r+|\chi|\right)}{2}}\right] \tag{11}$$

Equation (11) has been established for the case $\chi_r<0$. We easily get the perturbation as follow

$$\begin{aligned}B_n(\tau)=&\left\{b_1\exp\left[i\left(Kn+\left(\lambda_r\pm\sqrt{\frac{\left(\chi_r+|\chi|\right)}{2}}\right)\tau\right)\right]\right.\\&\left.+b_2^*\exp\left[-i\left(Kn+\left(\lambda_r\pm\sqrt{\frac{\left(\chi_r+|\chi|\right)}{2}}\right)\tau\right)\right]\right\}\\&\times\exp\left\{-\left[\lambda_i\pm\sqrt{\frac{\left(-\chi_r+|\chi|\right)}{2}}\right]\tau\right\}\end{aligned} \tag{12}$$

where K, b_1 and b_2 are the wave number and the complex constants, respectively.

The amplitude B_n will be unbounded as

$\tau \to +\infty$ if and only if: $\lambda_i \pm \sqrt{\frac{(-\chi_r + |\chi|)}{2}} < 0$, in order to get this relation, it is necessary that $\lambda_i < 0$. Because, $-\sqrt{\frac{(-\chi_r + |\chi|)}{2}} < 0$, the relation, $\lambda_i - \sqrt{\frac{(-\chi_r + |\chi|)}{2}} < 0$, holds and from this inequality we can easily derive the following inequality,

$$|\phi_0|^2 \le |\phi_0|_{cr}^2 = \left|\frac{4P_i \sin^2(K/2)\cos(k)}{Q_i}\right| \quad (13)$$

Relation (13) represents the amplitude threshold $|\phi_0|_{cr}^2$, for the MI versus the wave number k of the carrier wave and the K of the perturbation for the dissipative coefficients: $\alpha_1 = 2.6710 \times 10^4\ \Omega^{-1}\cdot F^{-1}$ (see **Figure 4**).

Assume that the necessary condition

$\lambda_i - \sqrt{\frac{(-\chi_r + |\chi|)}{2}} < 0$, is satisfied, then we can write the inequality $2\lambda_i^2 < -\chi_r$, that is

$$\left(P_r - 3D|\phi_0|^2\right)\left[\left(Q_r + 4D\cos(K)\right) - 6D|\phi_0|^2 \cos(k)\right] \sin^2(K/2)\cos(k) > 0 \quad (14)$$

Relation (14) represents the MI criterion associated with the DCGL equation with higher-order nonlinearities.

This result is the generalized Discrete Lange and Newell criterion for Stokes waves.

The growth rate of the perturbation is given by Ω_i. This quantity has been plotted in **Figure 5**.

From this figure, one can see that our system can be really stable unstable (the two branches).

3. Generation of Intrinsic Localized Modes

Let us check the theoretical predictions concerning the existence of MI in the system. So, to further explore MI, we compute numerical simulations. In particular, our results are based on the theory of linear stability analysis. However, we know that the linear stability analysis is limited because it can only predict the onset of instability and does not tell us anything about the long-time dynamical behavior of the system when the instability grows. When the perturbation amplitude grows large enough compared to that of the initial wave, the numerical analysis must be adopted. To further confirm that the linear instability analysis given above can correctly describe the initial stage of instability, we have performed numerical simulations of Equation (1). A fourth-order Runge-Kutta algorithm has been used. A normalized integration time step $\Delta t = 5 \times 10^{-3}$ is used for numerical simulations. Similarly, the number of cells N is chosen to be equal to 1600 and we have used periodic boundary conditions so that we do not encounter the wave reflection at the end of the line. The parameters of the system are choosen in accordance with **Figure 2** as well as with Equation (14). At the input of the line, we apply a slowly modulated signal located at $n_0 = 100$,

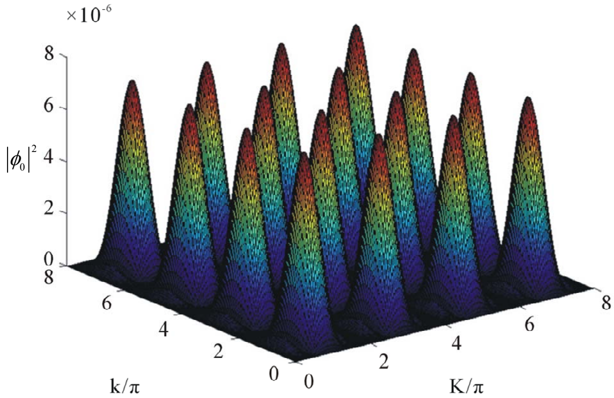


Figure 4. Treshold amplitude on the (K, k) plane. P_i = 0.0141, Q_i = −1.6681.

