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Dynamics of a Discrete Predator-Prey System with Beddington-DeAngelis Function Response

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ABSTRACT

This paper discusses the dynamic behaviors of a discrete predator-prey system with Beddington-DeAngelis function response. We first show that under some suitable assumption, the system is permanent. Furthermore, by constructing a suitable Lyapunov function, a sufficient condition which guarantee the global attractivity of positive solutions of the system is established.

Keywords: Discrete; Beddington-DeAngelis Functional Response; Permanence; Global Attractivity

1. Introduction

Since the end of the 19th century, many biological models have been established to illustrate the evolutionary of species, among them, predator-prey models attracted more and more attention of biologists and mathematicians. There are many different kinds of predator-prey models in the literature. In 1975, Beddington [1] and DeAngelis [2] proposed the predator-prey system with the Beddington-DeAngelis functional response as follows

$$\begin{cases} x' = x\left(a - bx - \dfrac{cy}{m_1 + m_2 x + m_3 y}\right), \\ y' = y\left(-d + \dfrac{fx}{m_1 + m_2 x + m_3 y}\right). \end{cases} \tag{1.1}$$

Recently, Li and Takeuchi [3] proposed the following model with both Beddington-DeAngelis functional response and density dependent predator

$$\begin{cases} x' = x\left(a - bx - \dfrac{cy}{m_1 + m_2 x + m_3 y}\right), \\ y' = y\left(-d - ey + \dfrac{fx}{m_1 + m_2 x + m_3 y}\right), \end{cases} \tag{1.2}$$

and discussed the dynamic behaviors of the model.

On the other hand, when the size of the population is rarely small or the population has non-overlaping generation, the discrete time models are more appropriate than the continuous ones. Discrete time models can also provide efficient computational models of continuous models for numerical simulations.

In [4], Qin and Liu studied the dynamic behavior of the following discrete time competitive system

$$\begin{cases} x(n+1) \\ = x(n)\exp\left\{a(n) - b(n)x(n) - \dfrac{c(n)y(n)}{y(n)+1}\right\}, \\ y(n+1) \\ = y(n)\exp\left\{-d(n) - e(n)y(n) + \dfrac{f(n)x(n)}{1+x(n)}\right\}, \end{cases} \tag{1.3}$$

In [5], Wu and Li considered the following discrete time predator-prey system with hassell-varley type functional response

$$\begin{cases} x(n+1) \\ = x(n)\exp\left\{a(n) - b(n)x(n) - \dfrac{c(n)y(n)}{m(n)y^r(n) + x(n)}\right\}, \\ y(n+1) \\ = y(n)\exp\left\{-d(n) + \dfrac{f(n)x(n)}{m(n)y^r(n) + x(n)}\right\}, \end{cases} \tag{1.4}$$

some sufficient conditions for the permanence and global attractivity of system (1.4) are obtained. For more work on this direction, one could refer to [6-14].

Based on the above discussion, in this paper, we consider the discrete analogous of (1.2), one can easily derive the discrete analogue of system (1.2), which takes the form of

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$$\begin{cases} x(n+1)=x(n)\exp\left\{a(n)-b(n)x(n)-\dfrac{c(n)y(n)}{m_1(n)+m_2(n)x(n)+m_3(n)y(n)}\right\}, \\ y(n+1)=y(n)\exp\left\{-d(n)-e(n)y(n)+\dfrac{f(n)x(n)}{m_1(n)+m_2(n)x(n)+m_3(n)y(n)}\right\}. \end{cases} \tag{1.5}$$

In this paper, we always assume that $\{a(n)\}$, $\{b(n)\}$, $\{c(n)\}$, $\{d(n)\}$, $\{e(n)\}$, $\{f(n)\}$, $\{m_1(n)\}$, $\{m_2(n)\}$, $\{m_3(n)\}$ are all positive bounded sequences and

$$0<a^l\le a(n)\le a^u,\quad 0<b^l\le b(n)\le b^u,$$

$$0<c^l\le c(n)\le c^u,\quad 0<d^l\le d(n)\le d^u,$$

$$0<e^l\le e(n)\le e^u,\quad 0<f^l\le f(n)\le f^u,$$

$$0<m_i^l\le m_i(n)\le m_i^u,\quad i=1,2,3.$$

Here, for any bounded sequence $\{f(n)\}$,

$$f^u=\sup_{n\in N} f(n),\quad f^l=\inf_{n\in N} f(n).$$

From the view point of biology, we will focus our discussion on the positive solutions of system (1.4). So it is assumed that the initial conditions of (1.4) are of the form

$$x(0)>0,\quad y(0)>0. \tag{1.6}$$

It is easily to see that the solutions of (1.4) with the initial condition (1.5) are defined and remain positive for all $k\in N$.

2. Permanence

DEFINITION 2.1. System (1.5) is said to be permanent, if there are positive constants r_1, r_2, R_1, R_2 such that each positive solution $(x(n), y(n))$ of system (1.5) satisfies

$$r_1\le \liminf_{n\to\infty} x(n)\le \limsup_{n\to\infty} x(n)\le R_1,$$

$$r_2\le \liminf_{n\to\infty} y(n)\le \limsup_{n\to\infty} y(n)\le R_2.$$

LEMMA 2.1. [6] Assume that $\{x(n)\}$ satisfies $x(n)>0$ and

$$x(n+1)\le x(n)\exp\{a(n)-b(n)x(n)\}$$

for all $n\ge n_0$, where $\{a(n)\}$, $\{b(n)\}$ are positive sequences. Then

$$\limsup_{n\to\infty} x(n)\le \frac{\exp(a^u-1)}{b^l}.$$

LEMMA 2.2. [6] Assume that $\{x(n)\}$ satisfies

$$x(n+1)\ge x(n)\exp\{a(n)-b(n)x(n)\}, n\ge n_0$$

$$\limsup_{n\to\infty} x(n)\le D \quad\text{and}\quad x(n_0)>0,$$

where $\{a(n)\}$, $\{b(n)\}$ are positive sequences. Then

$$\liminf_{n\to\infty} x(n)\ge \frac{a^l\exp(a^l-b^uD)}{b^u}.$$

LEMMA 2.3. Assume that $\dfrac{f^l}{m_2^u}-d^u>0$ holds, then for any positive solution $(x(n), y(n))$ of system (1.4), one has

$$\limsup_{n\to\infty} x(n)\le G_1=\frac{\exp(a^u-1)}{b^l}$$

and

$$\limsup_{n\to\infty} y(n)\le G_2=\frac{1}{e^l}\exp\left(\frac{f^u}{m_2^l}-d^l-1\right).$$

Proof. Let $(x(n), y(n))$ be any positive solution of system (1.5), from the first equation of (1.5), it follows that

$$\begin{aligned} x(n+1)&=x(n)\exp\left\{a(n)-b(n)x(n)-\frac{c(n)y(n)}{m_1(n)+m_2(n)x(n)+m_3(n)y(n)}\right\} \\ &\le x(n)\exp\{(a(n)-b(n)x(n)\}. \end{aligned}$$

By Lemma 2.1, we obtain

$$\limsup_{n\to\infty} x(n)\le G_1=\left(\exp(a^u-1)\right)/b^l.$$

Similarly, from the second equation of (1.5), it follows that

$$\begin{aligned} y(n+1)&=y(n)\exp\left\{-d(n)-e(n)y(n)+\frac{f(n)x(n)}{m_1(n)+m_2(n)x(n)+m_3(n)y(n)}\right\} \\ &\le y(n)\exp\left\{\frac{f(n)}{m_2(n)}-d(n)-e(n)y(n)\right\}\le y(n)\exp\left\{\frac{f^u}{m_2^l}-d^l-e^l y(n)\right\}. \end{aligned}$$

Under the assumption $\frac{f^l}{m_2^u} - d^u > 0$, by Lemma 2.1, we obtain

$$\limsup_{n\to\infty} y(n) \le G_2 = \frac{1}{e^l}\exp\left(\frac{f^u}{m_2^l} - d^l - 1\right).$$

This completes the proof of Lemma 2.3.