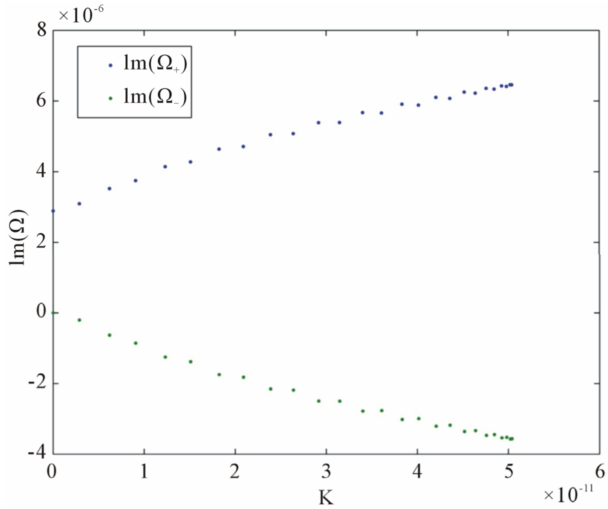
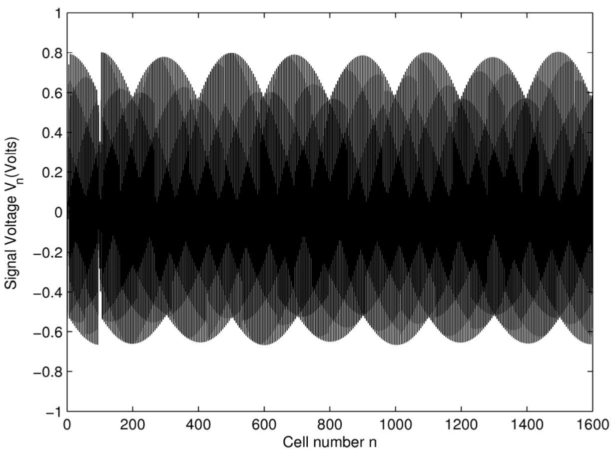


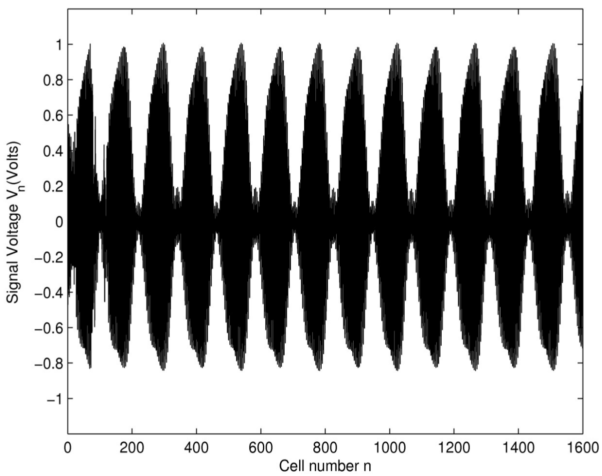
Figure 5. Imaginary part of Ω for $k = \pi$.

$$V(t) = V_0\left(1 + m_0 \cos(2\pi f_m t)\right)\cos(2\pi f_p t) \quad (15)$$

where, V_0 is the amplitude of the unperturbed plane wave, m_0 designates the modulation rate and f_m the frequency of modulation. MI has been analyzed for lattices with respect to discrete breathers. As a specific example, we use the following value $V_0 = 0.90\ V$, $f_p = 800$ kHz, $m_0 = 0.01$ and $f_m = 8$ kHz. Then, we launch solution (15) in the network. As time goes on, the modulation increases and the continuous wave breaks into a periodic pulse or envelope soliton train as shown in **Figure 6(a)** at time t = 1000. A soliton is a localized wave form that travels along the system with constant velocity and underformed shape. It is well known that in transmission media supporting solitons, any input pulse with a duration greater than soliton width tends to dissolve into a superposition of solitons. In this regard, a sinusoidal signal fed to the NLTL will progressively decompose into multiple solitons per cycle, and harmonics of the input frequency will be obtained at the output as viewed in **Figure 6(b)**. This figure has been obtained for the parameters f_m = 8 kHz, f_p = 750 kHz and k = 0.9π , one sees that as time goes on, the modulation increases and the initial continuous wave breaks into a periodic pulses soliton train at time t = 375. The amplitude of the wave generated by wave motion is modulated in the form of a train of small amplitude with a short wavelength. Each component of the train has the shape of a soliton like object.



(a)



(b)

Figure 6. Space evolution of the amplitude showing the dynamics of the pulse. The pulse undergoes periodic oscillations in the vicinity of the stable intersite configuration. V_0 = 0.90 V, m_0 = 0.01. (a) At time t = 1000 for f_p = 800 kHz, and f_m = 8 kHz; (b) At time t = 375 for f_m = 8 kHz and f_p = 750 kHz.