LEMMA 2.4. Assume that $h_1 > 0,\ h_2 > 0$. Then for any positive solution $(x(n), y(n))$ of system (1.5), one has

$$\liminf_{n\to\infty} x(n) \ge g_1,\ \limsup_{n\to\infty} y(n) \ge g_2,$$

where

$$h_1 = a^l - c^u / m_3^l,$$

$$h_2 = -d^u + f^l g_1 \big/ \left(m_1^u + m_2^u G_1 + m_3^u G_2\right),$$

$$g_1 = \frac{h_1 \exp\left(h_1 - b^u G_1\right)}{b^l},\ g_1 = \frac{h_2 \exp\left(h_2 - e^u G_2\right)}{e^u}.$$

Proof. Let $(x(n), y(n))$ be any positive solution of system (1.5), from the first equation of (1.5), it follows that

$$\begin{aligned} x(n+1) &= x(n)\exp\left\{a(n) - b(n)x(n) - \frac{c(n)y(n)}{m_1(n) + m_2(n)x(n) + m_3(n)y(n)}\right\} \\ &\ge x(n)\exp\left\{a(n) - \frac{c(n)}{m_3(n)} - b(n)x(n)\right\} \ge x(n)\exp\left\{a^l - \frac{c^u}{m_3^l} - b^u x(n)\right\} \\ &= x(n)\exp\left\{h_1 - b^u x(n)\right\}. \end{aligned}$$

Under the assumption $h_1 > 0$, By Lemma 2.2 and Lemma 2.3, we obtain

$$\liminf_{n\to\infty} x(n) \ge g_1 = \frac{h_1 \exp\left(h_1 - b^u G_1\right)}{b^u}.$$

Similarly, from the second equation of (1.5) and Lemma 2.3, it follows that

$$\begin{aligned} y(n+1) &= y(n)\exp\left\{-d(n) - e(n)y(n) + \frac{f(n)x(n)}{m_1(n) + m_2(n)x(n) + m_3(n)y(n)}\right\} \\ &\ge y(n)\exp\left\{-d^u - e^u y(n) + \frac{f^l g_1}{\left(m_1^u + m_2^u G_1 + m_3^u G_2\right)}\right\} = y(n)\exp\left\{h_2 - e^u y(n)\right\}. \end{aligned}$$

By Lemma 2.2 and Lemma 2.3, we have

$$\liminf_{n\to\infty} y(n) \ge g_1 = \frac{h_2 \exp\left(h_2 - e^u G_2\right)}{e^u}.$$

From Lemma 2.3 and Lemma 2.4, we obtain the following theorem.

THEOREM 2.1. Assume that

$$-d^u + f^l / m_2^u > 0,\ \ a^l - c^u / m_3^l > 0 \tag{2.1}$$

$$\frac{f^l g_1}{m_1^u + m_2^u G_1 + m_3^u G_2} - d^u > 0 \tag{2.2}$$

hold, then system (1.5) is permanent.

3. Global Attractivity

This section devotes to study the global attractivity of the positive solution of system (1.5).

DEFINITION 3.1. A positive solution $\left(x^*(n), y^*(n)\right)$ of system (1.5) is said to be globally attractive if each other positive solution $(x(n), y(n))$ of (1.5) satisfies

$$\lim_{n\to\infty}\left|x(n) - x^*(n)\right| = 0,\ \lim_{n\to\infty}\left|y(n) - y^*(n)\right| = 0.$$

THEOREM 3.1. In addition to (2.1) and (2.2), assume further that there exist positive constants α, β and δ such that

$$\alpha \min\left\{b^l, \frac{2}{G_1} - b^u\right\} - \frac{\alpha c^u G_2^{2/3}\left(m_2^u\right)^{1/3}}{9\left(m_1^l m_3^l g_1\right)^{2/3} g_2^{1/3}} - \frac{\beta f^u \left(m_1^u\right)^{1/3}}{9\left(m_2^l m_3^l g_1 g_2\right)^{2/3}} - \frac{\beta f^u G_2^{2/3}\left(m_3^u\right)^{1/3}}{9\left(m_1^l m_2^l g_1\right)^{2/3} g_2^{1/3}} > \delta \tag{3.1}$$

and

$$\beta\min\left\{e^l,\frac{2}{G_1}-e^u\right\}-\frac{\beta f^u G_1^{2/3}\left(m_3^u\right)^{1/3}}{9\left(m_1^l m_2^l g_2\right)^{2/3} g_1^{1/3}}-\frac{\alpha c^u\left(m_1^u\right)^{1/3}}{9\left(m_2^l m_3^l g_1 g_2\right)^{2/3}}-\frac{\alpha c^u G_1^{2/3}\left(m_2^u\right)^{1/3}}{9\left(m_1^l g_2\right)^{2/3} g_1^{1/3}}>\delta. \tag{3.2}$$

Then the positive solution of system (1.5)is globally attractive.

Proof. From (3.1) and (3.2), there exists an enough small positive constant $\varepsilon<\min\{g_1/2, g_2/2\}$ such that

$$\begin{aligned}&\alpha\min\left\{b^l,\frac{2}{G_1+\varepsilon}-b^u\right\}-\frac{ac^u\left(G_2+\varepsilon\right)^{2/3}\left(m_2^u\right)^{1/3}}{9\left[m_1^l m_3^l\left(g_1-\varepsilon\right)\right]^{2/3}\left(g_2-\varepsilon\right)^{1/3}}-\frac{\beta f^u\left(m_1^u\right)^{1/3}}{9\left[m_2^l m_3^l\left(g_1-\varepsilon\right)\left(g_2-\varepsilon\right)\right]^{2/3}}\\&-\frac{\beta f^u\left(G_2+\varepsilon\right)^{2/3}\left(m_3^u\right)^{1/3}}{9\left[m_1^l m_2^l\left(g_1-\varepsilon\right)\right]^{2/3}\left(g_2-\varepsilon\right)^{1/3}}>\delta\end{aligned} \tag{3.3}$$

and

$$\begin{aligned}&\beta\min\left\{e^l,\frac{2}{G_2+\varepsilon}-e^u\right\}-\frac{\beta f^u\left(G_1+\varepsilon\right)^{2/3}\left(m_3^u\right)^{1/3}}{9\left[m_1^l m_2^l\left(g_2-\varepsilon\right)\right]^{2/3}\left(g_1-\varepsilon\right)^{1/3}}-\frac{\alpha c^u\left(m_1^u\right)^{1/3}}{9\left[m_2^l m_3^l\left(g_1-\varepsilon\right)\left(g_2-\varepsilon\right)\right]^{2/3}}\\&-\frac{\alpha c^u\left(G_1+\varepsilon\right)^{2/3}\left(m_2^u\right)^{1/3}}{9\left[m_1^l\left(g_2-\varepsilon\right)\right]^{2/3}\left(g_1-\varepsilon\right)^{1/3}}>\delta.\end{aligned} \tag{3.4}$$

For any positive solutions $\left(x_1(k),y_1(k)\right)$ and $\left(x_2(k),y_2(k)\right)$ of system (1.4), it follows from Lemma 2.3 and Lemma 2.4 that

$$\begin{aligned}&\liminf_{n\to\infty} x_i(k)\ge g_1\\&\liminf_{n\to\infty} y_i(k)\ge g_2\\&\limsup_{n\to\infty} x_i(k)\le G_1\\&\limsup_{n\to\infty} y_i(k)\le G_2\ (i=1,2).\end{aligned} \tag{3.5}$$

In view of (3.5), for above ε, there exists an integer $k_1>0$ such that, for all $k>k_1$,

$$\begin{aligned}&g_1-\varepsilon\le x_i(k)\le G_1+\varepsilon\\&g_2-\varepsilon\le y_i(k)\le G_2+\varepsilon\quad(i=1,2)\end{aligned} \tag{3.6}$$

Let

$$V_1(k)=\left|\ln x_1(k)-\ln x_2(k)\right|,$$

$$A=m_1(k)+m_2(k)x_2(k)+m_3(k)y_2(k),$$

$$B=m_1(k)+m_2(k)x_1(k)+m_3(k)y_1(k).$$

From the first equation of system (1.5), we have

$$\begin{aligned}\Delta V_1(k)&=V_1(k+1)-V_1(k)=\left|\ln x_1(k+1)-\ln x_2(k+1)\right|-\left|\ln x_1(k)-\ln x_2(k)\right|\\&\le\left|\ln x_1(k)-\ln x_2(k)-b(k)[x_1(k)-x_2(k)]\right|-\left|\ln x_1(k)-\ln x_2(k)\right|+c(k)\left|\frac{m_2(k)y_1(k)\left[x_1(k)-x_2(k)\right]}{AB}\right|\\&+c(k)\left|\frac{m_1(k)\left[y_1(k)-y_2(k)\right]}{AB}\right|+c(k)\left|\frac{m_2(k)x_1(k)\left[y_2(k)-y_1(k)\right]}{AB}\right|\end{aligned}$$