Figure 7 shows the development of nonlinear wave packets with a slowly varying envelope in space with regard to a given carrier wave with frequency f_p = 750 kHz, f_m = 16 kHz and the modulated wave number k = 0.9π at time t = 750. One sees the appearence of envelope solitons related to the existence of MI in the NLTL. One obtains an interesting phenomenon: the wave displays an oscillating and breathing wave behavior. In nonlinear physical systems with discrete symmetry, which is considered to be as fundamental as the concepts of solitons, dissipative structures, etc. in modern nonlinear science the concept of bushes of normal modes could be applied. The phenomenon observe in **Figure 7** can be also explained by the theory of "bushes" of nonlinear normal modes [35-37]. Since the symmetry-determined bushes are valid for any of monatomic chains and, in some sense, they can be applied to multiatomic chains as well, one can describe these phenomena as bushes. As an indivisible nonlinear object, the bush exists because of force interactions between the modes contained in it. Apparently, bushes of modes play an important role in many physical phenomena of current interest [35-37].

4. Conclusions

In this work, we have introduced the generalized discrete complex Ginzburg-Landau equation with nearest-neighbor nonlinearities in the nonlinear discrete transmission lattices. The appearence of MI has been investigated and the generalized discrete Lange-Newell proposed. The theoretical findings have been numerically tested through direct simulations and solitonic excitations of the pulse train have been generated. The theory of "bushes" of nonlinear normal modes has been also point out.

The MI is the first step in the generation of soliton like excitations in physical systems. Therefore the study of

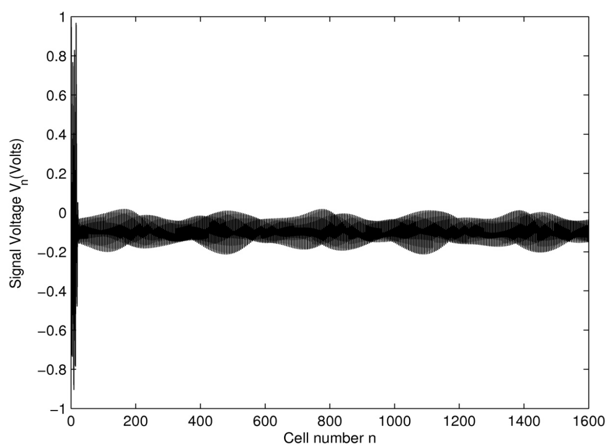


Figure 7. Desintegration of the initial periodic solution into a wave train at t = 750 for k = 0.9π, f_p = 750 kHz and f_m = 16 kHz.

the conditions in which this phenomenon takes place is of special importance. This result is very useful for either the investigation of nonlinear transmission lines or of there similar physical problems, such as nonlinearity, in the plasma, dusty plasma, Bose-Einstein condensates, etc.

Finally, it is important to mention that, in recent years, the development in NLTL has demonstrated its capacity to work as signal processing tools. To cite only very few examples, it has been demonstrated that the nonlinear uniform electrical line can be used for extremely wide band signal shaping applications [2] as well as a wave form equalizer in the compensation scheme for signal distortion caused by optical fiber polarization dispersion mode. Moreover, it is also possible to use NLTLs in the scheme for controlling the amplitude (amplification) and the delay of ultrashort pulses through the coupled propagation of the solitonic and dispersive parts, which is important in that it enables the characterization of high-speed electronic devices such as hetero-junction field effect transistor or resonant tunneling diodes, and raises the possibility of establishing future ultra-high signal processing technologies. Besides its practical interests, it is well known that NLTLs are convenient tools for the study of wave propagation in nonlinear dispersive media. In particular, they provide a useful way to check how the nonlinear excitation behaves inside the nonlinear medium and to model the strange properties of new systems.

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Appendix

$$\lambda_r = -2\left(P_r - 4D|\phi_0|^2\right)\sin(K)\sin(k)$$

$$\lambda_i = Q_i|\phi_0|^2 - 4P_i\sin^2(K/2)\cos(k)$$

$$\begin{aligned}\chi_r =\ & 16\left(P_r - 4D|\phi_0|^2\right)^2\sin^4(K/2)\cos^2(k) - 16D^2|\phi_0|^4\sin^4(K/2)\cos^2(k) \\ & - 4P_i^2\sin^2(K)\sin^2(k) + 4D^2|\phi_0|^2\sin^2(K)\sin^2(k) - Q_i^2|\phi_0|^4 - 8\left(P_r - 3D|\phi_0|^2\right)Q_r|\phi_0|^2 \\ & \times\sin^2(K/2)\cos(k) + 48\left(P_r - 3D|\phi_0|^2\right)\times D|\phi_0|^2\sin^2(K/2)\cos^2(k) - 32\left(P_r - 3D|\phi_0|^2\right) \\ & \times D|\phi_0|^2\sin^2(K/2)\cos(k) + 64\left(P_r - 3D|\phi_0|^2\right)\times D|\phi_0|^2\sin^4(K/2)\cos(k)\end{aligned}$$

$$\begin{aligned}\chi_i =\ & 8\left(P_r - 4D|\phi_0|^2\right)P_i\sin^2(K/2)\sin(2k)\sin(K) - 4P_iQ_r\sin(K)\sin(k) \\ & + 12P_iD|\phi_0|^2\sin(K)\sin(2k) - 8P_iD|\phi_0|^2\sin(2K)\sin(k)\end{aligned}$$