By the mean value theorem, we have

$$x_1(k)-x_2(k)=\exp\left[\ln x_1(k)\right]-\exp\left[\ln x_2(k)\right]=\xi_1(k)\left|\ln x_1(k)-\ln x_2(k)\right|, \tag{3.7}$$

where $\xi_1(k)$ lies between $x_1(k)$ and $x_1(k)$. It follows from (3.7) that

$$\begin{aligned}\Delta V_1(k)\ &\le-\left(\frac{1}{\xi_1(k)}-\left|\frac{1}{\xi_1(k)}-b(k)\right|\right)\left|x_1(k)-x_2(k)\right|+\frac{c(k)m_2^{1/3}(k)y_1^{2/3}(k)\left|x_1(k)-x_2(k)\right|}{9m_1^{2/3}(k)m_3^{2/3}(k)x_1^{1/3}(k)x_2^{1/3}(k)y_2^{1/3}(k)}\\&+\frac{c(k)m_1^{1/3}(k)\left|y_2(k)-y_1(k)\right|}{9m_2^{2/3}(k)m_3^{2/3}(k)x_1^{1/3}(k)x_2^{1/3}(k)y_1^{1/3}(k)y_2^{1/3}(k)}+\frac{c(k)m_2^{1/3}(k)x_1^{21/3}(k)\left|y_2(k)-y_1(k)\right|}{9m_1^{2/3}(k)x_2^{1/3}(k)y_1^{1/3}(k)y_2^{1/3}(k)},\end{aligned}$$

and so, for $k > k_1$

$$\Delta V_1(k) \le -\min\left\{b^l, \frac{2}{G_1+\varepsilon} - b^u\right\}\left|x_1(k) - x_2(k)\right| + \frac{c^u\left(m_2^u\right)^{1/3}(G_2+\varepsilon)^{2/3}\left|x_1(k)-x_2(k)\right|}{9\left[\left(m_1^l m_3^l\right)(g_1-\varepsilon)\right]^{2/3}(g_2-\varepsilon)^{1/3}} + \frac{c^u\left(m_1^u\right)^{1/3}\left|y_2(k)-y_1(k)\right|}{9\left[m_2^l m_3^l(g_1-\varepsilon)(g_2-\varepsilon)\right]^{2/3}} + \frac{c^u\left(m_2^u\right)^{1/3}(G_1+\varepsilon))^{2/3}\left|y_2(k)-y_1(k)\right|}{9\left[m_1^l(g_2-\varepsilon)\right]^{2/3}(g_1-\varepsilon)^{1/3}}. \tag{3.8}$$

Let

$$V_2(k) = \left|\ln y_1(k) - \ln y_2(k)\right|.$$

From the second equation of system (1.5), we have

$$\begin{aligned}\Delta V_2(k) &= V_2(k+1) - V_2(k) \\ &= \left|\ln y_1(k+1) - \ln y_2(k+1)\right| - \left|\ln y_1(k) - \ln y_2(k)\right| \\ &\le \left|\ln y_1(k) - \ln y_2(k) - e(k)\left[y_1(k) - y_2(k)\right]\right| \\ &- \left|\ln y_1(k) - \ln y_2(k)\right| \\ &+ f(k)\left|\frac{m_3(k)x_1(k)\left[y_2(k)-y_1(k)\right]}{AB}\right| \\ &+ f(k)\left|\frac{m_1(k)\left[x(k)-x_2(k)\right]}{AB}\right| \\ &+ f(k)\left|\frac{m_3(k)y_1(k)\left[x_1(k)-x_2(k)\right]}{AB}\right|.\end{aligned}$$

By the mean value theorem, we have

$$y_1(k) - y_2(k) = \exp\left(\ln y_1(k)\right) - \exp\left(\ln y_2(k)\right) = \xi_2(k)\left[\ln y_1(k) - \ln y_2(k)\right], \tag{3.9}$$

where $\xi_2(k)$ lies between $y_1(k)$ and $y_2(k)$. It follows from (3.9) that

$$\Delta V_2(k) \le -\left[\frac{1}{\xi_2(k)} - \left|\frac{1}{\xi_2(k)} - e(k)\right|\right]\left|y_1(k) - y_2(k)\right| + \frac{f(k)m_3^{1/3}(k)x_1^{2/3}(k)\left|y_2(k)-y_1(k)\right|}{9m_1^{2/3}(k)m_2^{2/3}(k)x_2^{1/3}(k)y_1^{1/3}(k)y_2^{1/3}(k)} + \frac{f(k)m_1^{1/3}(k)\left|x_1(k)-x_2(k)\right|}{9m_2^{2/3}(k)m_3^{2/3}(k)x_1^{1/3}(k)x_2^{1/3}(k)y_1^{1/3}(k)y_2^{1/3}(k)} + \frac{f(k)m_3^{1/3}(k)y_1^{2/3}(k)\left|x_1(k)-x_2(k)\right|}{9m_1^{2/3}(k)m_2^{2/3}(k)x_1^{1/3}(k)x_2^{1/3}(k)y_2^{1/3}(k)},$$

and so, for $k > k_1$

$$\Delta V_2(k) \le -\min\left\{e^l, \frac{2}{G_2+\varepsilon} - e^u\right\}\left|y_1(k) - y_2(k)\right| + \frac{f^u\left(m_3^u\right)^{1/3}(G_1+\varepsilon)^{2/3}\left|y_2(k)-y_1(k)\right|}{9\left[m_1^l m_2^l(g_2-\varepsilon)\right]^{2/3}(g_1-\varepsilon)^{1/3}} + \frac{f^u\left(m_1^u\right)^{1/3}\left|x_1(k)-x_2(k)\right|}{9\left[m_2^l m_3^l(g_1-\varepsilon)(g_2-\varepsilon)\right]^{2/3}} + \frac{f^u\left(m_3^u\right)^{1/3}(G_2+\varepsilon)^{2/3}\left|x_1(k)-x_2(k)\right|}{9\left[m_1^l m_2^l(g_1-\varepsilon)\right]^{2/3}(g_2-\varepsilon)^{1/3}}. \tag{3.10}$$

Now we define a Lyapunov function as follows:

$$V(k) = \alpha V_1(k) + \beta V_2(k).$$

Calculating the difference of $V(k)$ along the solution of system (1.5), for $k > k_1$, it follows from (3.8) and (3.10) that

$$\begin{aligned}\Delta V(k) = \alpha\Delta V_1(k) + \beta\Delta V_2(k) \le &-\left[\alpha\min\left\{b^l, \frac{2}{G_1+\varepsilon} - b^u\right\} - \frac{\alpha c^u(G_2+\varepsilon)^{2/3}\left(m_2^u\right)^{1/3}}{9\left[\left(m_1^l m_3^l\right)(g_1-\varepsilon)\right]^{2/3}(g_2-\varepsilon)^{1/3}}\right. \\ &\left.- \frac{\beta f^u\left(m_1^u\right)^{1/3}}{9\left[m_2^l m_3^l(g_1-\varepsilon)(g_2-\varepsilon)\right]^{2/3}} - \frac{\beta f^u(G_2+\varepsilon)^{2/3}\left(m_3^u\right)^{1/3}}{9\left[m_1^l m_2^l(g_1-\varepsilon)\right]^{2/3}(g_2-\varepsilon)^{1/3}}\right]\left|x_1(k) - x_2(k)\right| \\ &-\left[\beta\min\left\{e^l, \frac{2}{G_2+\varepsilon} - e^u\right\} - \frac{\beta f^u(G_1+\varepsilon)^{2/3}\left(m_3^u\right)^{1/3}}{9[m_1^l m_2^l(g_2-\varepsilon)]^{2/3}(g_1-\varepsilon)^{1/3}} - \frac{\alpha c^u\left(m_1^u\right)^{1/3}}{9[m_2^l m_3^l(g_1-\varepsilon)(g_2-\varepsilon)]^{2/3}} - \frac{\alpha c^u(G_1+\varepsilon)^{2/3}\left(m_2^u\right)^{1/3}}{9[m_1^l(g_2-\varepsilon)]^{2/3}(g_1-\varepsilon)^{1/3}}\right] \\ &\cdot\left|y_2(k) - y_1(k)\right|.\end{aligned}$$

It follows from (3.3) and (3.4) that

$$\Delta V(k) \le -\delta\left[\left|x_1(k)-x_2(k)\right|+\left|y_2(k)-y_1(k)\right|\right].$$

Summating both sides of the above inequalities from k_1 to k, we have

$$\sum_{i=k_1}^{k}\Delta V(i) \le -\delta\sum_{i=k_1}^{k}\left[\left|x_1(i)-x_2(i)\right|+\left|y_1(i)-y_2(i)\right|\right].$$

Which implies

$$\sum_{i=k_1}^{k}\left[\left|x_1(i)-x_2(i)\right|+\left|y_1(i)-y_2(i)\right|\right] \le \frac{V(k_1)}{\delta}.$$

Then

$$\sum_{i=k_1}^{\infty}\left[\left|x_1(i)-x_2(i)\right|+\left|y_1(i)-y_2(i)\right|\right] < +\infty.$$

Therefore,

$$\lim_{x\to\infty}\left[\left|x_1(i)-x_2(i)\right|+\left|y_1(i)-y_2(i)\right|\right] = 0.$$

That is

$$\lim_{k\to\infty}\left|x_1(k)-x_2(k)\right| = 0, \lim_{k\to\infty}\left|y_1(k)-y_2(k)\right| = 0.$$

This completes the proof of Theorem 3.1.

REFERENCES

[1] J. R. Beddington, "Mutual Interference between Parasites or Predators and Its Effect on Searching Efficiency," *Journal of Animal Ecology*, Vol. 44, No. 3, 1975, pp. 331-340. doi:10.2307/3866

[2] D. L. DeAngelis, R. A. Goldstein and R. V. O'Neil, "A Model for Trophic Interaction," *Ecology*, Vol. 56, No. 4, 1975, pp. 881-892. doi:10.2307/1936298

[3] H. Y. Li and Y. Takeuchi, "Dynamics of the Density Dependent Predator-Prey System with Beddington-DeAngelis Functional Response," *Journal of Mathematical Analysis and Application*, Vol. 374, No. 4, 2011, pp. 644-654. doi:10.1016/j.jmaa.2010.08.029

[4] W. J Qin, Z. J. Liu and Y. P. Chen, "Permanence and Global Stability of Positive Periodic Solutions of a Discrete Competitive System," *Discrete Dynamics in Nature and Society*, 2009, Article ID 830537.

[5] R. X. Wu and Lin Li, "Permanence and Global Attractivity of Discrete Predator-Prey System with Hassell-Varley Type Functional Response," *Discrete Dynamics in Nature and Society, Applications*, Vol. 299, No. 2, 2004, pp. 357-374.

[6] F. Chen, "Permanence and Global Stability of Nonautonomous Lotka-Volterra System with Predator Prey and Deviating Arguments," *Applied Mathematics and Computation*, Vol. 173, No. 2, 2006, pp. 1082-1100. doi:10.1016/j.amc.2005.04.035

[7] F. Chen, "Permanence and Global Attractivity of a Discrete Multispecies Lotka-Volterra Competition Predator-Prey Systems," *Applied Mathematics and Computation*, Vol. 182, No. 1, 2006, pp. 3-12. doi:10.1016/j.amc.2006.03.026

[8] F. Chen, "Permanence of a Discrete n-Species Food-Chain System with Time Delays," *Applied Mathematics and Computation*, Vol. 182, No. 1, 2007, pp. 719-726. doi:10.1016/j.amc.2006.07.079

[9] F. Chen, "Permanence for the Discrete Mutualism Model with Time Delays," *Mathematical and Computer Modelling*, Vol. 47, No. 3-4, 2008, pp. 431-435. doi:10.1016/j.mcm.2007.02.023

[10] Y.H. Fan, W.T. Li, "Permanence for a Delayed Discrete Ratio-Dependent Predator-Prey System with Holling Type Functional Response," *Journal of Mathematical Analysis*, 009, Article ID: 323065.

[11] L. Chen, J. Xu and Z. Li, "Permanence and Global Attractivity of a Delayed Discrete Predator-Prey System with General Holling-Type Functional Response and Feedback Controls," *Discrete Dynamics in Nature and Society*, 2008, Article ID 629620.

[12] J. Yang, "Dynamics Behaviors of a Discrete Ratio-Dependent Predator-Prey System with Holling Type III Functional Response and Feedback Controls," *Discrete Dynamics in Nature and Society*, Vol. 2008, Article ID: 186539.

[13] X. Li and W. Yang, "Permanence of a Discrete Predator-Prey Systems with Beddington-DeAngelis Functional Response and Feedback Controls," 2008, Article ID 149267.

[14] M. Fan and K. Wang, "Periodic Solutions of a Discrete Time Nonautonomous Ratio-Dependent Predator-Prey System," *Mathematical Computer Modelling*, Vol. 35, No. 9-10, 2002, pp. 951-961. doi:10.1016/S0895-7177(02)00062-6